physics

FOR SCIENTISTS AND ENGINEERS

a strategic approach

THIRD EDITION

randall d. knight

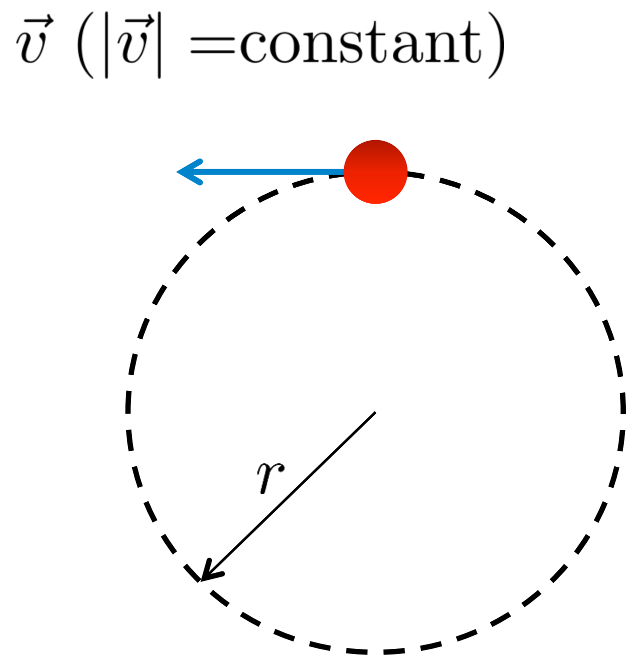
ALWAYS LEARNING

PEARSON

Chapter 8. Newton's Laws for Circular Motion

Chapter Goal: To learn how to apply Newton's Laws to circular motion.

Dynamics of Uniform Circular Motion (Sec 8.1 – 8.2)

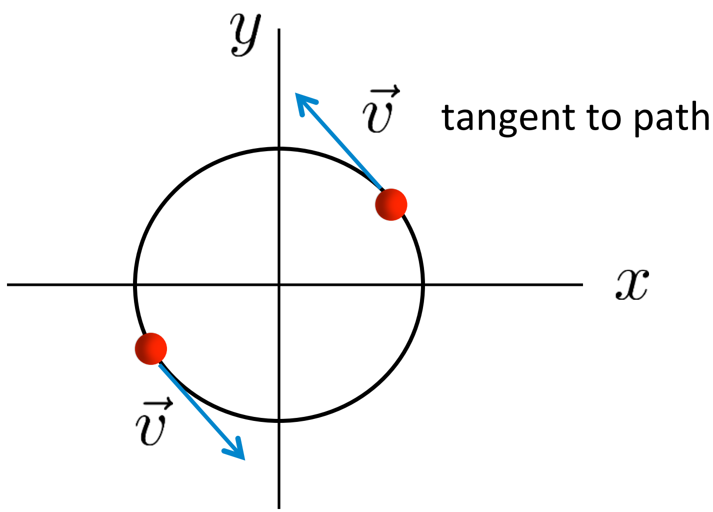


We've already studied the kinematics of UCM; let's review that now:

Uniform Circular Motion (Sec 8.1 – 8.2)

(from Chap 4)

Uniform Circular Motion is a special case of 2D motion where an object moves in a circular path at **constant speed**.



Note: The speed, $v =$ constant

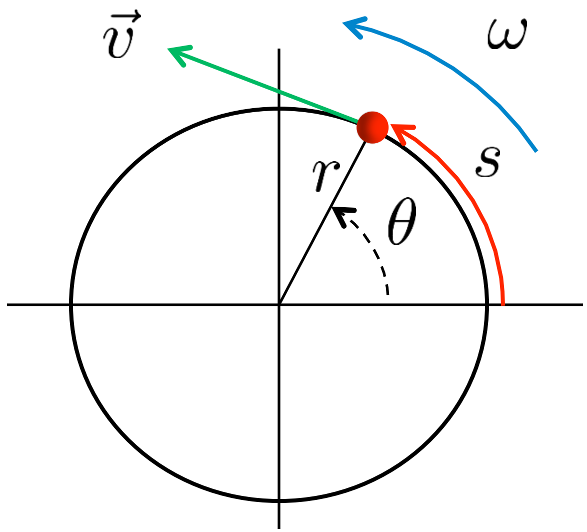
But, the velocity, $\vec{v} \neq$ constant

Thus, there must be an acceleration

Relations Between Linear and Angular Quantities for UCM

(from Chap 4)

For UCM:



$$\text{Arc Length } s = r\theta$$

$$\text{Speed, } v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\text{Period} = \text{time to complete one circle}$$

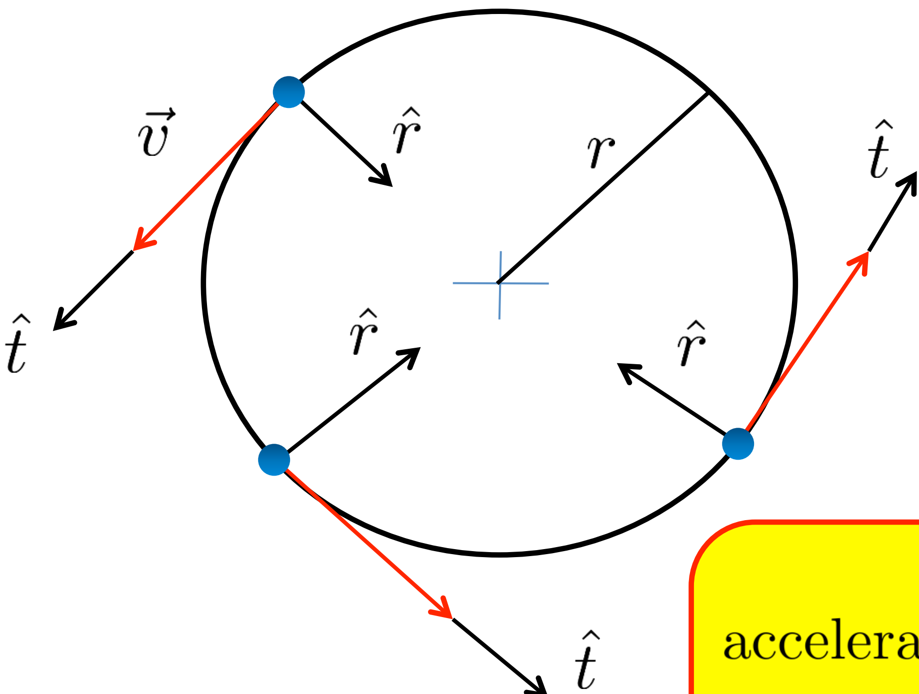
$$\text{Period } T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

(see equation sheet)

Acceleration for Uniform Circular Motion

(this is the really important part of this section) (from Chap 4)

For an object in uniform circular motion (UCM), the speed is constant, but the velocity continuously changes direction. **Thus, there must be an acceleration.**



$\hat{r} =$ The instantaneous radial direction, Always toward the center of the circle

$\hat{t} =$ The instantaneous tangential direction, Always tangent to the circle.

For UCM:

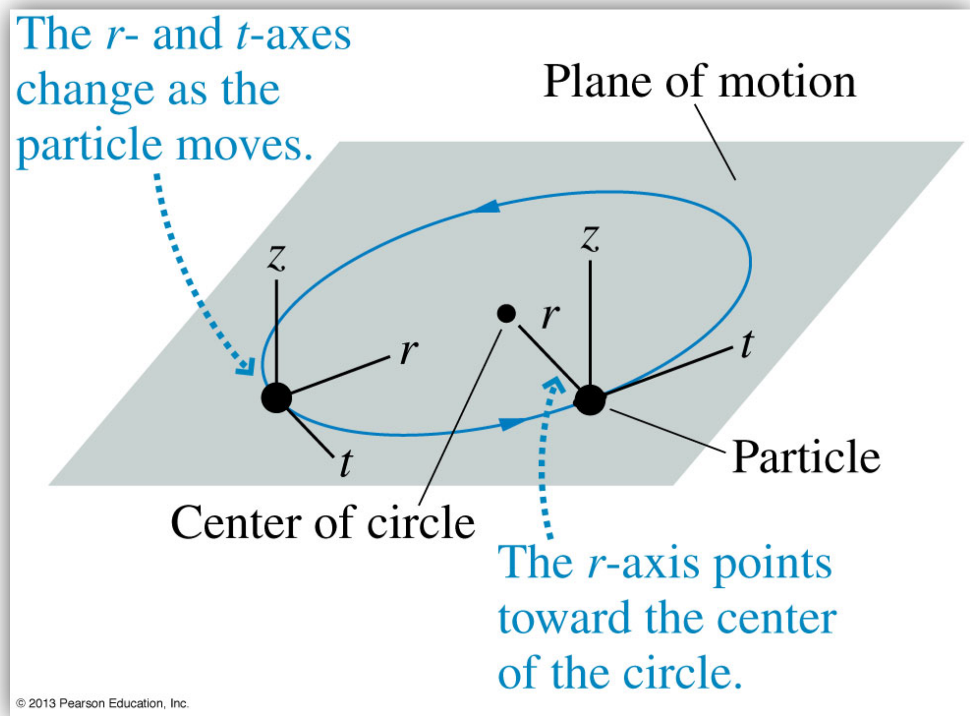
$\vec{v}$ is in the tangential direction

$\vec{a}$ is in the radial direction

$$\text{acceleration, } \vec{a} = \left(\frac{v^2}{r}, \text{towards the center } \underbrace{\text{(or radially inward)}}_{\text{"Centripetal"}} \right)$$

Dynamics of UCM (Sec 8.1 – 8.2)

To analyze UCM problems, we want to make full use of a convenient rotating Coordinate system that your author calls the *rtz-coordinate system*.

