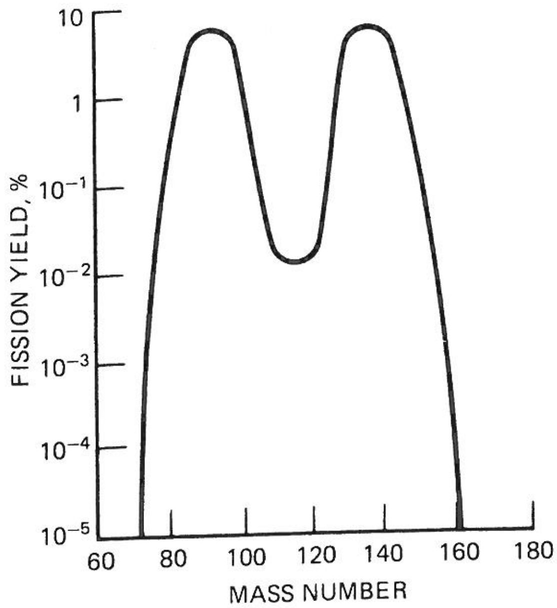

FIGURE 10-2
The mass distribution of fission fragments. It is unlikely (by about 2 orders of magnitude) that a nucleus undergoing fission induced by neutrons of a few MeV will divide into two equal (symmetric) fragments.

P10-5 AN ACTUAL FISSION CHAIN FORMED FROM UNEQUAL MASS FRAGMENTS AND A CALCULATION OF THE ENERGY RELEASED IN THE FISSION THAT FORMED THEM

The maxima in the distribution of fission fragments occur at mass numbers of about 95 and 140, rather than at 118, as would be expected if the fragment distribution were symmetric. The actual situation can be described as follows:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow X + Y + 2 \text{ or } 3 \text{ neutrons} \qquad \text{(P10-16)}$$

The sum of the atomic numbers of X and Y must equal 92, the number of protons in uranium. Figures 10-1 and 10-2 show the unstable fission-fragment nuclei in the vicinity of atomic numbers 35 and 55. For symmetric fission (which occurs rarely), X = Y = element 46 = palladium. A probable combination is X = element 37 = rubidium, and Y = element 55 = cesium. We choose X = element 37 = rubidium, and Y = element 55 = cesium. If we assume that 2 neutrons are emitted, then the fragments X and Y have a total mass of 234. We choose arbitrarily 90 for the mass of the rubidium and 144 for the mass number of cesium. Other pairs of mass numbers are also possible as well as other pairs of elements as long as the atomic numbers of the fragments add to 92, the original atomic number of the uranium 236 parent nucleus. The mass numbers of the fragments total 234 since two neutrons are emitted.

$$^{236}_{92}\text{U} \rightarrow {}^{90}_{37}\text{Rb} + {}^{144}_{55}\text{Cs} + 2^{1}_{0}n \qquad \text{(P10-7)}$$

The cesium 144, that is the heavy fragment of this pair, has a half-life of about one second and decays by a series of six beta emissions forming radioactive isotopes of mass number 144 in turn of barium, lanthanum, cerium, and praseodymium, which beta-decays to stable neodymium.

We now follow the decay of the $^{90}_{37}$Rb nucleus that seeks stability by emission of three beta particles with accompanying gamma rays and antineutrinos.

$$^{90}_{37}\text{Rb} \xrightarrow{2.6\ \text{min}} {}^{90}_{38}\text{Sr} + \underset{6.3\ \text{MeV}}{{}^{0}_{-1}e} + \underset{\text{antineutrino}}{{}^{0}_{0}\bar{\nu}} \qquad \text{(P10-8)}$$

$$^{90}_{38}\text{Sr} \xrightarrow{28\ \text{yr}} {}^{90}_{39}\text{Y} + \underset{0.55\ \text{MeV}}{{}^{0}_{-1}e} + \underset{\text{antineutrino}}{{}^{0}_{0}\bar{\nu}} \qquad \text{(P10-9)}$$

$$^{90}_{39}\text{Y} \xrightarrow{64\ \text{h}} {}^{90}_{40}\text{Zr} + \underset{2.3\ \text{MeV}}{{}^{0}_{-1}e} + \underset{\text{antineutrino}}{{}^{0}_{0}\bar{\nu}} \qquad \text{(P10-10)}$$

The strontium 90 fission product is particularly dangerous because of its long half-life and because it is deposited in the bone. It also gives rise to a daughter, yttrium 90, which has a very energetic beta ray that will produce additional biological damage at the site of the strontium 90.

