Some Remarks about Part II of the Text

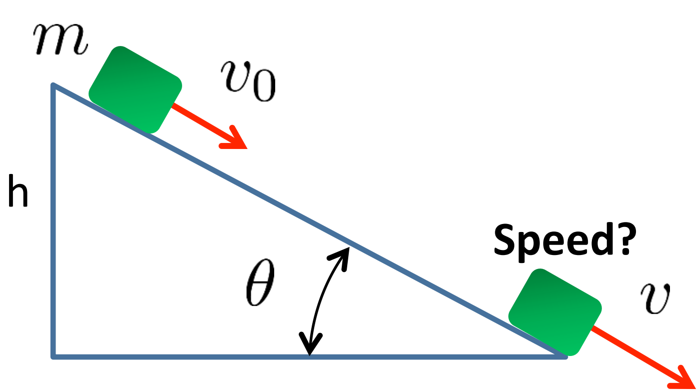
Up to this point in Physics 191, we have concentrated on:

Kinematics: How things move $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ and $\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$

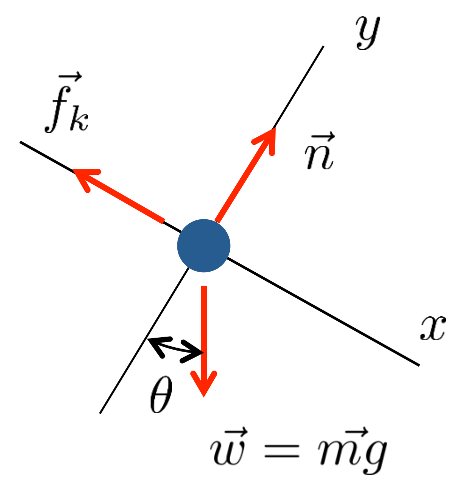
Dynamics: Why things move $\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$

(So, all of mechanics is nothing more than solving 2^{nd} order differential equations!)

Here's something we can handle:



FBD:



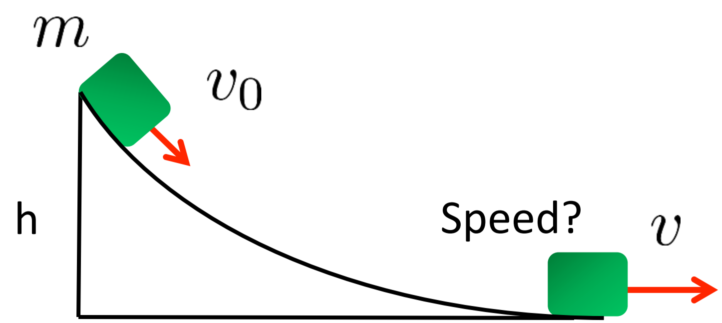
As we've done many times, for no friction: $a_x = g\sin\theta$

Or, if there is friction: $a_x = g(\sin\theta - \mu_k\cos\theta)$

Either way, we can solve for the speed

Some Remarks about Part II of the Text

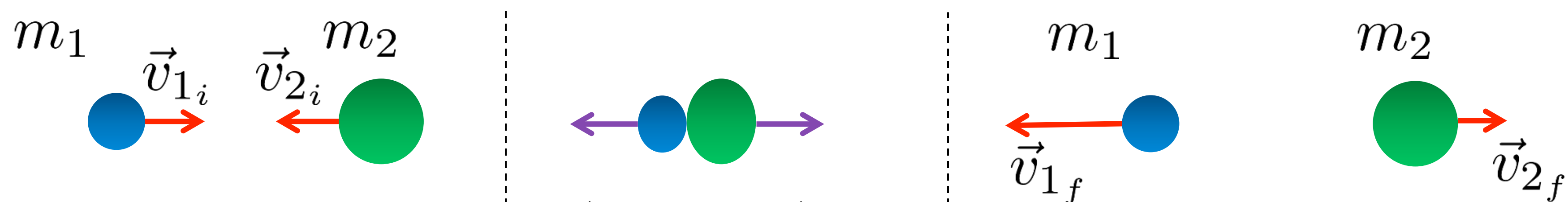
But, what if we had this problem:



Here, the FBD is not the same at every point on the slide.

This means that the **acceleration is not constant.**

Or, how about this, a collision:



$\text{Force on } m_1 \text{ and } m_2 = 0$

$\vec{F}_{2\text{on}1} - \vec{F}_{1\text{on}2}$

(some complicated function of time - watch)

$\text{Force on } m_1 \text{ and } m_2 = 0$

$\text{Find: } \vec{v}_{1_f} \text{ and } \vec{v}_{2_f}$

Both of these examples can be solved using Newton's Laws, but they're very very difficult! We need a better way. **Fortunately, there is one, and it's really easy!**

<u>Some Remarks about Part II of the Text</u>

Remember, we started Physics 191 with the observation:

"Everything moves"

And we've spent the entire semester learning how to describe that motion. We're now ready to add to that observation:

"Everything moves, but in all processes, some quantities stay the same, i.e. are <u>conserved</u>."

A **<u>Conservation Law</u>** tells us that *something* stays the same, and we can use that to solve many types of problems very easily. The trick is to find out what that <u>something</u> is.

We'll concentrate on the two conservation laws:

<u>Conservation of Energy</u> (Chapters 9 & 10)

<u>Conservation of Momentum</u> (Chapter 11)

These will become essential tools that we can use to solve all kinds of problems that would otherwise be very difficult.

One Last Remark about Part II of the Text

Your author points out that these conservation laws are actually more fundamental than Newton's Laws*; this is true:

In the realm of the very small (i.e. atoms), Newton's 2nd Law fails, but Energy and Momentum conservation are still valid:

Quantum Mechanics (end of PHY191)

Also, in the realm of the very fast (near the speed of light), both Newton's Laws and the rules of kinematics fail, but with more complete definitions of momentum and energy, the conservation laws are still valid:

Special Relativity (end of PHY192)

*Newton did not use the ideas of momentum and energy.

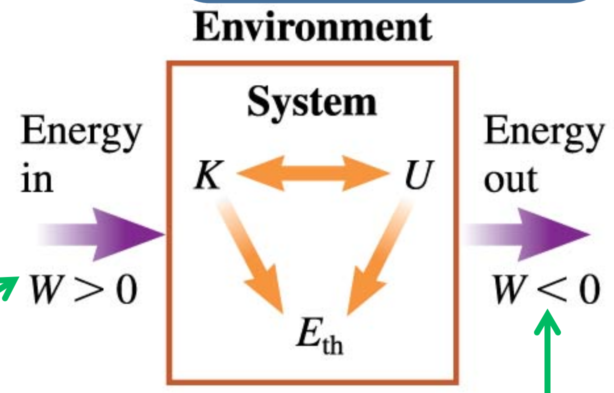
MODEL 9.1

Sec 9.1

Kinetic Energy, K
Potential Energy, U
Thermal Energy, E_{th}
$E_{sys} = K + U + E_{th}$

Basic energy model (For Mechanical Systems)

Energy is a property of the system.



- Energy is *transformed* within the system without loss. (i.e. $K + U + E_{th}$ = constant)
- Energy is *transferred* to and from the system by forces from the environment.
  - The forces do *work* on the system.
  - $W > 0$ for energy added.
  - $W < 0$ for energy removed.
- The energy of an *isolated system*—one that doesn't interact with its environment—does not change. We say it is *conserved*.
- The energy principle is $\Delta E_{sys} = W_{ext}$.
- Limitations: Model fails if there is energy transfer via thermal processes (heat).