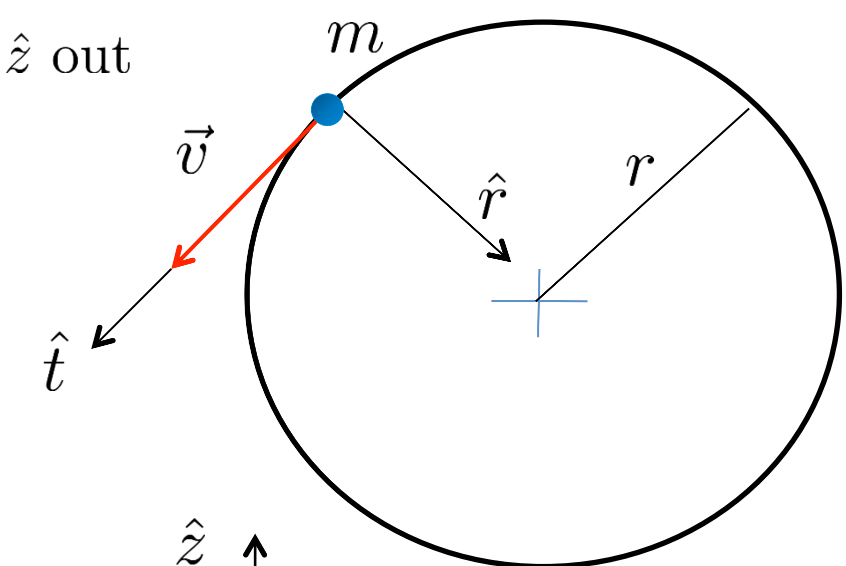


$$\hat{r} = \text{radial axis}$$
$$\hat{t} = \text{tangential axis}$$
$$\hat{z} \perp \hat{r} \text{ and } \hat{t}$$

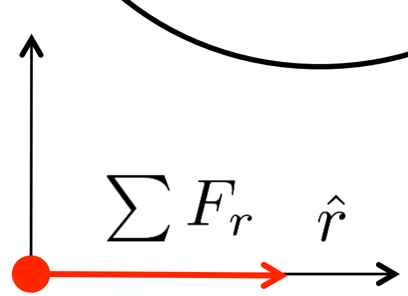
Note: for UCM problems, I put hats on the coordinate labels to distinguish them from other r, t, & z's

Also: for UCM problems, we aren't free to choose any coordinates; **the radial direction MUST always be toward the center of rotation.**

Dynamics of UCM (Sec 8.2)



FBD:



For UCM, must have:

$$a_r = \frac{v^2}{r}$$
$$a_t = 0$$
$$a_z = 0$$

$$\sum F_z = ma_z = 0$$
$$\sum F_t = ma_t = 0$$
$$\sum F_r = ma_r = \frac{mv^2}{r}$$

Therefore, for UCM, must be a **<u>centrally directed force</u>** (*the "centripetal force"*):

$$\vec{F} = \left(\frac{mv^2}{r}, \text{toward the center of rotation} \right)$$

But . . . <u>This is not a new force</u>! In problems, the centripetal force will be caused by familiar forces like friction, tension, normal force, etc.

Objective: To analyze *F* = *ma* for UCM (Sec 8.2)

Q: WHICH FORCES SUPPLY THE CENTRIPETAL ACCELERATION?

Pendulum, Ball rotating on string: [Weight, Tension]

Roller Coasters: Loop-da-loop!! [Normal force, Weight]

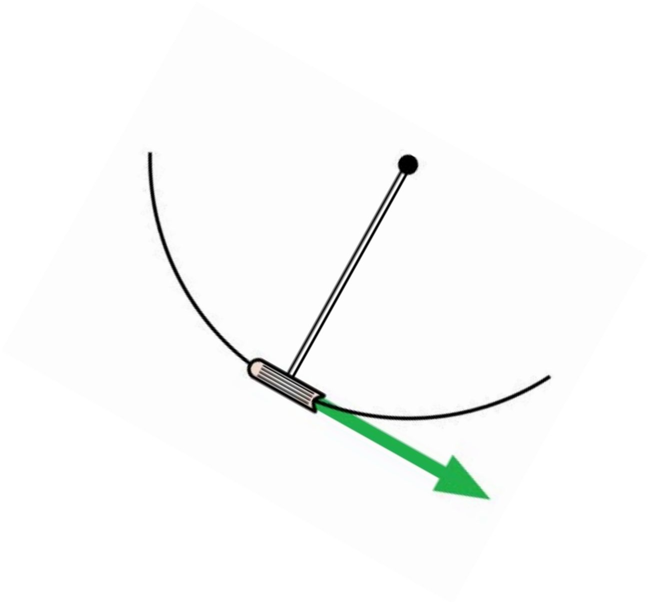
Water in bucket: Let's get wet! [Normal force, Weight]

Human Centrifuge at COSI: Atomic Wedgie [Normal force]

Making a turn: Why banked highways? [Friction, Normal force]

Whiteboard problem 8-1 (Sec 8.2 – 8.4)

A physics textbook is tied to a piece of string and swings back and forth as a pendulum. Draw the free-body diagram for the book at the point shown.



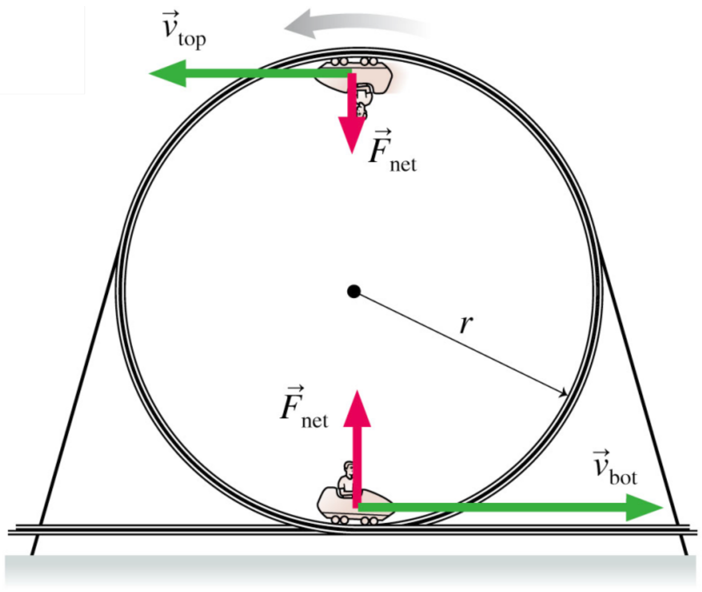
Whiteboard problem 8-2: Draw f.b.d. at top & bottom of loop

(Sec 8.2, 8.4, 8.5)

The figure shows a roller-coaster going around a vertical loop-the-loop of radius r.

<u>Q: What's the minimum speed required at the top **in order to barely complete the circle**?</u>

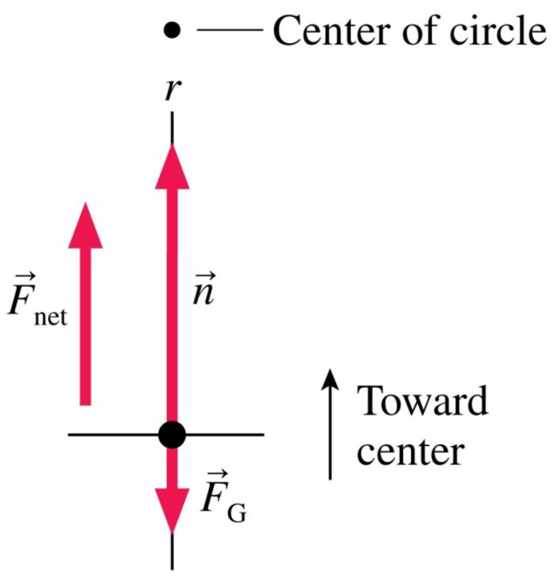
WATCH DEMO



Because the car is moving in a circle, there must be a net force toward the center of the circle.

