Chapter 4 Lecture

physics

FOR SCIENTISTS AND ENGINEERS

a strategic approach

THIRD EDITION

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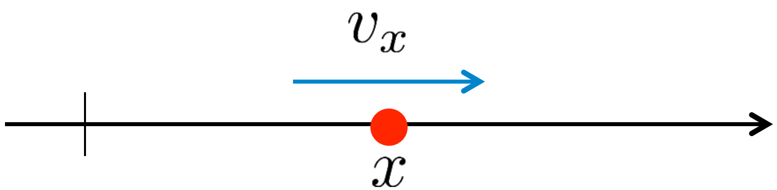
QUICK REVIEW – What we've done so far…

A quick review:

So far, we've looked at 1D motion along the x-axis (Chapter 2):



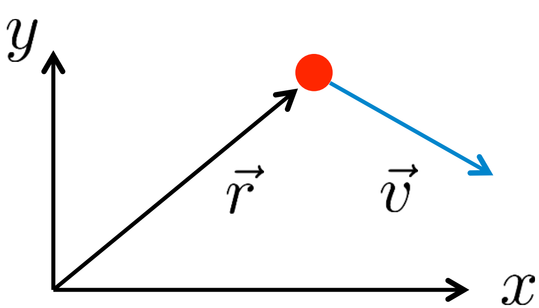
The kinematic equations for motion with constant acceleration

$$v_{fs} = v_{is} + a_s \Delta t$$

$$s_f = s_i + v_{is} \Delta t + \frac{1}{2} a_s (\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

Then, we extended this to 2D (Chapter 4, Sections 4.1 – 4.2):



x-motion

$$x_f = x_i + v_{x_i} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$v_{x_f} = v_{x_i} + a_x \Delta t$$

$$v_{x_f}^2 = v_{x_i}^2 + 2a_x \Delta x$$

$$\Delta x = x_f - x_i$$

$$\Delta t = t_f - t_i$$

y-motion

$$y_f = y_i + v_{y_i} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$v_{y_f} = v_{y_i} + a_y \Delta t$$

$$v_{y_f}^2 = v_{y_i}^2 + 2a_y \Delta x$$

$$\Delta y = y_f - y_i$$

For projectiles: $a_x = 0$, $a_y = g$ downward

Now consider **Circular Motion** (Sec. 4.4-4.6)

Circular Motion: Another important example of 2D motion

- Consider a ball on a roulette wheel.
- It moves along a circular path of radius r.
- Other examples of circular motion are a satellite in an orbit or a ball on the end of a string.

Preview of Terminology for Circular Motion

<u>2D Motion, continued…Circular Motion (Sections 4.4 – 4.6)</u>

The Language of Circular Motion

- Angular quantities like ω, *a*, θ, etc.
- Radian

Circular motion with constant angular acceleration

Very similar to the usual equations for "linear motion with constant acceleration" you know and love

Tangential acceleration and…something new…``RADIAL acceleration"

- Radial acceleration is a.k.a "Centripetal" acceleration
- Circular motion with constant angular velocity…is there any acceleration? Answer: YES!