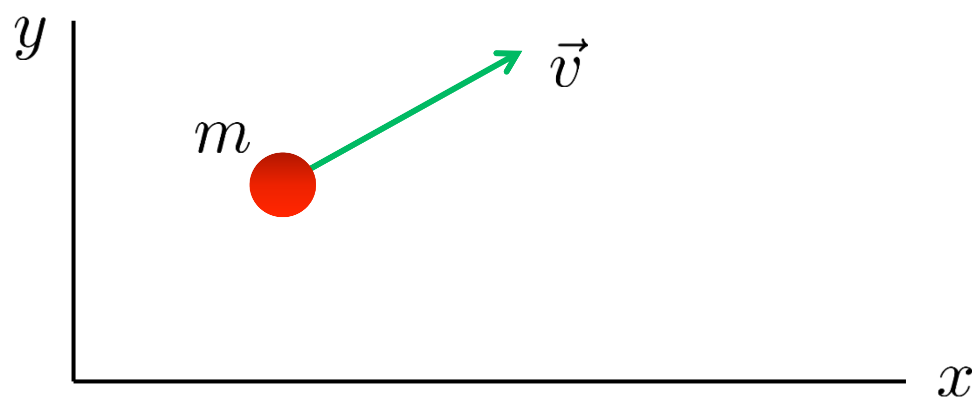
Exercise 1

We'll develop and refine this model as we go through Chapters 9 & 10, and we'll come back to it in PHY192 when we'll include heat.

Sec 9.2 – WORK AND KINETIC ENERGY

Energy – the Basics- Kinetic Energy

Kinetic Energy = The Energy of Motion.



$$\textbf{Kinetic Energy}, K \equiv \frac{1}{2}mv^2$$

$$\text{Units: } 1\ \frac{kg\ m^2}{s^2} \equiv 1\ Joule\ (J)$$

Note: **Kinetic Energy (KE) is a Scalar**. Also, the v in the equation is the speed of the object. **Therefore, KE is always a positive quantity.**

Another Note: As we'll see soon, you can obtain the Kinetic Energy from the velocity vector as :

$$K = \frac{1}{2}m\ \vec{v} \cdot \vec{v}$$

Whiteboard Problem: 9-1

At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/h?

The Concept of Work (Sec 9.1 - 9.2)

How do you make an object speed up or slow down, i.e. change it's kinetic energy?
In the language of Chapter 6, you exert a Force on it.
In the language of Energy, you do work on it: positive work increases the speed; negative work decreases the speed.

What is $\Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$?

Consider this in 1D:

Kinematics and Dynamics:

$$v_1^2 = v_0^2 + 2a_x\Delta x$$

$$v_1^2 - v_0^2 = 2a_x(x_1 - x_0)$$

$$\underbrace{\tfrac{1}{2}m(v_1^2 - v_0^2)}_{\Delta K} = \underbrace{ma_x}_{F_x}(x_1 - x_0)$$

$$\Delta K = F_x(x_1 - x_0)$$

Your author does this for a 1D variable force:

(details on p. 209-210)

$$\Delta K = \int_{x_i}^{x_f} F_x dx = \text{Work}$$

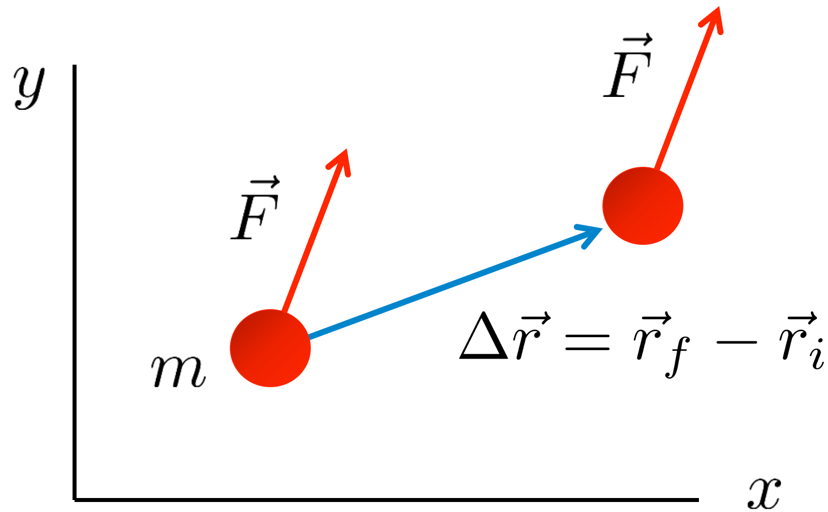
i.e. The Change in KE = "Force x displacement" = **The Work done by the Force**

Note of caution: Work = Force x displacement is a very special case. It is valid only when the force is constant and parallel to the displacement.

What is This Thing Called "Work" (Sec 9.3)

(it seems pretty important)

Work done by a Constant Force:



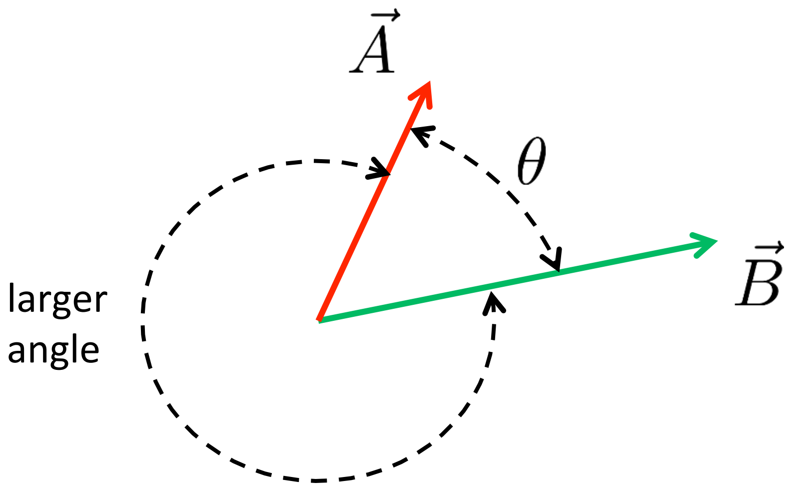
$$\text{Work (done on } m \text{ by } \vec{F} \text{ moving } m \text{ by } \Delta\vec{r}) = \vec{F}\cdot\Delta\vec{r}$$

$$i.e. \quad \boxed{\text{for } \vec{F} = \text{constant:} \qquad W = \vec{F}\cdot\Delta\vec{r}} \qquad [\text{Units: } Nm = \text{Joule}]$$

But, what is this thing? **It's called a "Dot Product" or "Scalar Product"**

Dot Product (Sec 9.3)

(sometimes called a Scalar Product)



$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Where: A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$

θ is the smaller of the two angles between $\vec{A}$ and $\vec{B}$

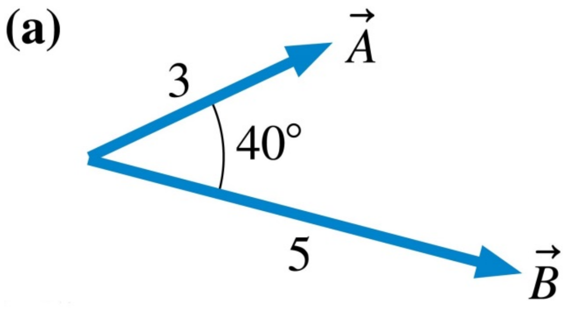
So, the Dot Product produces a **scalar** that can be positive, negative, or zero.

What does the dot product mean in words?

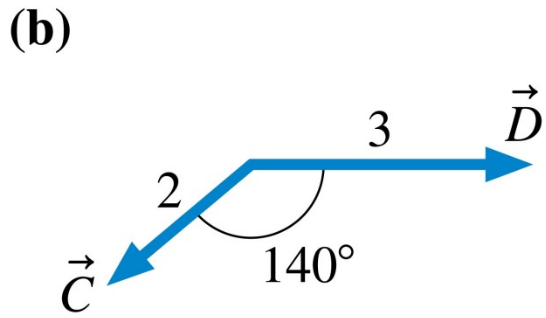
$\vec{A} \cdot \vec{B}$ is a measure of how much of $\vec{A}$ points in the direction of $\vec{B}$, and vice versa.