The energy given to the fission fragments can be calculated from the masses of the initial nucleus and its fragments.* The released energy arises from the coulomb repulsion of the positively charged fragments which gives them large kinetic energies (on a nuclear scale) as they fly apart. See Chapter 12 for a discussion of the repulsive force between particles of similar electric charge, in this case the nuclear protons.

The energy released in forming the fission fragments will be the mass difference between the left-hand side of Eq. (P10-7) and its right-hand side multiplied by the square of the velocity of light according to Einstein's equation $E = mc^2$. The following are the masses involved:

Uranium 236:	236.045663 amu (atomic mass units)
Rubidium 90:	89.914752 amu
Cesium 144:	143.931108 amu
Neutron:	1.008665 amu

An *atomic mass unit* is equal to 1.661×10^{-27} kg. If, for example, the mass of the neutron were exactly its mass number, 1.000, then the neutron mass would be

*The masses were calculated from mass excesses that were calculated by G. T. Garvey et al., *Review of Modern Physics*, Vol. 41, No 4, pp. S1–S80, 1969.

1.661×10^{-27} kg. The mass of the neutron is in fact a little larger. One can see from the above numbers that uranium 236 has an excess of mass over its mass number, whereas rubidium 90 and cesium 144 have a deficiency, since they have more binding energy (see Figure 9-10).

Adding up the masses on the left-hand side taking into account that two neutrons are emitted, the total mass after fission of the uranium 236 is 235.863190. It is necessary to keep so many significant figures (see Chapter 8) here because when 235.863190 is subtracted from 236.045663 to find the mass difference three significant figures are lost. This difference is in fact 0.182473. The energy released to form the fragments is therefore:

$$E = (0.182473 \text{ amu} \times 1.661 \times 10^{-27} \text{ kg/amu}) \times (2.99793 \times 10^{8})^{2} \text{ J}$$

(Here we take a more exact velocity of light than usual because of the significant figures in the calculation.)

$$= 2.7240 \times 10^{-11} \text{ J}$$
$$= 2.7240 \times 10^{-11} \text{ J} \times 1 \text{ MeV}/(1.602 \times 10^{-13} \text{ J}) = 170.0 \text{ MeV}$$

This result is in accord with Table 11-2, which shows the distribution of fission energy. Here we have calculated only the energy associated with the kinetic energy of the fission fragments. The masses of the rubidium 90 and cesium 144 isotopes are still larger than the same mass numbers for stable nuclei. These isotopes then beta-decay to reach stability, changing their excess neutrons into protons spontaneously. In so doing they will release additional energy in the form of beta and gamma rays from the fission products and their accompanying antineutrinos. We now calculate this additional energy release by finding the difference in mass between rubidium 90 and the stable nucleus it reaches after three beta decays, zirconium 90. Similarly we calculate the energy release for cesium 144 and the stable nucleus it reaches, neodymium 144, after six beta decays.

The new masses needed are:

Zirconium 90:	89.904625 amu
Neodymium 144:	143.91004 amu

The beta decay of rubidium 90 then releases an atomic mass unit difference of 89.914752 − 89.904625 amu or 0.010095 amu. Using the direct energy conversion that 1 amu $\times c^2 = 931.2$ MeV, this is equivalent to 9.43 MeV. If we are interested in the energy release in MeV, the use of direct conversion from amu to MeV avoids calculation of the difference in nuclear masses in kilograms as in the previous example. Similarly the beta decay of cesium 144 releases a difference of 143.91004 amu = 143.931108 amu or 0.021068 amu. This is equal to 19.62 MeV. The total energy released in the beta decay processes from both fragments is therefore 29.05 MeV. This total includes the energies of all the electrons, antineutrinos, and gamma rays emitted. This figure compares favorably with the typical value of 23 MeV found in Table 11-2.

Therefore, the total energy released, including both kinetic energy of the fission fragments and their subsequent beta decay, is 170.0 MeV + 29.05 MeV = 199.1 MeV. This total energy release is the value for this particular choice of fragments and is close to the typical value of 200 MeV often cited as the energy released in fission (see Table 11-2).