Loop-the-Loop: What's going on at the bottom? (Sec 8.4 - 8.5)

- The figure shows the roller-coaster free-body diagram at the *bottom* of the loop.
- Since the net force is toward the center (upward at this point), $n > F_G$.
- This is why you "feel heavy" at the bottom of the valley on a roller coaster.

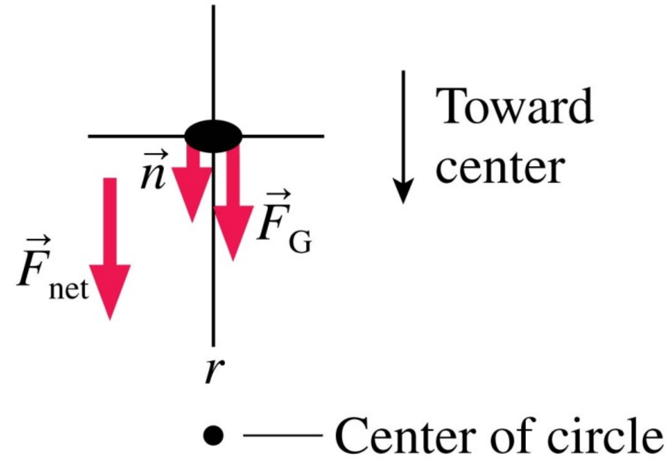


$$\sum F_r = n_r + (F_G)_r = n - mg = ma_r = \frac{m(v_{bot})^2}{r}$$

$$n = mg + \frac{m(v_{bot})^2}{r}$$

- The normal force at the bottom is *larger* than mg.

Loop-the-Loop: What's going on at the top? (Sec 8.4 - 8.5)



- The figure shows the roller-coaster free-body diagram at the *top* of the loop.
- The track can only push on the wheels of the car, it cannot pull, therefore $\vec{n}$ presses *downward.*
- The car is still moving in a circle, so the net force is also downward:

$$\sum F_r = n_r + (F_G)_r = n + mg = \frac{m(v_{top})^2}{r}$$

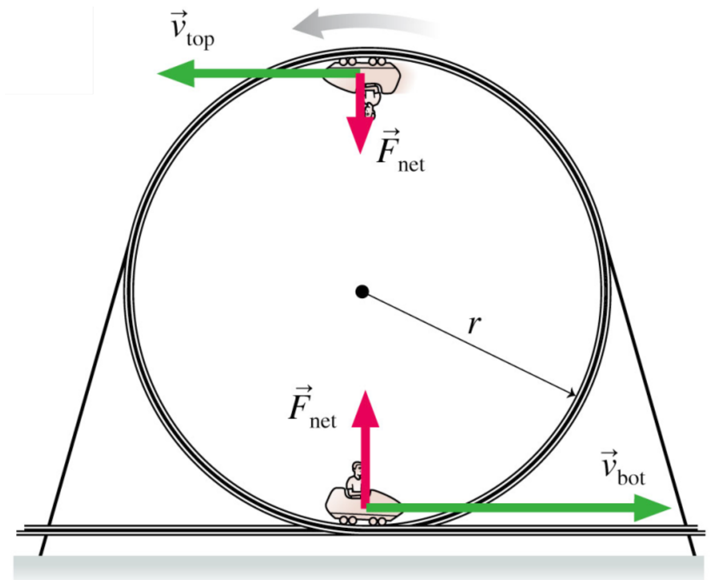
$$n = \frac{m(v_{top})^2}{r} - mg$$

- The normal force at the at the top can exceed mg if v_{top} is large enough.

Loop-the-Loop (Sec 8.4 - 8.5)

- At the top of the roller coaster, the normal force of the track on the car is:

$$n = \frac{m(v_{\text{top}})^2}{r} - mg$$



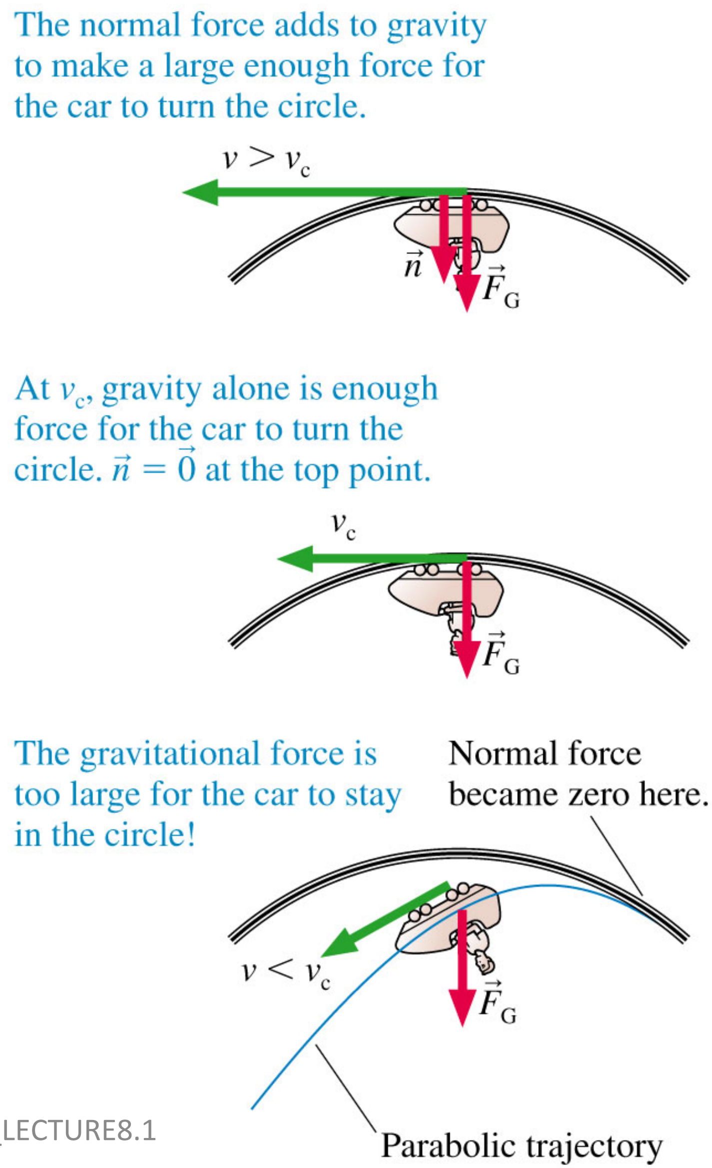
- As v_{top} decreases, there comes a point when *n* reaches zero.
- The speed at which $n = 0$ is called the *critical speed*:

$$v_c = \sqrt{\frac{rmg}{m}} = \sqrt{rg}$$

- This is the slowest speed at which the car can complete the circle without falling off the track near the top.

Loop-the-Loop (Sec 8.4 – 8.5)

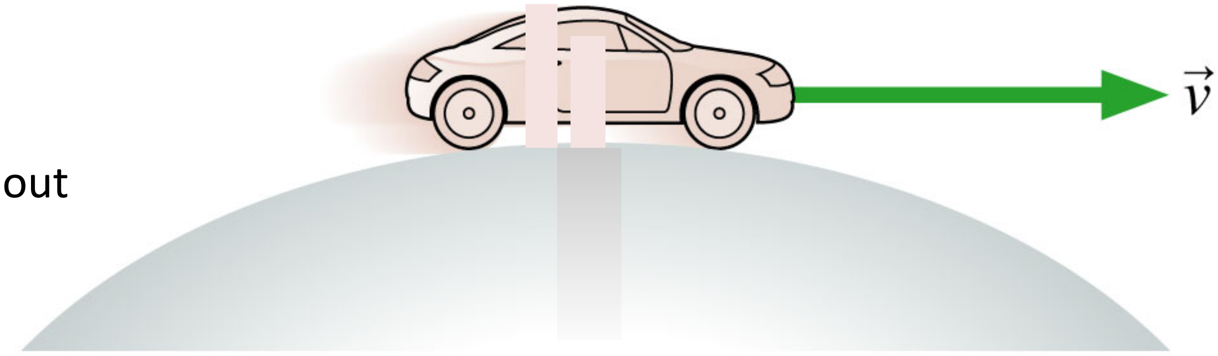
A roller-coaster car at the top of the loop.