Whiteboard Problem: 9-2

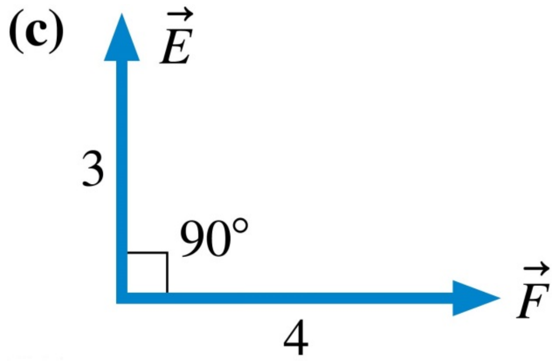
Evaluate the dot product of the following pairs of vectors:



Answer: 11.5



Answer: -4.60



Answer: 0.0

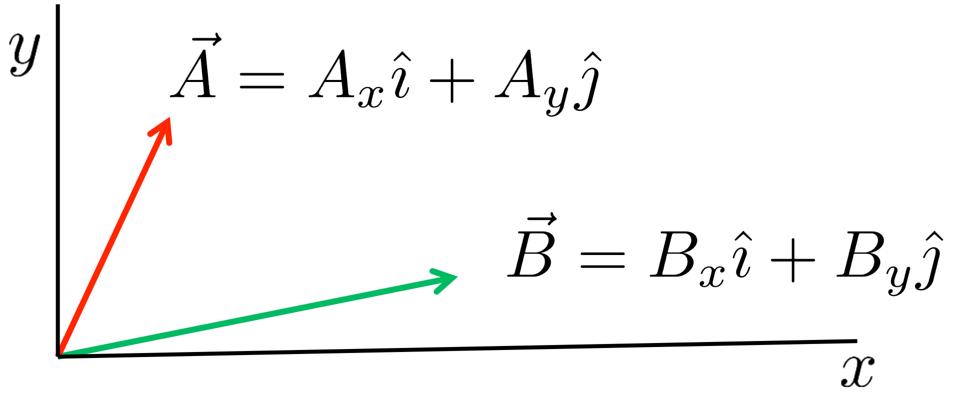
(d) Evaluate the dot product of:

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} \quad \text{and} \quad \vec{B} = 2\hat{\imath} - 6\hat{\jmath}$$

Maybe we need another way to do a Dot Product

Another Way to do Dot Products

Suppose we know the components of the vectors:



It is easy to show that:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

Note: The two formulae for the Dot Product are equally important and useful. If you know the angle between the vectors, you use one; if you know the components, you use the other.

YOU HAVE TO KNOW AND BE ABLE TO USE BOTH!

Whiteboard Problem: 9-2 (continued)

(d) Evaluate the dot product of:

Answer:

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} \;\; \text{and} \;\; \vec{B} = 2\hat{\imath} - 6\hat{\jmath}$$

$$\vec{A}\cdot\vec{B} = (3)(2) + (4)(-6) = -18$$

(e)

$$\vec{D} = 3\hat{\imath} - 2\hat{\jmath} \;\; \text{and} \;\; \vec{F} = 6\hat{\imath} + 4\hat{\jmath}$$

$$\vec{D}\cdot\vec{F} = (3)(6) + (-2)(4) = 10$$

In fact, this gives us a way to find the angle between two vectors;

e.g. for the first two vectors in part (d) above:

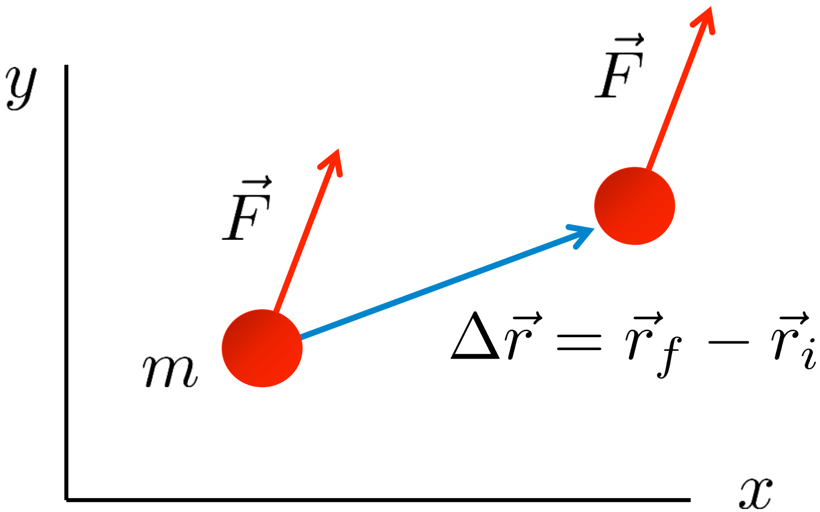
$$\vec{A}\cdot\vec{B} = -18 = AB\cos\theta$$

$$A = \sqrt{3^2 + 4^2} = 5.0 \qquad B = \sqrt{2^2 + 6^2} = 6.324$$

$$\text{So, } \theta = \cos^{-1}\left[\frac{-18}{(5)(6.324)}\right] = 124.7^\circ$$

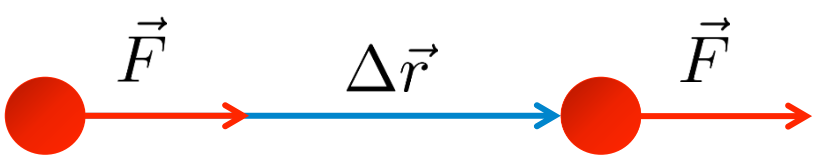
Back to Work done by a Force (Sec 9.3)

For a <u>Constant Force</u>, we had:



$$W = \vec{F} \cdot \Delta\vec{r} \quad (\vec{F} = \text{constant})$$

Note: Now we see how **"Work = Force x Distance" is just a <u>special case</u>** for a constant force parallel to the displacement. Consider:

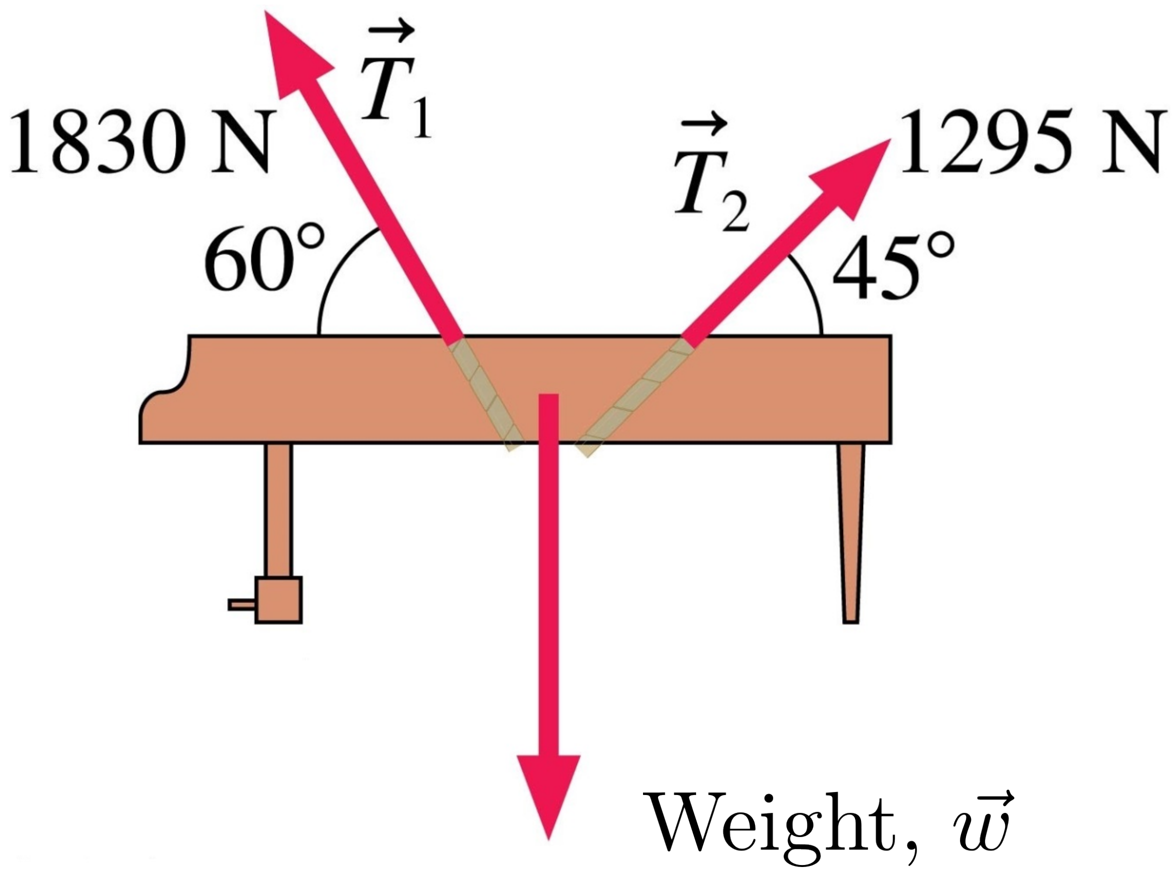


$$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos(0^\circ) = F\Delta r$$

Whiteboard Problem: 9-3

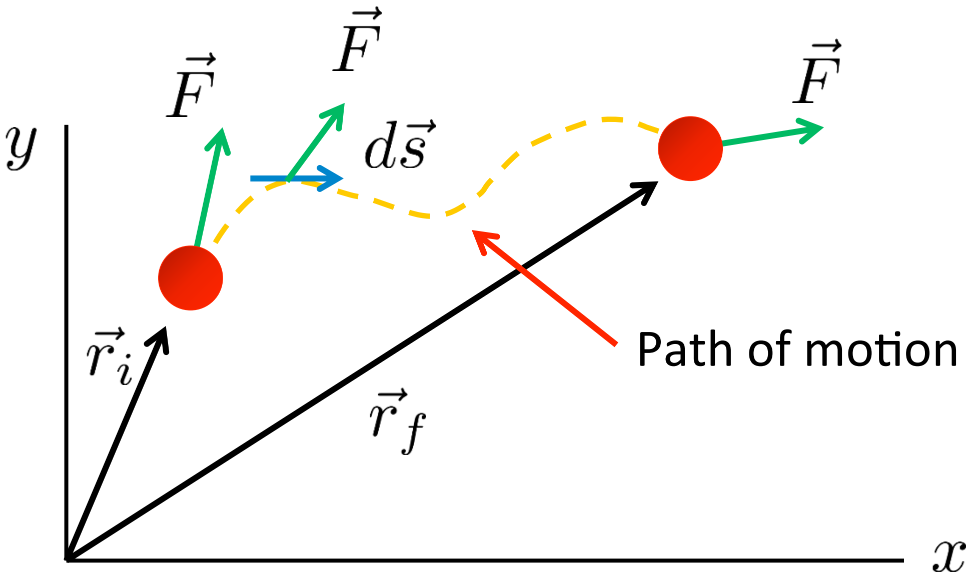
The two ropes below are used to lower a 255 kg piano 5.0 m from a second story window to the ground at constant speed.

How much work is done by each of the three forces?



Work done by a Variable Force (Sec 9.3)

General Definition of Work done by a Force:



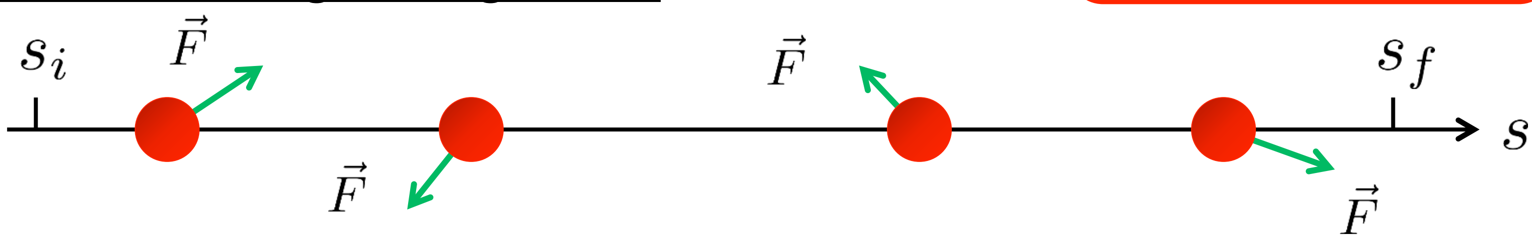
At every point along the path, an infinitesimal amount of work is done:

$$dW = \vec{F} \cdot d\vec{s}$$

So, just add these up for the total work done by F in going from the initial to the final point:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s}$$

For motion along a straight line:



Only that part of the Force parallel to the s-axis (i.e. the s-component) does any work.

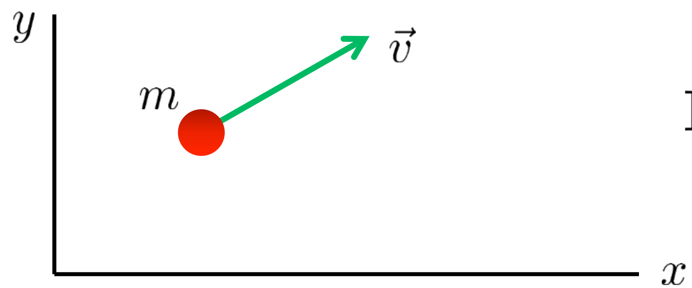
$$W \text{ (by } \vec{F} \text{ going } s_i \text{ to } s_f) = \int_{s_i}^{s_f} F_s ds = [\text{area under the } F_s \text{ vs. } s \text{ curve}]$$

<u>Whiteboard Problem 9-4</u>

A person pushes horizontally on a heavy box and slides it across the level floor at constant velocity. The person pushes with a force of 75.0 N for the first 5.0m, at which time he begins to tire. The force he exerts then starts to decrease linearly from 75.0 N to 0.0 N across the remaining 3.0 m.

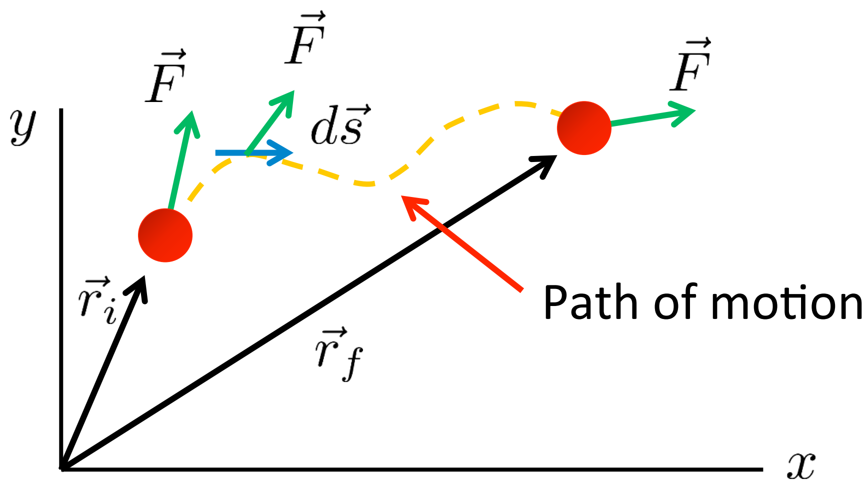
How much work did the person do on the box?

