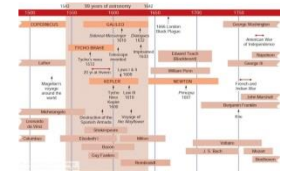
Kepler's Laws of Planetary Motion

Johannes Kepler would be first person to understand how the planets moved . . . it would remain for Isaac Newton to explain why.

Before we discuss Kepler's Laws, let's look at a timeline of what was going on at the end of the 16th century (*i.e. ~1600*):



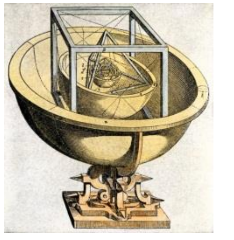
Tycho Brahe (1546 – 1601)

- Danish nobleman who was the last great pre-telescope observer *(yes, it has been claimed that he had a gold nose; some say silver, but after a 2010 exhumation, the report is brass!)*
- Was able to measure the position of planets on the sky to within 1 arc minute.
- Tycho had proposed a "hybrid theory" of the Solar System that had the Sun going around the Earth, but the planets orbiting the Sun. *(Couldn't give up the perfect circles and some geocentrism!)*

- After his death in 1601, his extremely accurate data (~ 20 years worth) would be used by his assistant, **Johannes Kepler**, to establish his **Laws of Planetary Motion**.

Kepler's Laws of Planetary Motion

Johannes Kepler (1571 – 1630)



- A German mathematician, was an early convert to the Copernican Heliocentric Theory.

- Before he worked with Tycho, Kepler had his own ideas regarding the geometry an structure of the Solar System
  . . . *where did this come from?*

- After Tycho's death in 1601, Kepler obtained Tycho's planetary data which would lead him to his first two laws in 1609 and his third law in 1619.

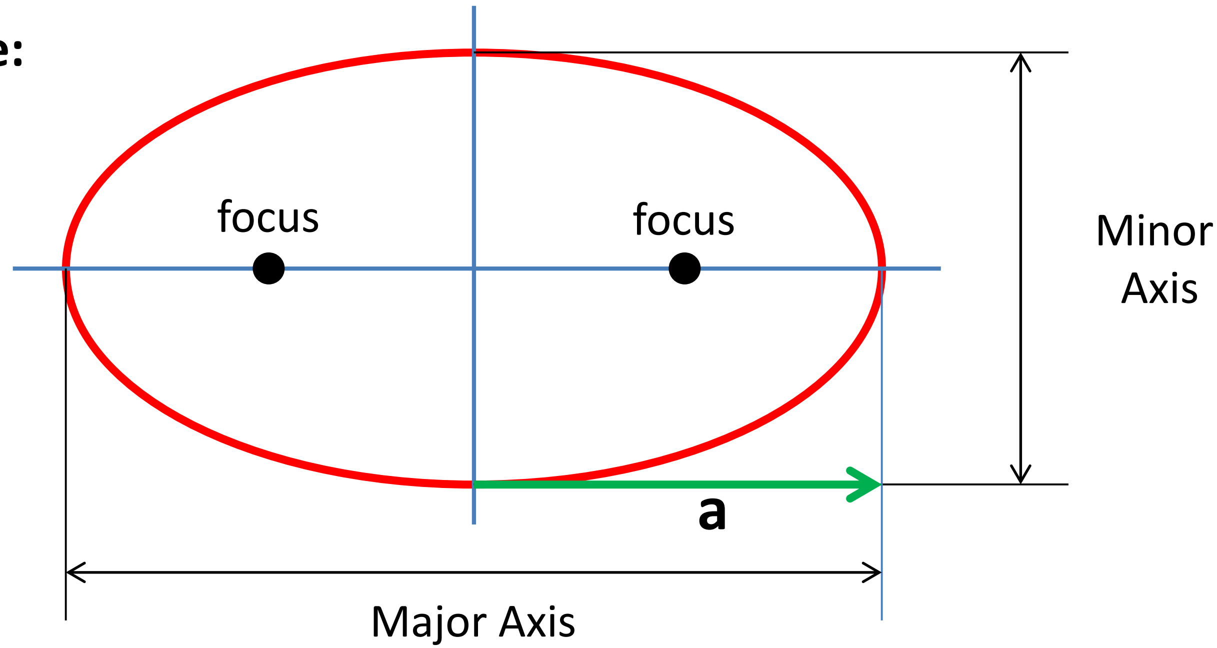
- Essentially, his method for the first two laws was simply "trial and error." He just kept changing the shapes and speeds of the planets in their orbits until he found something that matched Tycho's observations.

Kepler's First Law

Planetary orbits are Ellipses with the Sun at one focus.