Loop-the-Loop: Stop-to-think (Sec 8.4 – 8.5)

An out-of-gas car is rolling over the top of the hill at speed v. At this instant,

a. $n > F_g$
b. $n < F_g$
c. $n = F_g$
d. We can't tell about n without knowing v

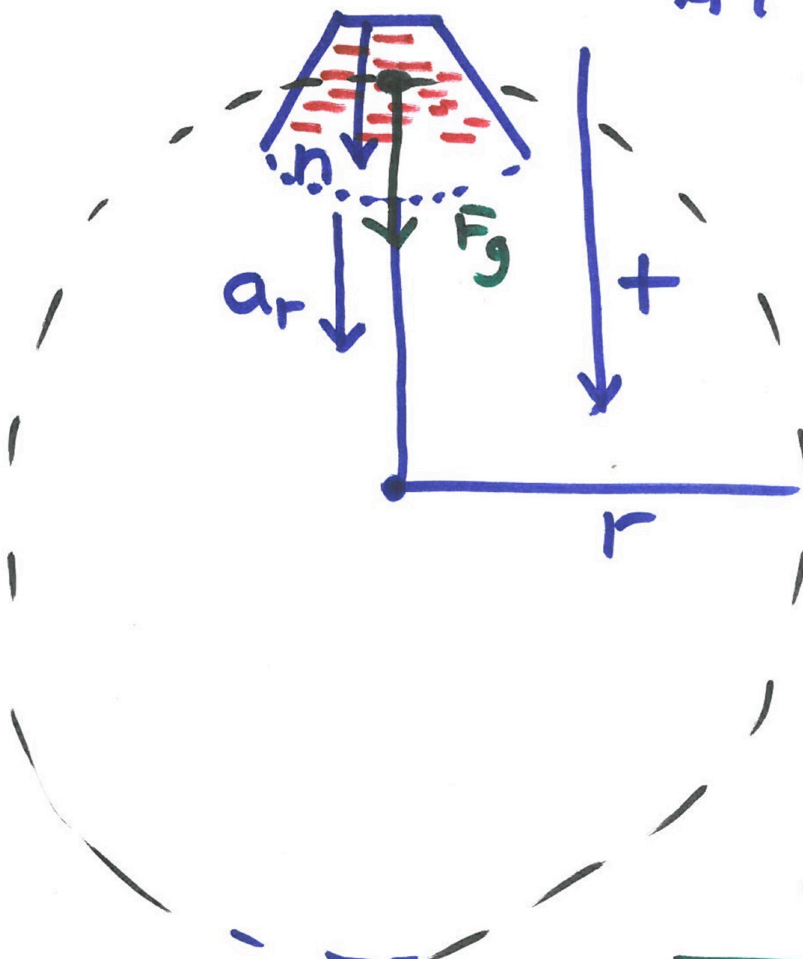


Q: What keeps the water in the bucket? (Sec 8.4)

WATCH DEMO. Draw f. b. d. at the top.

Q: What's the minimum velocity required at the top to complete the circle w/o water falling?

LOOP-D A-LOOP



AT TOP: $F_{net} = ma$

$$n + F_g = ma_r = \frac{mv^2}{r}$$

$$n + mg = \frac{mv^2}{r}$$

WHAT'S MINIMUM VELOCITY AT TOP REQUIRED TO COMPLETE CIRCLE?

THAT OCCURS WHEN

$$n = 0$$

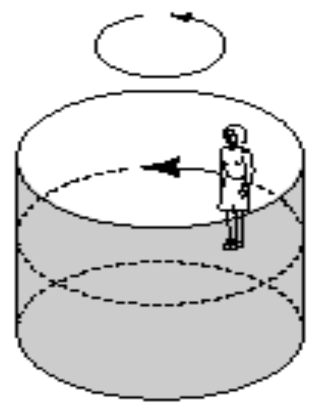
THEN $\frac{m v_{min}^2}{r} = mg$

$$v_{min}^2 = gr$$

$$v_{min} = \sqrt{gr}$$

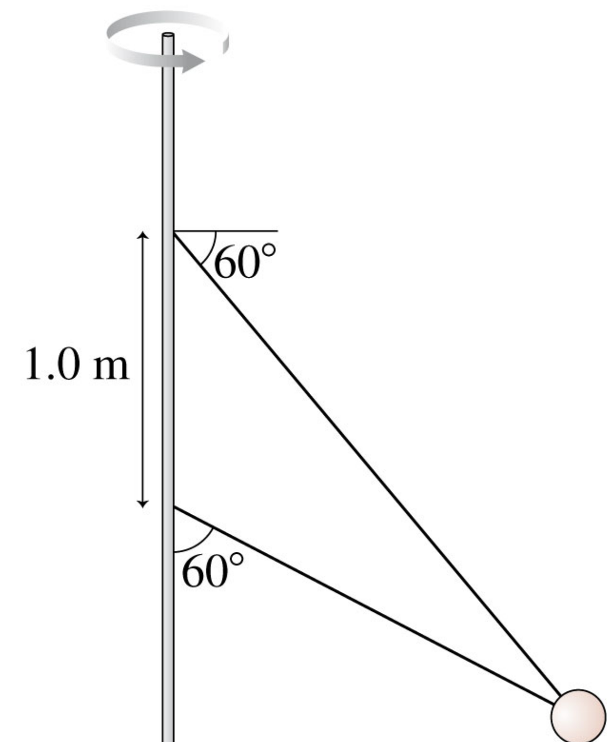
Whiteboard Problem 8.3 (Sec 8.4)

You are pinned to the wall in the Human Centrifuge at COSI. In other words, your back is pinned to the wall of a rapidly rotating cylinder such that you don't fall even when the floor suddenly drops away beneath your feet! **Draw your f.b.d.**



Whiteboard Problem 8.4, Problem 8-45

Two wires are tied to the 2.0 kg sphere as shown, and the sphere is revolved in a horizontal circle at constant speed *such that the tension in the wires is the same*. **Find the tension in the wires.**



Whiteboard Problem 8-5

EXAMPLE 8.3 **Turning the corner I**

What is the maximum speed with which a 1500 kg car can make a left turn around a curve of radius 50 m on a level (unbanked) road without sliding? ——— **(Draw the FBD)**

MODEL Although the car turns only a quarter of a circle, we can model the car as a particle in uniform circular motion as it goes around the turn. Assume that rolling friction is negligible.

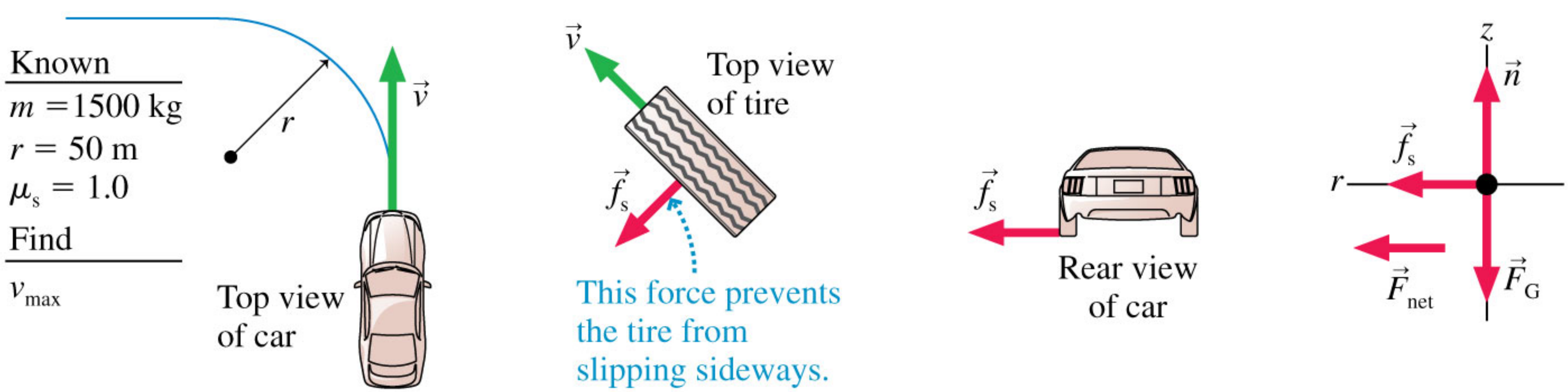
For rubber on concrete the coefficient of static friction = 1.0

Here's a guy on a motorcycle who didn't do this calculation.

WB 8.5 Making a turn on a flat road (Sec 8.4)

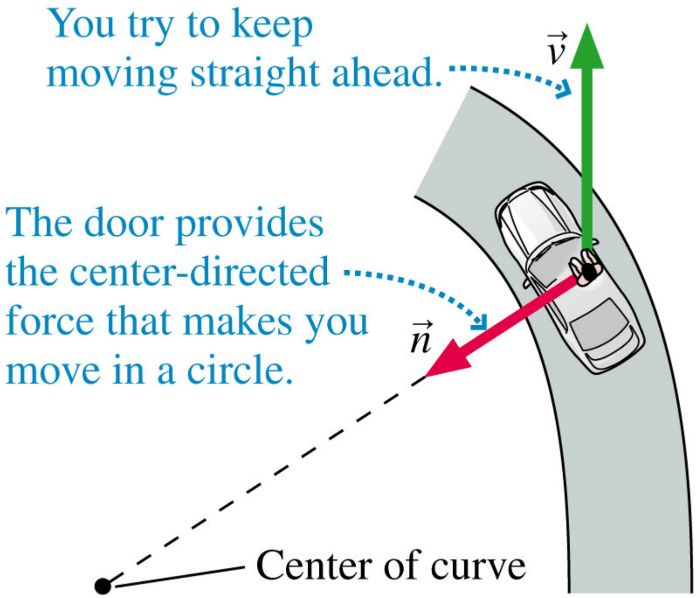
VISUALIZE

- The second figure below shows the top view of a tire as it turns a corner.
- The force that prevents the tire from sliding across a surface is *static friction*.
- Static friction pushes sideways on the tire, perpendicular to the velocity, since the car is not speeding up or slowing down.
- The free-body diagram, drawn from behind the car, shows the static friction pointing toward the center of the circle.