A Quick Recap



$$\textbf{Kinetic Energy}, \; K \equiv \frac{1}{2}mv^2$$

Work done by a Force:



General Expression:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s}$$

For motion along a line s:

$$W = \int_{s_i}^{s_f} F_s ds$$

For a Constant Force:

$$W = \vec{F} \cdot \Delta\vec{r}$$

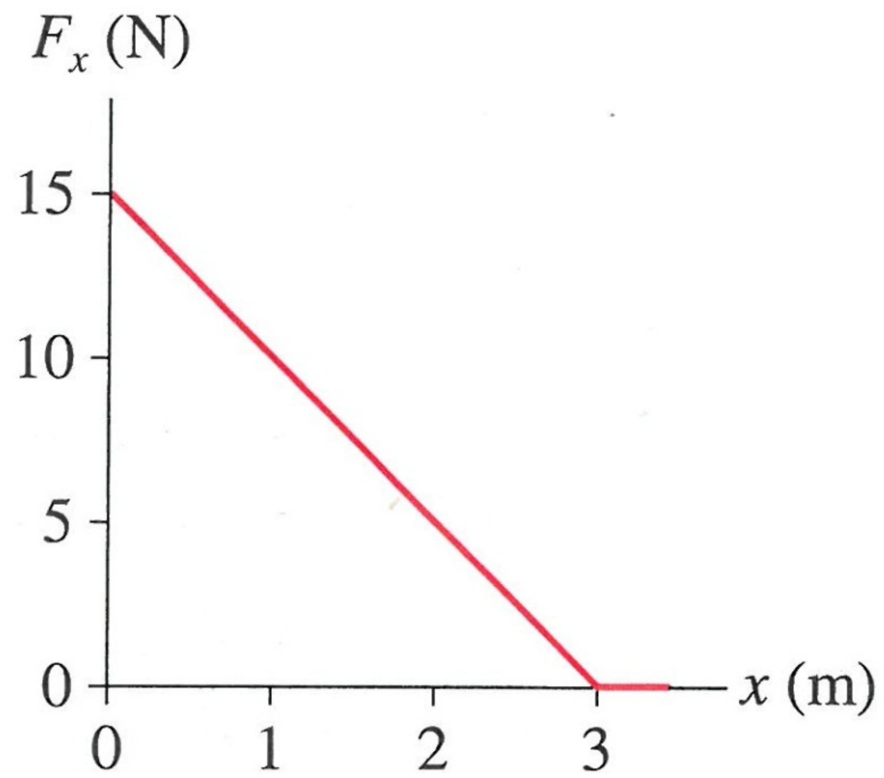
For a Constant Force parallel to the displacement:

$$W = F\Delta r$$

*****We're not going to be following the order of material in the text*****

Whiteboard Problem: 9-5

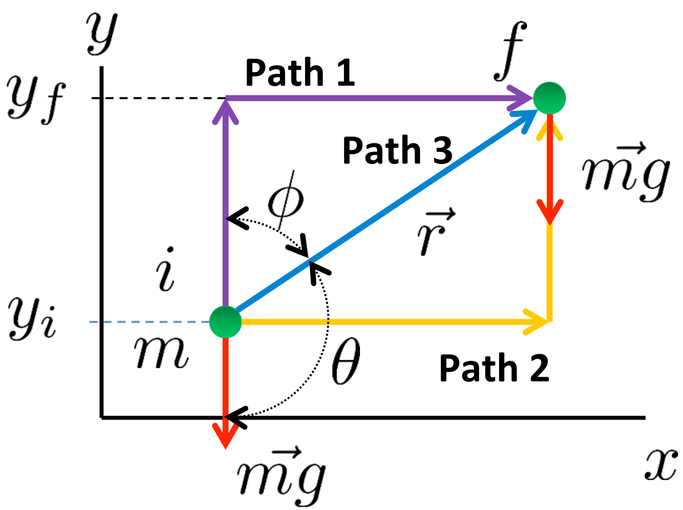
A 500 g particle moving along the x axis experiences the force shown below. The particle's velocity is 2.0 m/s at x = 0 m. **What is its speed at x = 3 m?**



Conservative Forces (Sec 10.7)

Definition: A **conservative* force** is a force for which the work done by the force in going from some initial point i to some final point f **does not depend on the path followed.**

By example: Gravity:



Work done by Gravity going i to f:

$$\text{Path 1: } W_g = -mg(y_f - y_i)$$

$$\text{Path 2: } W_g = -mg(y_f - y_i)$$

$$\begin{aligned}\text{Path 3: } W_g &= \vec{mg} \cdot \vec{r} \\ &= mgr\cos\theta \\ &= -mgr\cos\phi \\ &= -mg(y_f - y_i)\end{aligned}$$

So the work done by gravity does not depend on the path, just the change in y. So gravity is a conservative force.

*As we'll see, the name comes from the fact that conservative forces conserve the total mechanical energy.

Potential Energy (Sec 10.1)

Any Conservative Force can be associated with a Potential Energy:

For a system going from an initial state i to a final state f subject to a conservative force, F, **the change in potential energy is the negative of the work done by the conservative force, F.**

$$\Delta U = U_f - U_i \equiv -W_c(i \rightarrow f)$$

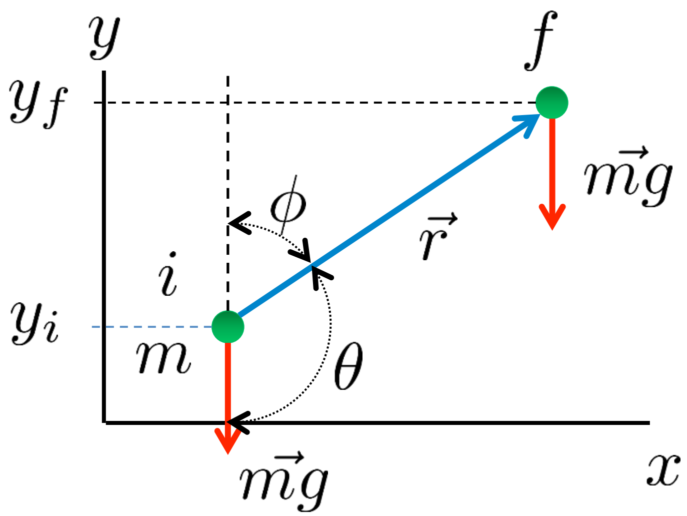
Work done by a conservative force

Since the work is path independent, the change in potential energy depends only on the initial and final points, not the path followed.

Note: we'll also get to nonconservative forces for which we can't associate a potential energy.

Gravitational Potential Energy (Sec 10.2)

Gravity is a conservative force, so we can define a potential energy for gravity:



Work done by gravity in going from i to f:

$$W_g = \vec{m}g \cdot \vec{r} = mgr\cos\theta$$
$$= -mgr\cos\phi$$
$$= -mg(y_f - y_i)$$

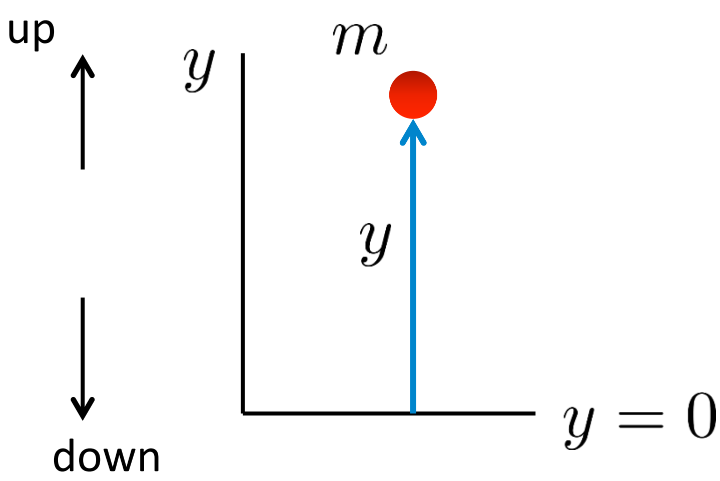
So, the Change in Gravitational Potential Energy is:

$$\Delta U_g = U_f - U_i = -W_g = mg(y_f - y_i)$$

And we say that the Gravitational Potential Energy depends just on the elevation:

$$U_g = mgy$$

More on Gravitational Potential Energy (PE) (Sec 10.2)



*If you hold a mass above the floor, it has the **Potential** to move – if you release it. Or, it has the **Potential** to acquire Kinetic Energy – again, if you release it.*

*That's the origin of term **"Potential Energy"** which is associated with a position and has the potential to be converted to kinetic energy.*

$$\textbf{Gravitational Potential Energy}^*, U_g = mgy \ [\text{Units} = \text{Joule}]$$

Important Note: you are free to set the $y = 0$ point and hence the $U_g = 0$ point anywhere you like. Only a change in PE (i.e. ΔU_g) has any physical meaning.