(no more perfect circles!)

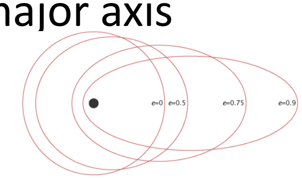
Geometry of an Ellipse:



Size of an ellipse is given by its **Semimajor Axis**, **a = Half the Major Axis**

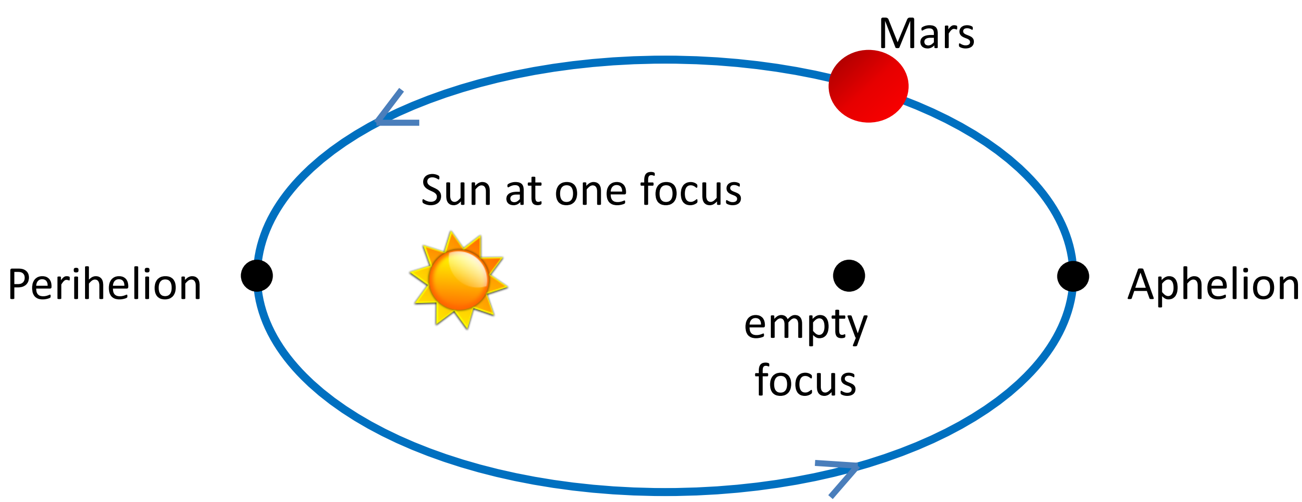
Shape of an ellipse is given by the **Eccentricity, e** = $\frac{\text{distance between foci}}{\text{major axis}}$

Note: $0 \leq e \leq 1.0$ Where e = 0 is a circle



Kepler’s First Law

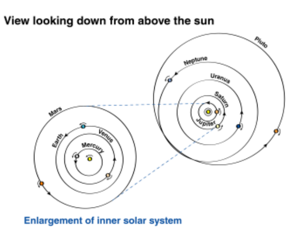
So, for Mars, Kepler’s 1st Law says the orbit is like this:



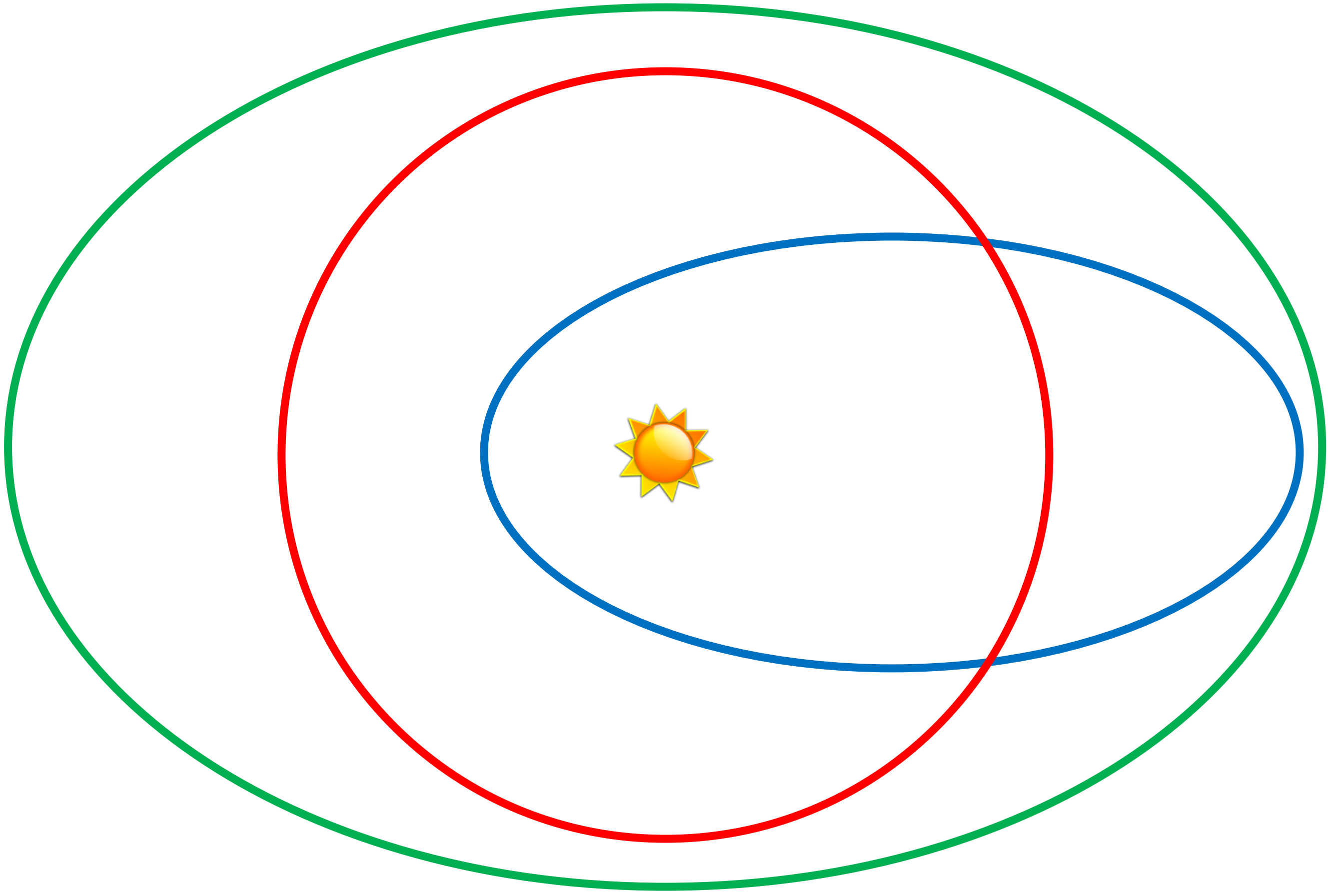
Note: this is a highly exaggerated eccentricity

The actual eccentricity of Mars’ orbit is 0.1 which would look like a circle if I could draw it.

Only the accuracy of Tycho’s data allowed Kepler to distinguish an ellipse of eccentricity 0.1 from a circle.



LC Question: Which orbit below is **not** a possible planetary orbit according to Kepler's First Law?

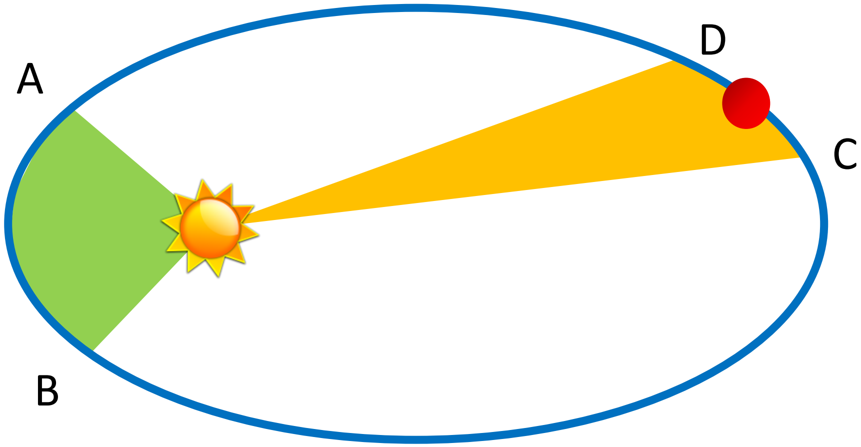


Answer: **The Green orbit because the Sun is not at a focus of the ellipse.**

Kepler's Second Law

Kepler's 1st Law gives the shapes of the orbits, but now that the orbits are not symmetric circles, the speed of the planet doesn't have to be constant.

Kepler's Second Law: **"A line drawn between the Planet and the Sun sweeps out equal areas in equal times as the Planet moves in its orbit."**



If the time for the planet to go A to B is the same as it is to go from C to D.

The green area is the same as the orange area.

What does this mean? (LC)

What does this mean? **The planet's orbital speed is faster when it's closer to the Sun and slower when it's further away.**
(In modern physics language, this is a result of angular momentum conservation.)

Kepler's Third Law

Kepler's 3rd Law (published in 1619) relates the size of planet's orbit to its orbital period.