Centrifugal Force? Is it real? Nope. (Sec 8.4)

- The figure shows a bird's-eye view of you riding in a car as it makes a left turn.
- The normal force from the door points *inward*, keeping you on the road with the car.



- You feel pushed toward the *outside* of the curve.
- The fictitious force which seems to push an object to the outside of a circle is called the **centrifugal force**.
- There really is no such force as a centrifugal force.

WB 8.5 Making a turn on a flat road (Sec 8.4)

EXAMPLE 8.4 Turning the corner I

SOLVE The maximum turning speed is reached when the static friction force reaches its maximum $f_{s\,max} = \mu_s n$. If the car enters the curve at a speed higher than the maximum, static friction will not be large enough to provide the necessary centripetal acceleration and the car will slide.

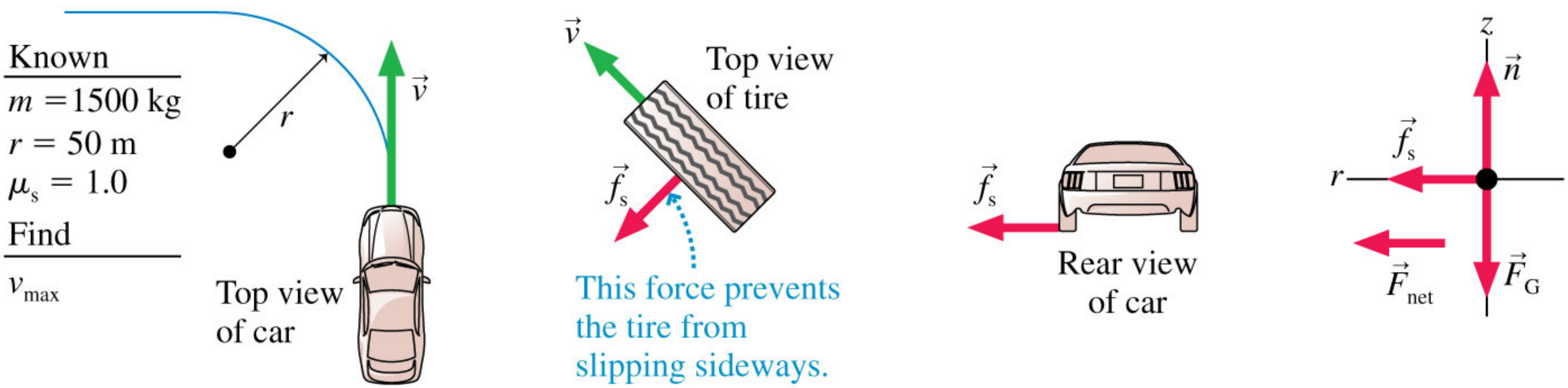
The static friction force points in the positive r-direction, so its radial component is simply the magnitude of the vector: $(f_s)_r = f_s$. Newton's second law in the rtz-coordinate system is

$$\sum F_r = f_s = \frac{mv^2}{r}$$

$$\sum F_z = n - mg = 0$$

From the radial equation, the speed is

$$v = \sqrt{\frac{rf_s}{m}}$$



WB 8.5 Making a turn on a flat road (Sec 8.4)

EXAMPLE 8.4 **Turning the corner I**

The speed will be a maximum when f_s reaches its maximum value:

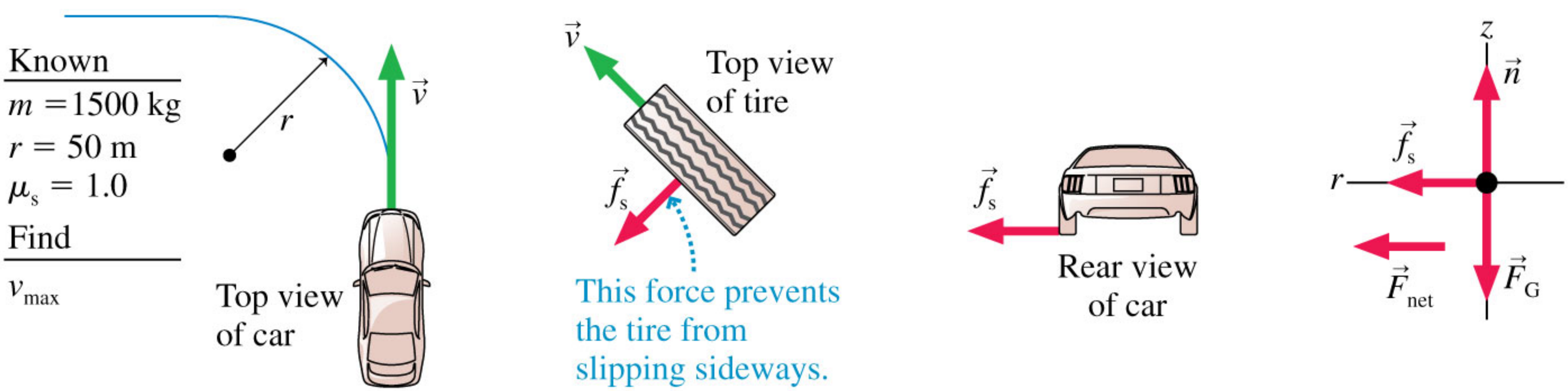
$$f_s = f_{s\,max} = \mu_s n = \mu_s mg$$

where we used $n = mg$ from the z-equation. At that point,

$$v_{max} = \sqrt{\frac{r f_{s\,max}}{m}} = \sqrt{\mu_s r g}$$

$$= \sqrt{(1.0)(50\text{ m})(9.80\text{ m/s}^2)} = 22\text{ m/s}$$

where the coefficient of static friction was taken from Table 6.1.



WB 8.5 Making a turn on a flat road (Sec 8.4)

EXAMPLE 8.4 Turning the corner I

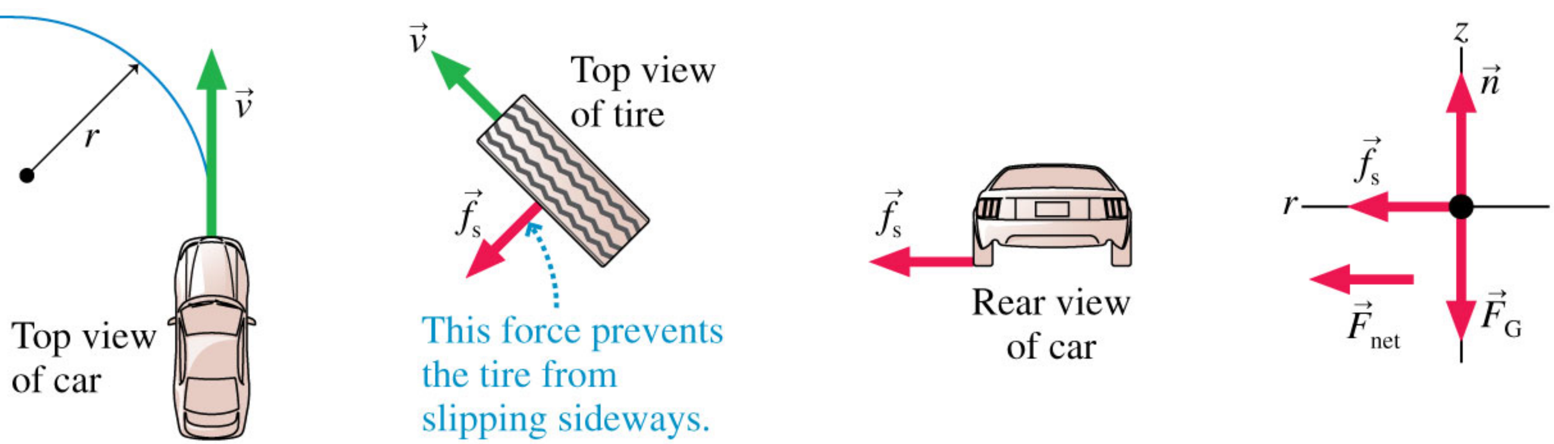
ASSESS 22 m/s ≈ 45 mph, a reasonable answer for how fast a car can take an unbanked curve. Notice that the car's mass canceled out and that the final equation for v_{max} is quite simple. This is another example of why it pays to work algebraically until the very end.