**This form for the gravitational PE is only valid near the surface of the Earth where g is fairly constant. We'll see a more general form in Chapter 13.*

Conservation of Mechanical Energy for Conservative Forces (Sec 10.4)

Define: **Mechanical Energy = Kinetic Energy + Potential Energy**

$$\text{Or,} \qquad E_{mech} = K + U$$

It is the **Mechanical Energy** that is conserved in processes where there are no applied or dissipative forces like friction.

Now, the **Work – Kinetic Energy Theorem** for a system going from an initial state, i, to a final state, f, is:

$$\Delta K = W_{\text{ext}}(i \to f) \qquad W_c(i \to f) = -\Delta U$$

So, for problems where only conservative forces are present:

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

Where: $\Delta \overset{\text{(always)}}{=}$ (final) - (initial)

In most problems, we'll have: $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$

$$\Delta U_g = mgy_f - mgy_i = mg(y_f - y_i)$$

I start every conservation of energy problem with this equation (*or it's more general form, that we'll see soon*). It's different than what's in the text – ***why do I do it this way?***
Every problem begins with exactly the same equation even with multiple forms of PE.
Also, including nonconservative forces, e.g. applied forces and friction,
is quite easy and very straight forward.

BOOK: $\underbrace{K_f + U_f}_{\text{final M.E.}} = \underbrace{K_i + U_i}_{\text{Initial M.E.}}$

WE USE THIS:

$$\underbrace{K_f - K_i}_{\Delta K} + \underbrace{U_f - U_i}_{\Delta U} = 0$$

$$\Delta K + \Delta U = 0$$

change in KE + change in PE = 0

WHY ARE WE BETTER?

- w/ $\Delta K + \Delta U = 0$ you do not get hung up on "where's the zero-point for U_i?"

- WHEN M.E. is not conserved (due to losses from friction, say) then $\Delta K + \Delta U = 0$ can be EASILY MODIFIED to describe the new situation. Not so easy w/ $K_i + U_i = K_f + U_f$!

Whiteboard Problem: 9 - 6

You're driving your car at 35 km/h when the road suddenly descends on a slope 15 m into a valley. You take your foot off the accelerator and coast down the hill. Just as you reach the bottom, you see a policeman hiding behind a speed limit sign that reads "70 km/h."

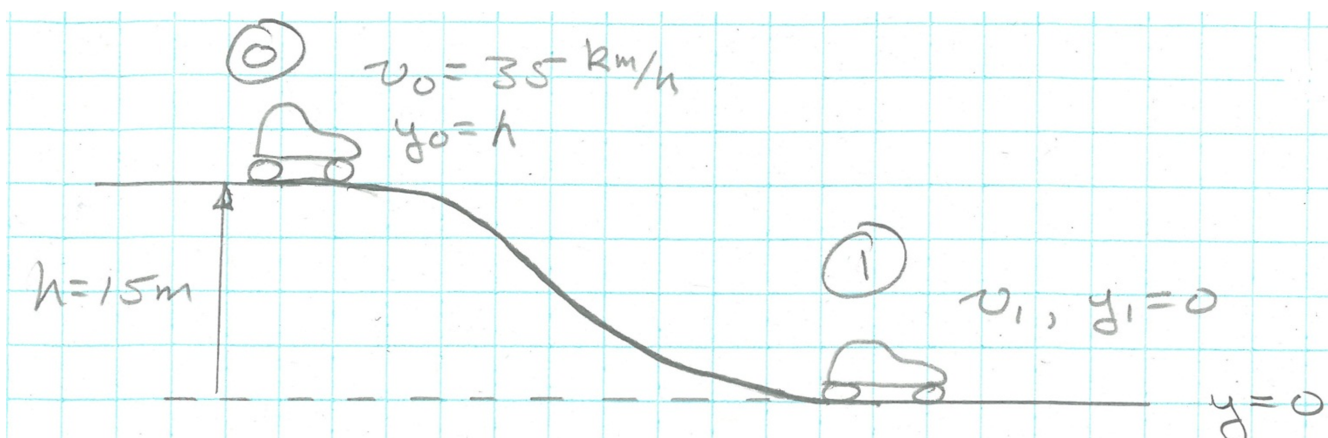
What is your speed when you pass the policeman? (LC)

Are you going to get a speeding ticket?

Could you do this problem using kinematics?

Only if you know that the hill is an incline plane – but it's not!

Your sketch should look like this



Whiteboard Problem 9-6

You're driving at 35 km/h when the road suddenly descends 15 m into a valley. You take your foot off the accelerator and coast down the hill. Just as you reach the bottom you see the policeman hiding behind the speed limit sign that reads "70 km/h." Are you going to get a speeding ticket? Neglect friction.

$0 \rightarrow 1: \quad \Delta K + \Delta V = 0$

Could you do this problem using kinematics?

Only if you know that the hill is an incline plane – but it's not!

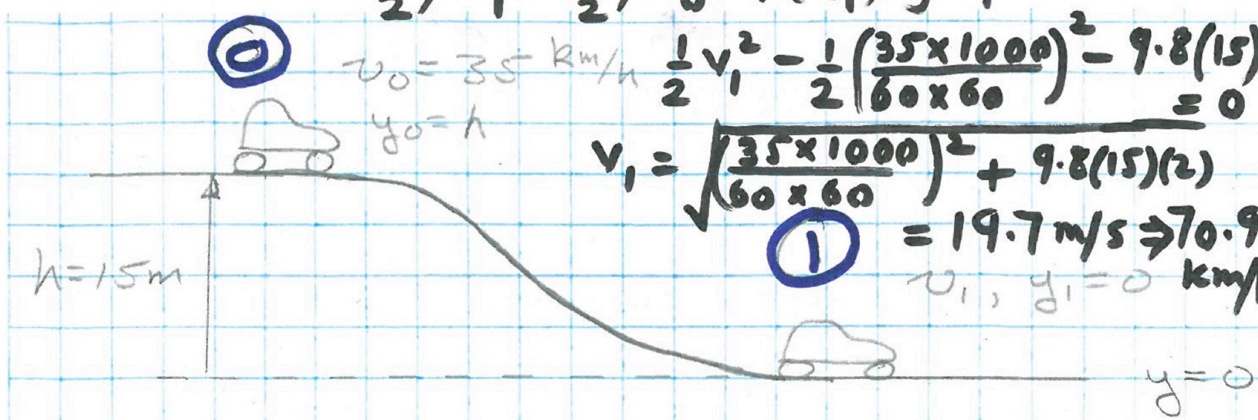
$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 + (-)|m g h| = 0$ (g = 9.8)

$\frac{1}{2} v_1^2 - \frac{1}{2}\left(\frac{35 \times 1000}{60 \times 60}\right)^2 - 9.8(15) = 0$

$v_1 = \sqrt{\left(\frac{35 \times 1000}{60 \times 60}\right)^2 + 9.8(15)(2)}$

$= 19.7 \text{ m/s} \Rightarrow 70.9 \text{ km/hr}$

Your sketch should look like this

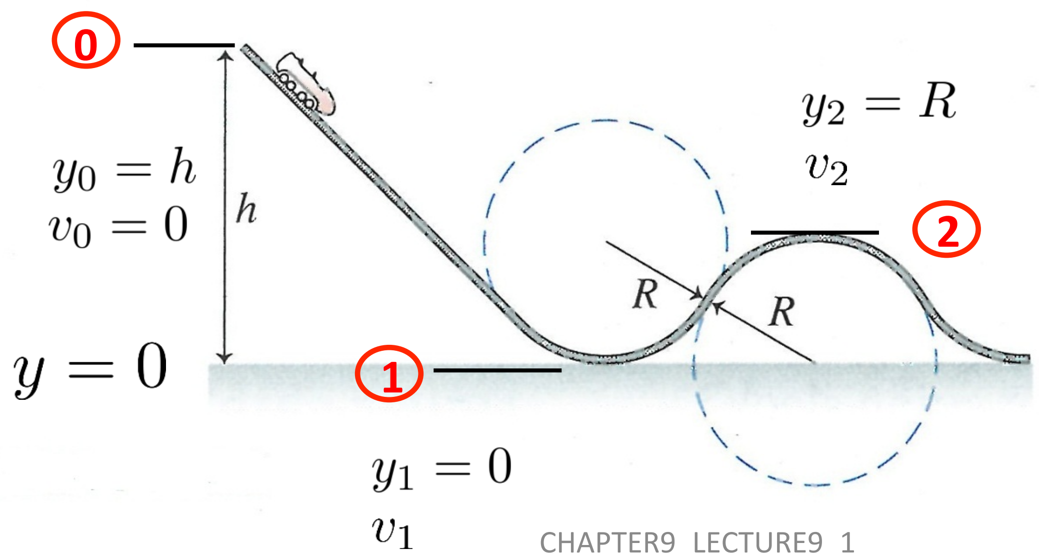


A Good Whiteboard Problem: 9-7

A roller coaster car on a frictionless track shown below starts from rest at height h. The track's valley and hill consist of circular shaped segments of radius R.