"The square of the Period of revolution of a planet is directly proportional to the cube of it's semimajor axis."

Where: Period (T) = time to complete one orbit

So symbolically, Kepler's 3rd says:

$$(\text{Period})^2 \propto (\text{Semimajor axis})^3$$

$$T^2 \propto a^3$$

Note: this is a proportionality; however, if we use units of years for period and Astronomical Units (AU) for semimajor axis, it is an equality.

For example: in Kepler's time is was well known that the period of Mars is 1.88 yr.

So:

$$(T \text{ in yr})^2 = (a \text{ in AU})^3$$

$$T^2 = a^3$$

$$a = (T^2)^{1/3} = (1.88^2)^{1/3} = 1.52 \; AU$$

LC Question: **In October 2017, an asteroid passed near the Earth. Astronomers know its orbital semimajor axis is 4 AU. What year will it pass the Earth again? (assuming its orbit is unperturbed by the Earth's gravity)**

Answer: **$a = 4$ AU and $T^2 = a^3$**

So: $a^3 = 64$

and $T = (64)^{½} = 8$ yr

Therefore the year will be 2017+ 8 = 2025

Kepler's Third Law

Since the orbital periods of all of the known planets could be measured, Kepler's 3rd Law could be used to understand the scale of the Solar System

But, note that this only gives the relative scale, not the absolute distances, *i.e. no one had any idea how far an Astronomical Unit really is.*


It would take more than 150 years for someone to actually measure the size of the AU. In 1769, **James Cook** and others observed the **transit of Venus** from multiple locations on the Earth.

Then, trigonometry could be used to finally establish the absolute scale of the Solar System:

1 Astronomical Unit = 1.5×10^8 kilometers

This was the first time that the Scale of the Known Universe could be accurately established – this is an idea that we'll come back to several times, i.e. how do we measure the scale of known Universe.

Some Remarks on Kepler's Laws:

1. **Kepler's Laws gave excellent predictions of planetary motion.** The predictions agreed exactly with observations – even after the adoption of the telescope.

2. **Kepler's Laws are Universal.** They are true not just for the planets in our Solar System; they hold for any two bodies orbiting each other. *(Should really be called Kepler's Laws of Orbital Motion.)*

3. **Kepler's Laws are Empirical.** They are a description based on observation that tells us how the planets move.

What do I mean by this?

Suppose you had never seen a clock:

By observation, you may arrive at a *"Kepler's Law" of Clocks*:

"The Big Hand makes 12 revolutions for the Little Hand's one"

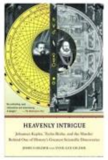
But why? You would have to look inside the clock.
Newton would look inside the clock of the Solar System.

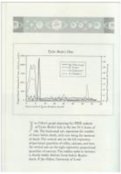
(summary)

Is This the Final Story of Tycho & Kepler?

The standard story of Tycho's death in 1601 is that due to a night of heavy drinking, he suffered from uremia . . . perhaps from a burst bladder. Ultimately this would cause kidney failure and death.


However, in their 2004 book, **Heavenly Intrigue**, Joshua and Anne-Lee Guilders proposed that Tycho was deliberately poisoned with Mercury *(Mercury poisoning mimics uremia and causes kidney failure.)*





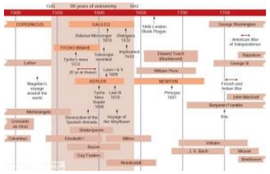
The Evidence: in 1901, Tycho was exhumed to move his burial site. Some samples of his beard hair were saved and later chemically analyzed. Notice the spike of Mercury 13 hours before he died.

The Suspect: **Johannes Kepler** had the means, motive, and opportunity to give Tycho lethal doses of Mercury in the weeks before his death.

Let's not get carried away:

In 2010, Tycho was exhumed again, and a more detailed analysis does not indicate foul play. The Mercury is on the outside of the hair meaning that it probably wasn't ingested. Conclusion: he likely died of a burst bladder!

Did Kepler and Galileo ever Communicate?



Yes, on at least two occasions.

In the late 1590's, Kepler sent Galileo a copy of his work on the Platonic Solids and the sizes of the planetary orbits. Galileo only responded with a brief reply of thanks.

In 1610, Galileo sent Kepler a copy of **The Sidereal Messenger** about his discoveries with the telescope. Kepler was extremely enthusiastic and built his own telescope; although due to his poor eyesight, he had trouble using it.

That same year, Galileo received a copy of Kepler's work **The New Astronomy** describing Kepler's first two Laws of Planetary motion.

The really odd thing is that Galileo never replied to Kepler about his Laws of Planetary Motion and never mentions them in any of his subsequent writings. **Why not?**