Known

$m = 1500$ kg
$r = 50$ m
$\mu_s = 1.0$

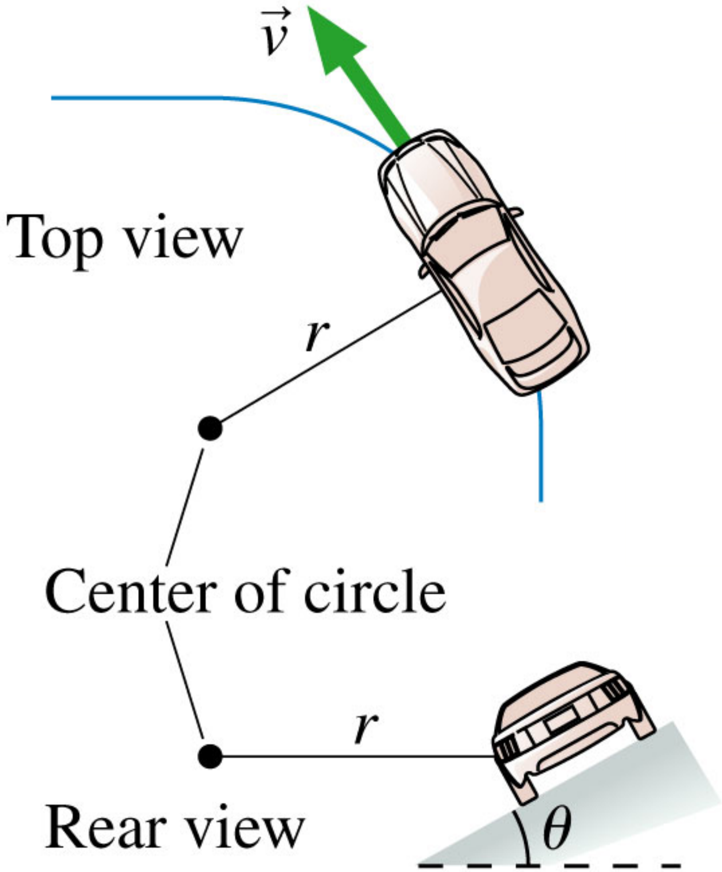
Find

v_{max}

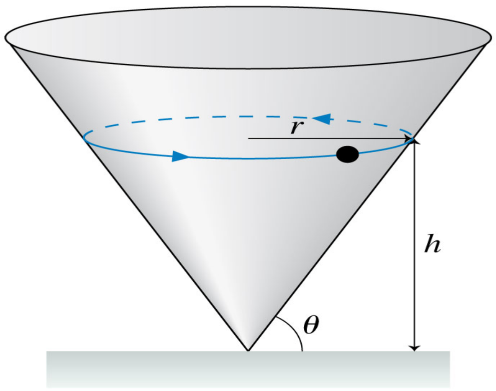


Whiteboard Problem 8.6: Banked highways (Sec 8.4)

Draw the f. b. d. for a car rounding the curve at the maximum possible speed without slipping.

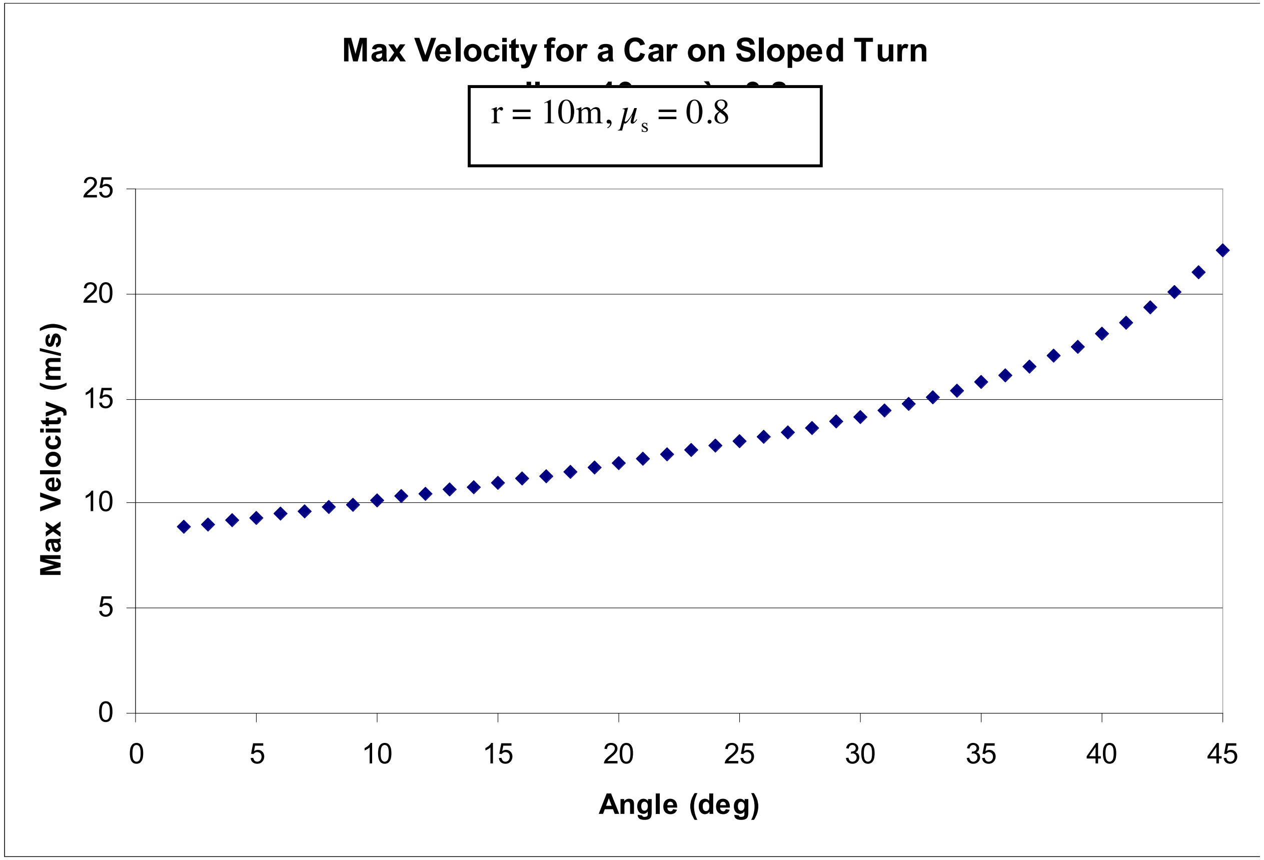


Important Point:
Where is the circle?



Q: What about rounding the curve @ the *minimum* possible speed?

How Banking helps… (Sec 8.4)



Nonuniform Circular Motion: Review Kinematics (Sec 8.5)

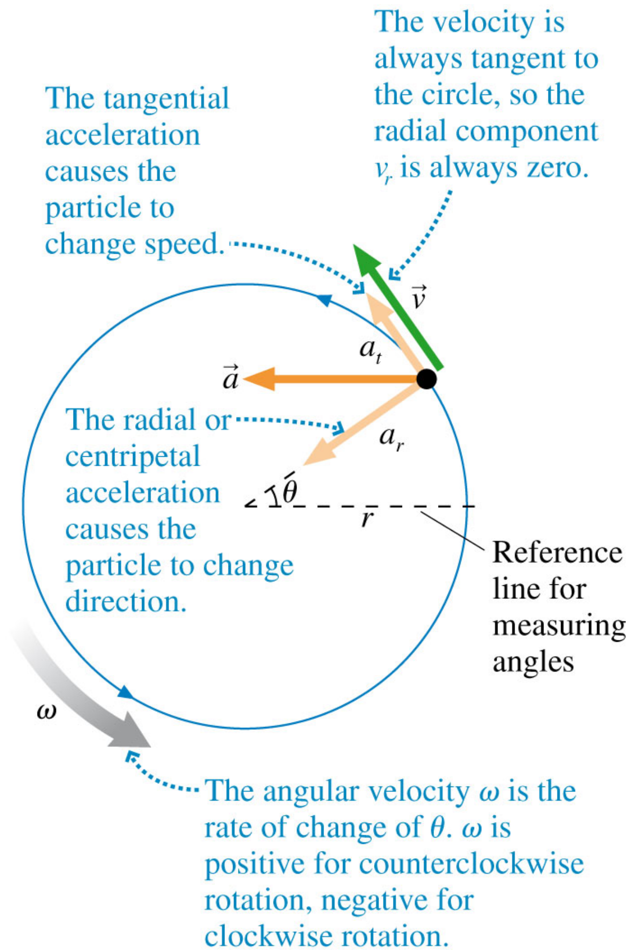
- The particle in the figure is speeding up as it moves around the circle.
- The tangential acceleration is:

$$a_t = \frac{dv_t}{dt}$$

$$a_t = r\alpha$$

- The centripetal acceleration is:

$$a_r = v^2/r = \omega^2 r$$



Nonuniform Circular Motion: Review Kinematics (Sec 8.5)