a) **What is the maximum height h_{max} (expressed as a multiple of R) from which the car can start so as not to fly off the track going over the hill?** Hint: First find the max speed for going over the hill.

b) **For the height found in part a, what is the apparent weight (in terms of the actual weight w) felt by the riders at the bottom of the valley?**

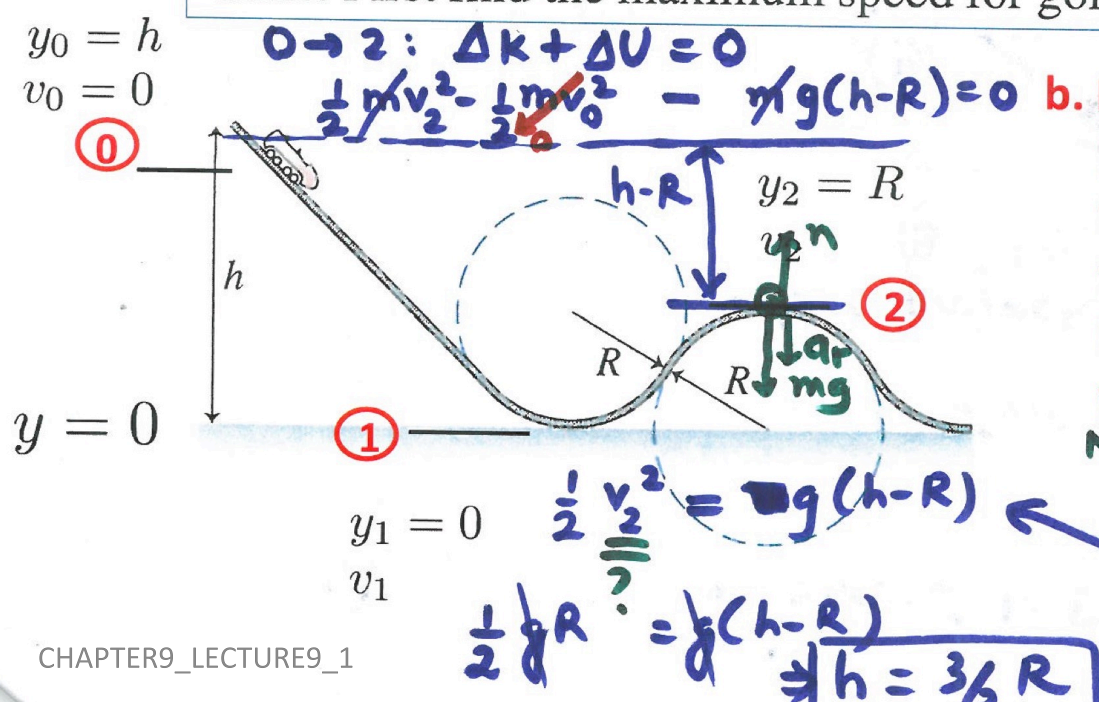


A Good Whiteboard Problem: 9-7

53. || A roller coaster car on the frictionless track shown in **FIGURE P10.53** starts from rest at height h. The track's valley and hill consist of circular-shaped segments of radius R.

a. What is the *maximum* height h_{max} from which the car can start so as not to fly off the track when going over the hill? Give your answer as a multiple of R.

Hint: First find the maximum speed for going over the hill.



b. For the height found in part a, what is the apparent weight felt by the riders at the bottom of the first loop (in terms of the actual weight, w)?

Newton's 2nd law @ "2":

$mg - n = ma_r = \frac{mv_{max}^2}{R}$, $n = 0$

$v_{max,2} = \sqrt{gR}$

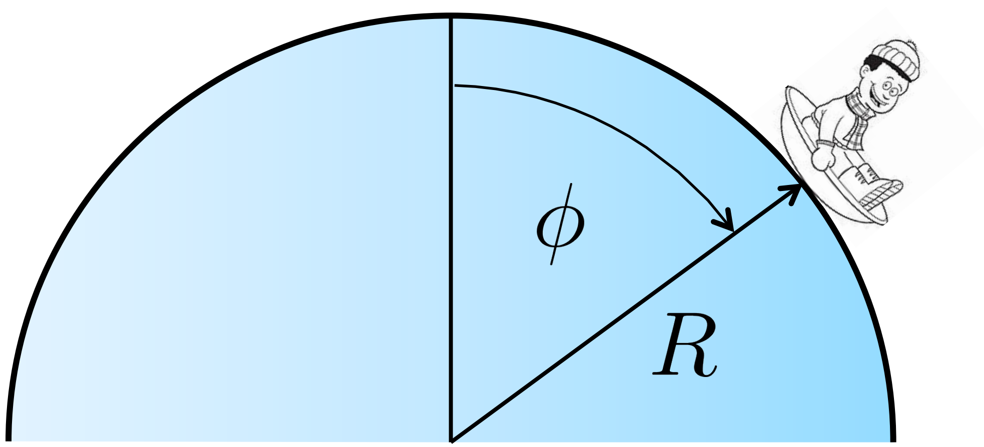
Table Challenge Problem

You are to solve the following problem as a group of your entire table. **Work together on your whiteboards and the wall whiteboards. When the group has arrived at an answer, write it below and turn this sheet in.** **Only your answer will be graded.**

Table:

Names:

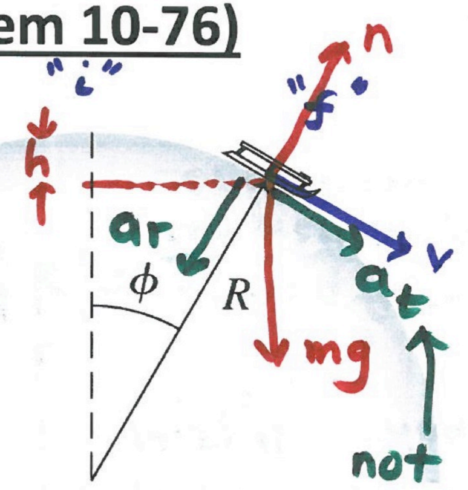
You are sledding on a hemispherical hill that is covered with frictionless snow. If you start from rest at the top of the hill, **at what angle (in degrees) does the sled fly off the hill?**



Answer: ____________

Discussion: Sledding on a Spherical Hill (HW problem 10-76)

76. A sled starts from rest at the top of the frictionless, hemispherical, snow-covered hill shown in FIGURE CP10.76.
   a. Find an expression for the sled's speed when it is at angle ϕ.
   b. Use Newton's laws to find the maximum speed the sled can have at angle ϕ without leaving the surface.
   c. At what angle ϕ_{max} does the sled "fly off" the hill?



not important for this problem

$\Delta K + \Delta U = 0$ [C.O.M.E.]

a) $\frac{1}{2}mv^2 - \frac{1}{2}m(0) - mgh = 0$

Find 'h' in terms of R, ϕ

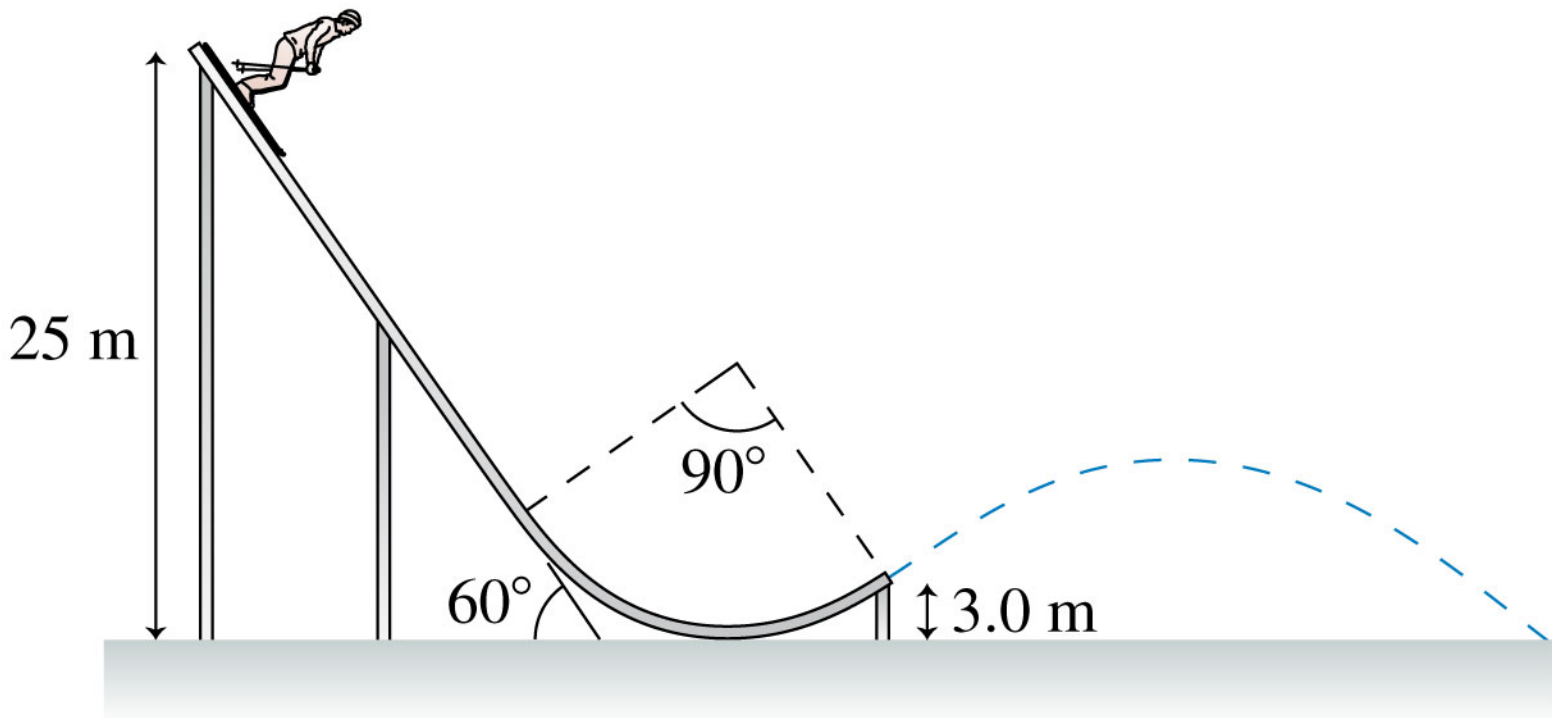
Solve for 'v' in terms of R, ϕ, g

b) • Draw f.b.d. for 'm' } $(F_{net})_r = ma_r$
   • Set $n = 0$ for v_{max}

Memorizing that $v_{max} = \sqrt{gR}$ won't work! ✗ (only true when at ~~n = mg~~ top ~~or bottom~~ of rollercoaster)

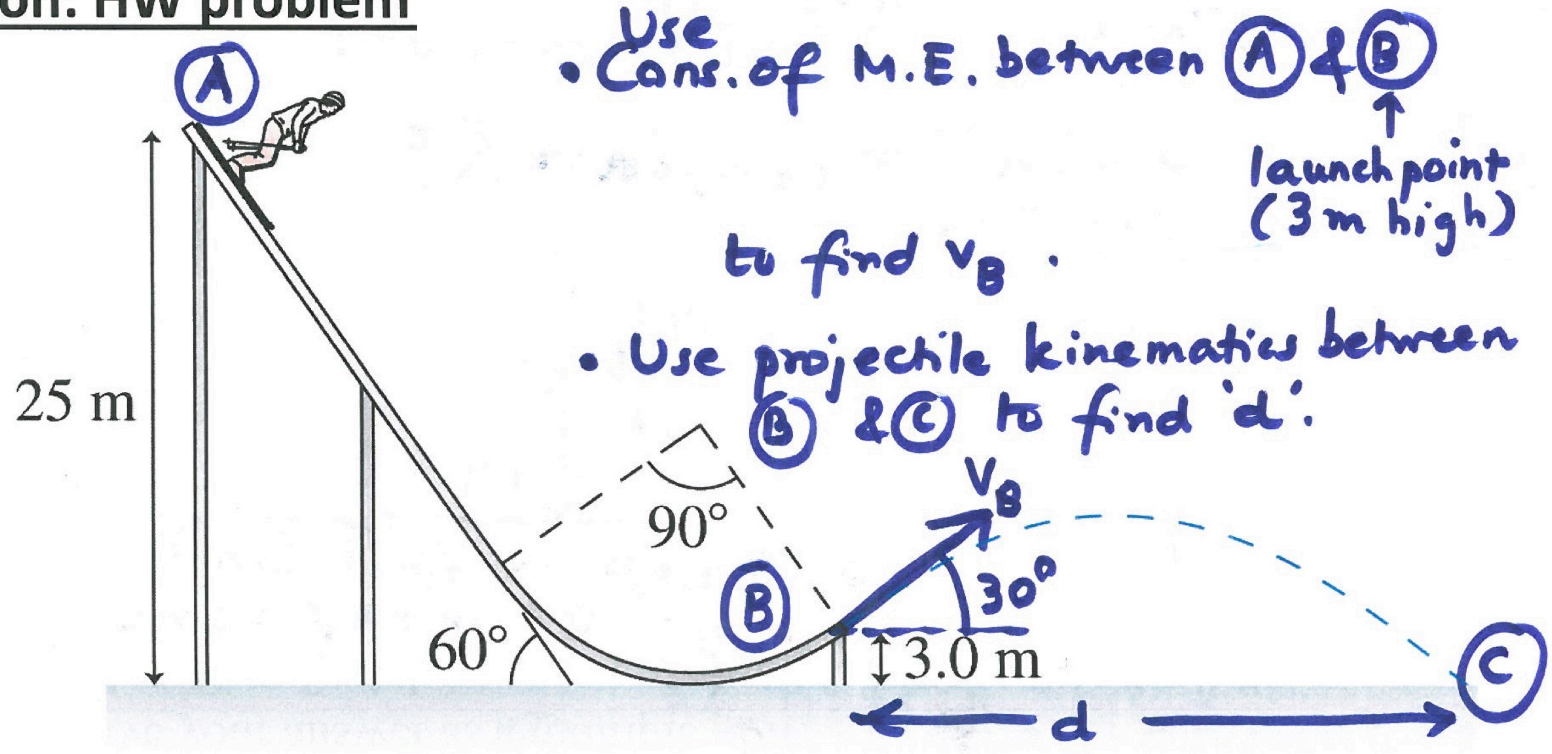
c) Set (a) = (b)

Discussion: HW problem



It's been a great day of new, frictionless snow. Julie starts at the top of the 60^0 slope shown in the figure. At the bottom, a circular arc carries her through a 90^0 turn, and she then launches off a 3.0 m high ramp. How far horizontally is her touchdown point from the end of the ramp?

Discussion: HW problem



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