- In terms of angular quantities, the equations of constant-acceleration kinematics are:

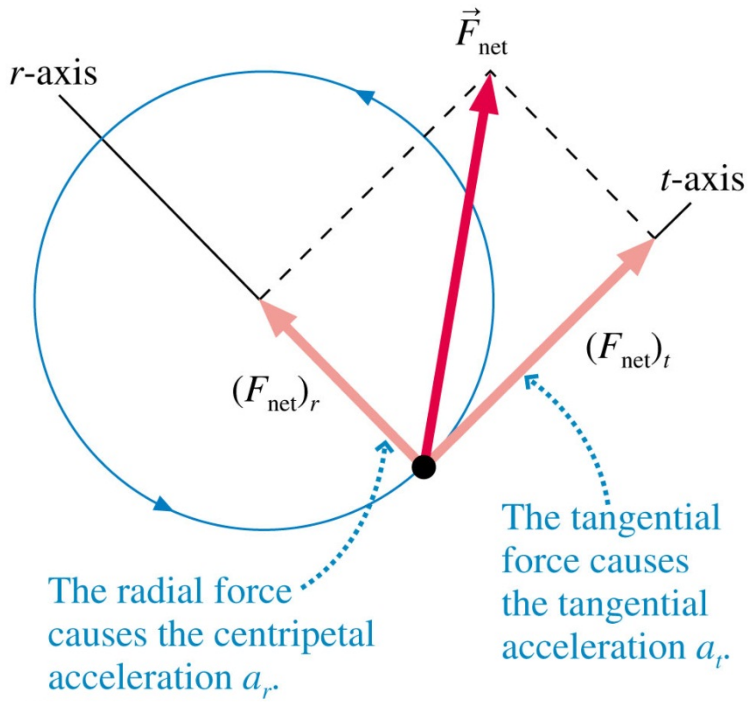
$$\theta_f = \theta_i + \omega_i \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

Dynamics of Nonuniform Circular Motion (Sec 8.5)

- A net force $\vec{F}_{\text{net}}$ is applied to a particle moving in a circle.
- $\vec{F}_{\text{net}}$ is likely to be a superposition of several forces, such as tension, thrust, friction, etc.



- The *tangential* component of the net force $(F_{\text{net}})_t$ creates a tangential acceleration and causes the particle to change speed.
- The *radial* component $(F_{\text{net}})_r$ is directed toward the *center*, creates a centripetal acceleration, and causes the particle to change direction.

Dynamics of Nonuniform Circular Motion

- Force and acceleration are related through Newton's second law:

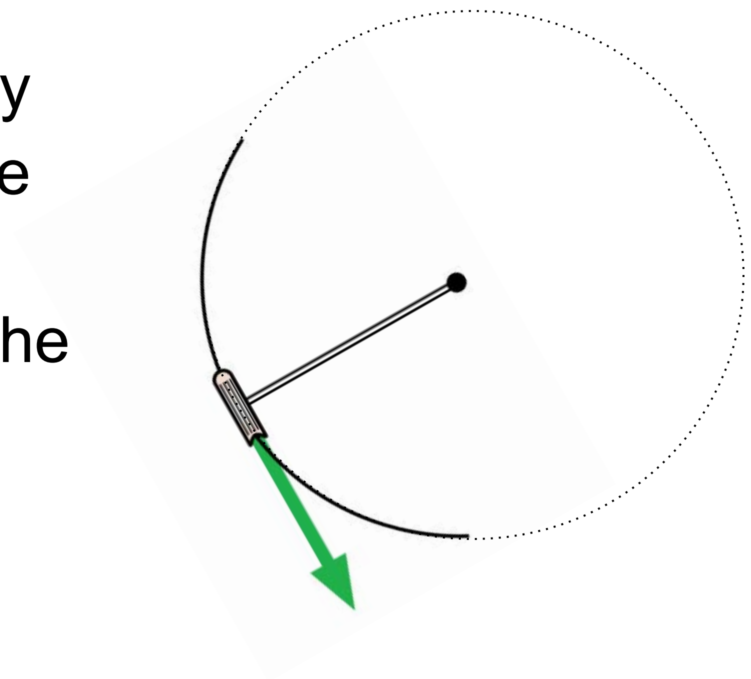
$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = ma_t$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

Example of non-uniform circular motion: Vertical Circle (Sec 8.5)

A physics textbook is tied to a piece of string and swung rapidly around in a vertical circle. Where do you think the string is most likely to break? By considering the free-body diagram for the book, can you explain why?



Whiteboard Problem 8.7: Problem 8-63 (Sec 8.5)

A 2.0 kg ball swings in a vertical circle on the end of an 80-cm-long string. The tension in the string is 20N when its angle from the highest point on the circle is $\theta = 30^0$.

a) What is the ball's speed when $\theta = 30^0$?

b) What are the magnitude & direction of the ball's acceleration when $\theta = 30^0$?

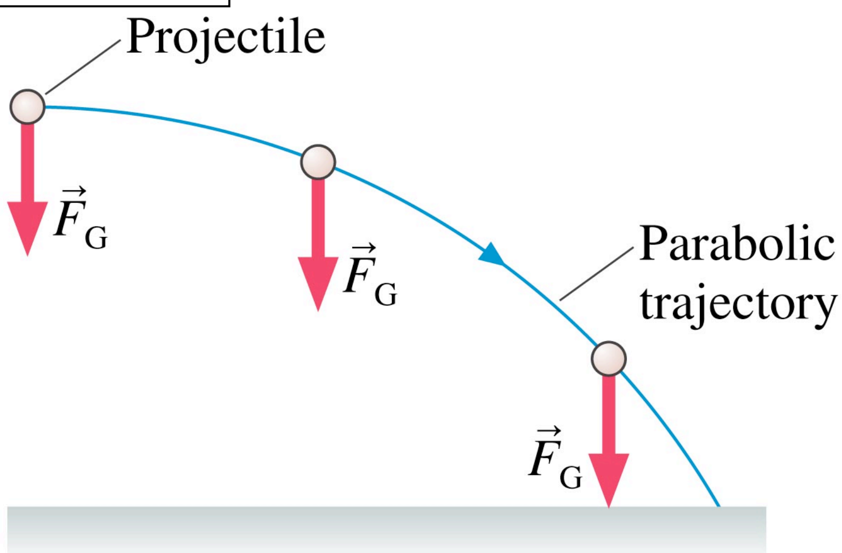
Gravity and Orbits: Newton's Law of Gravitation (Sec 8.3)

AN IMPORTANT FUNDAMENTAL APPLICATION OF NEWTON'S IDEAS

Newton's Law of Gravitation

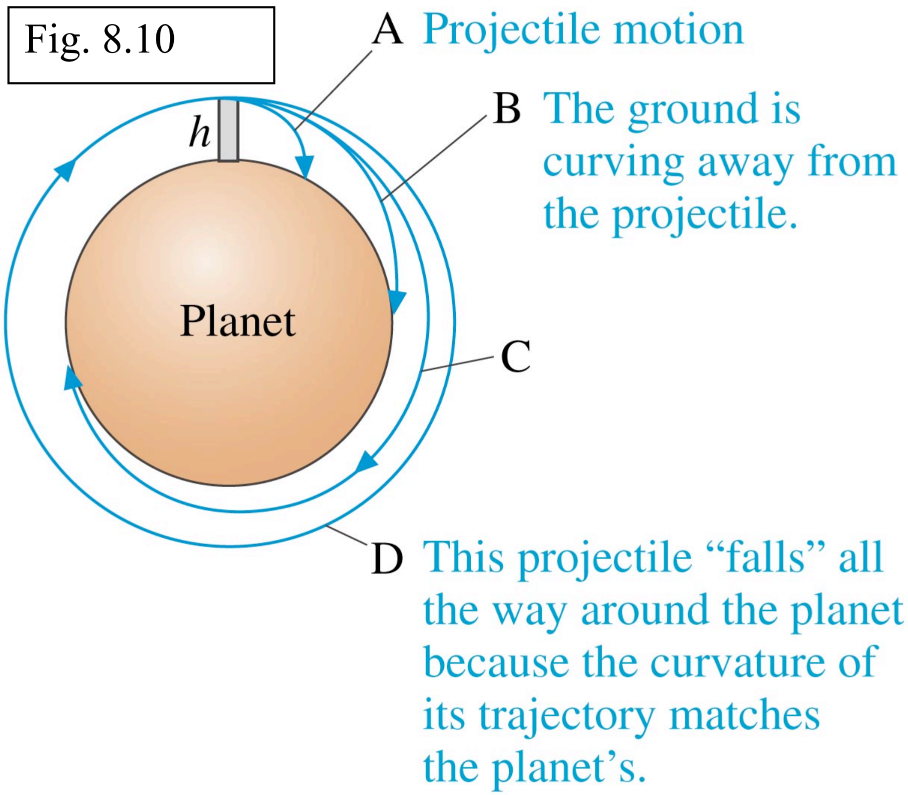
Q1: Imagine a space shuttle orbiting the Earth. Why are the astronauts inside weightless? Is it because there's no gravity there? NO! Well, then what's the reason?
Q2: Where does $g = 9.8\ \text{m/s}^2$ come from? From observations and measurement, you say. Yes, but what's the fundamental origin? By answering this question, we also understand why, and at what rate, the value of g steadily decreases with height.

Fig. 8.11 a)



Flat-earth approximation

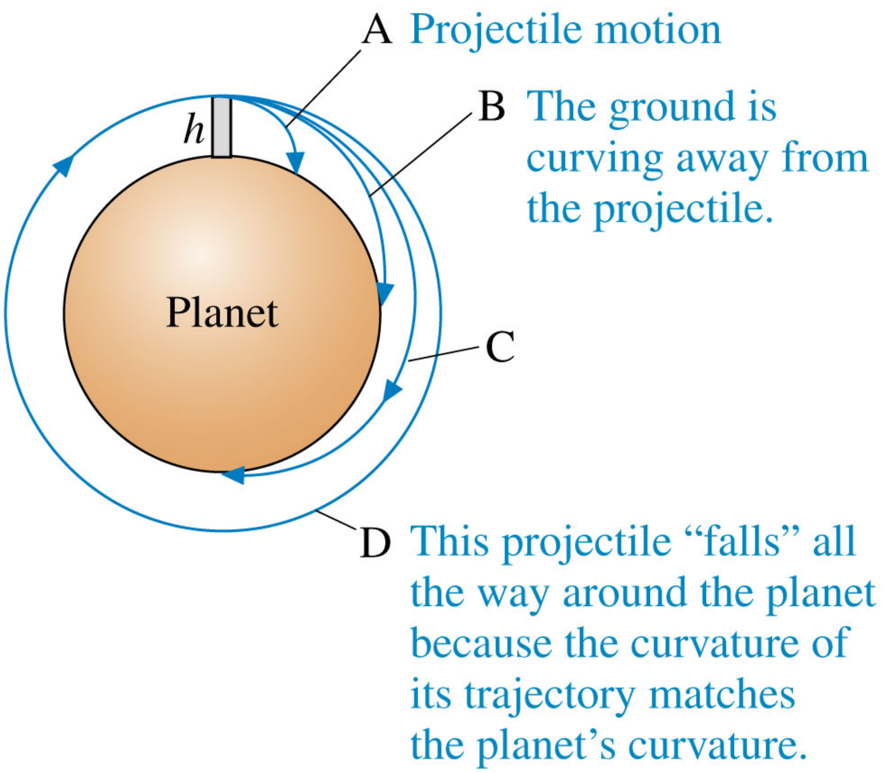
Fig. 8.10



So…the shuttle and the astronauts are always *in free fall* toward the center of the Earth! That's why they are weightless!!

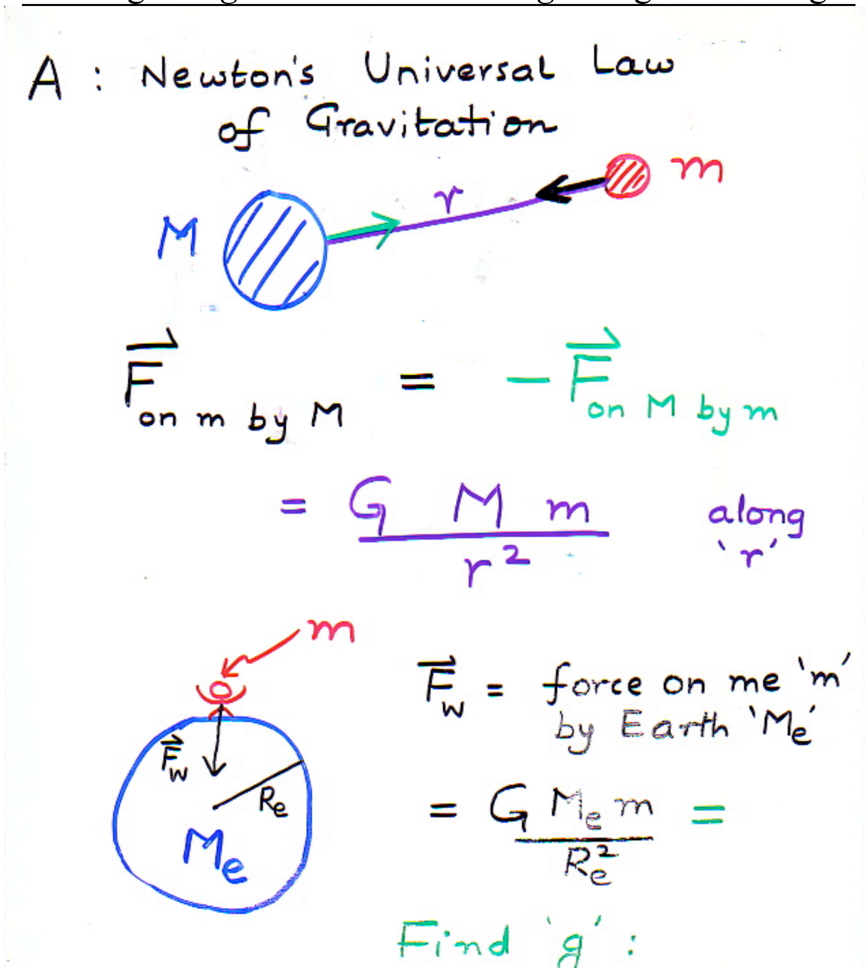
Circular Orbits (Sec 8.3)

- As the initial speed v_0 is increased, the range of the projectile increases as the ground curves away from it.
- Trajectories B and C are of this type.
- If v_0 is sufficiently large, there comes a point where the trajectory and the curve of the earth are parallel.
- In this case, the projectile "falls" but it never gets any closer to the ground!
- This is trajectory D, called an **orbit**.



The origin of *g* and how *g* changes with height (Sec 8.3 – 8.4)

The origin of $g = 9.8$ m/s^2 and how g changes with height



Gravity decreases with altitude. An increase in altitude from sea level to the top of Mount Everest (8,850m) causes a weight decrease of about 0.28%. It is a common misconception that astronauts in orbit are weightless because they have flown high enough to "escape" the Earth's gravity. In fact, at an altitude of 400 km (250 miles), equivalent to a typical orbit of the Space Shuttle, gravity is still nearly **90% as strong as at the Earth's surface**, and weightlessness actually occurs because orbiting objects are in free-fall.

Useful astronomical data for understanding "Origin of g":

TABLE 13.2 Useful astronomical data

Planetary body	Mean distance from sun (m)	Period (years)	Mass (kg)	Mean radius (m)
Sun	—	—	1.99×10^{30}	6.96×10^{8}
Moon	3.84×10^{8}*	27.3 days	7.36×10^{22}	1.74×10^{6}
Mercury	5.79×10^{10}	0.241	3.18×10^{23}	2.43×10^{6}
Venus	1.08×10^{11}	0.615	4.88×10^{24}	6.06×10^{6}
Earth	1.50×10^{11}	1.00	5.98×10^{24}	6.37×10^{6}
Mars	2.28×10^{11}	1.88	6.42×10^{23}	3.37×10^{6}
Jupiter	7.78×10^{11}	11.9	1.90×10^{27}	6.99×10^{7}
Saturn	1.43×10^{12}	29.5	5.68×10^{26}	5.85×10^{7}
Uranus	2.87×10^{12}	84.0	8.68×10^{25}	2.33×10^{7}
Neptune	4.50×10^{12}	165	1.03×10^{26}	2.21×10^{7}

*Distance from earth.

The origin of *g* and how *g* changes with height (Sec 8.3 – 8.4)

TABLE 13.1 Variation of g with height above the ground

Height h	Example	g (m/s^2)
0 m	ground	9.83
4500 m	Mt. Whitney	9.82
10,000 m	jet airplane	9.80
300,000 m	space shuttle	8.90
35,900,000 m	communications satellite	0.22

Gravity decreases with altitude. An increase in altitude from sea level to the top of Mount Everest (8,850m) causes a weight decrease of about 0.28%. It is a common misconception that astronauts in orbit are weightless because they have flown high enough to "escape" the Earth's gravity. In fact, at an altitude of 400 km (250 miles), equivalent to a typical orbit of the Space Shuttle, gravity is still nearly **90% as strong as at the Earth's surface**, and weightlessness actually occurs because orbiting objects are in free-fall.