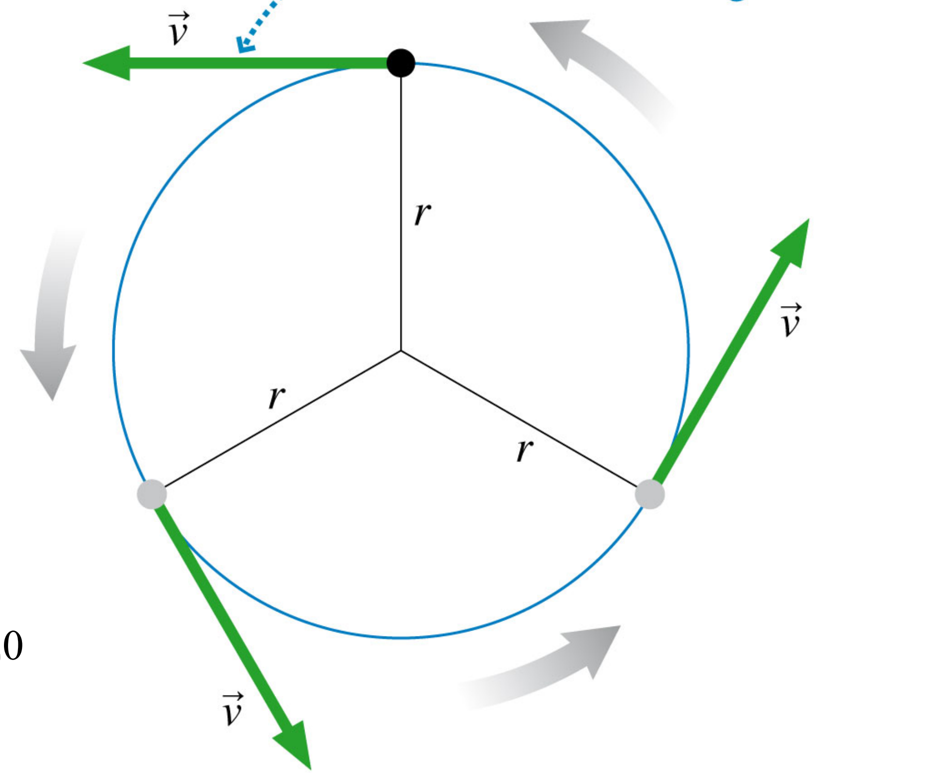
Total acceleration for particle in circular motion

First, look at **UNIFORM** Circular Motion (Sec 4.4)

- To begin the study of circular motion, consider a particle that moves at *constant speed* around a circle of radius r.
- This is called **uniform circular motion**.
- The time interval to complete one revolution is called the period, T.
- The period T is related to the speed v:

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$

$$2\pi \text{ radians} = 1 \text{ full revolution} = 360^0$$



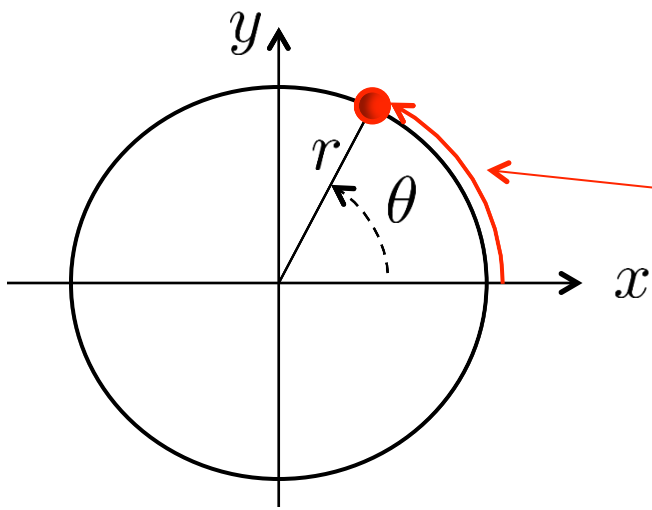
KEY POINT: The speed, $v = \text{constant}$

But, the velocity, $\vec{v} \neq \text{constant}$

Thus, there must be an acceleration

Angular Position and Velocity (Sec 4.4)

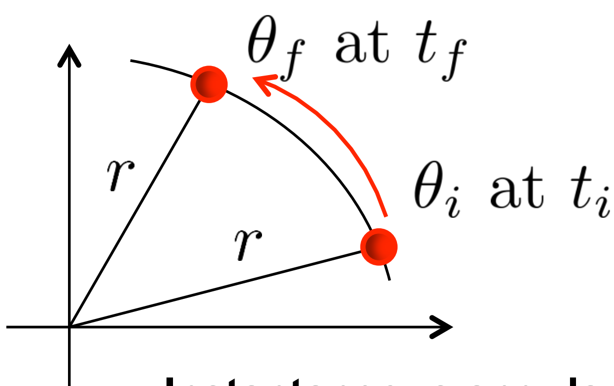
Circular motion can be described using x-y Cartesian coordinates, but it's messy. Angular coordinates work a lot better.



$$\text{Coordinates: } (r, \theta)$$

$$s = \text{arc length} = r\theta$$

where θ must be in radians



<u>Average angular velocity</u>
between points i and f:

Displacement in radial form

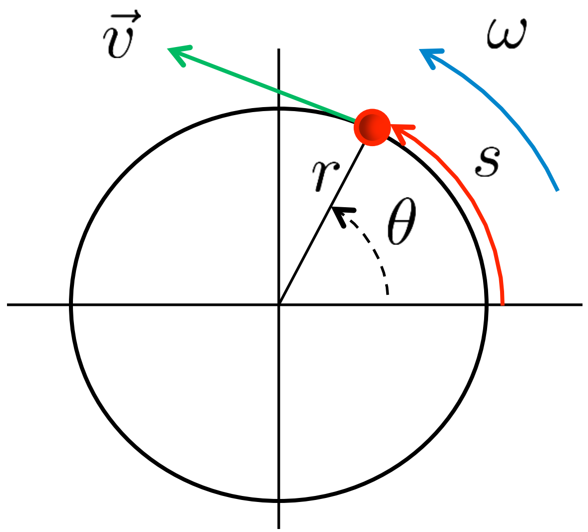
$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

(Units are radians/second)

<u>Instantaneous angular velocity</u>: $\omega = \dfrac{d\theta}{dt}$

(Sign Convention: CCW = positive and CW = negative)

For Uniform Circular Motion:



Quantity	Linear	Angular
Position	s	$s = r\theta$
Displacement	Δs	$\Delta\theta$
Speed	$v = \frac{ds}{dt}$	$v_t = r\frac{d\theta}{dt} = r\omega$

?

$$\text{Period} = \text{time to complete one circle}$$

$$\text{Period } T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

$$\left[\frac{\text{m}}{\text{m/s}} = \text{s}\right]$$

Graphs can be interpreted in similar fashion to linear motion

Tangential Velocity (Sec 4.4)

Demonstration

Recap: The tangential velocity of the students on the outside was much larger than the velocity of the students on the inside. This reinforces the following equation:

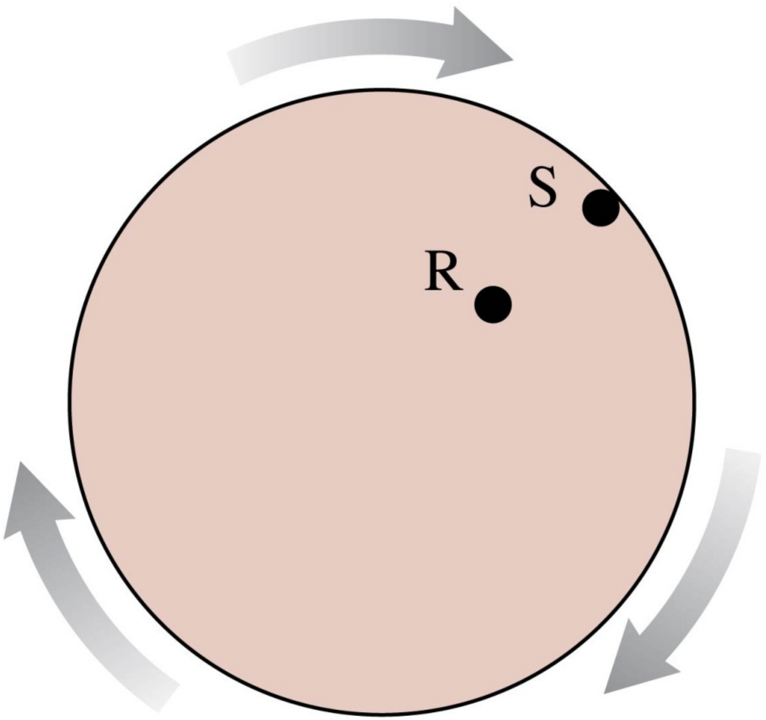
$$v_t = \omega r$$

where v_t is the tangential velocity, ω is angular velocity, and r is the radius.

In case you forget here is the link to a similar activity:
https://www.youtube.com/watch?v=FiQgPi0E4Y4

QuickCheck (Sec 4.4)

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is ______ that of Rasheed.



A. half

B. the same as

C. twice

D. four times

E. We can't say without knowing their radii.

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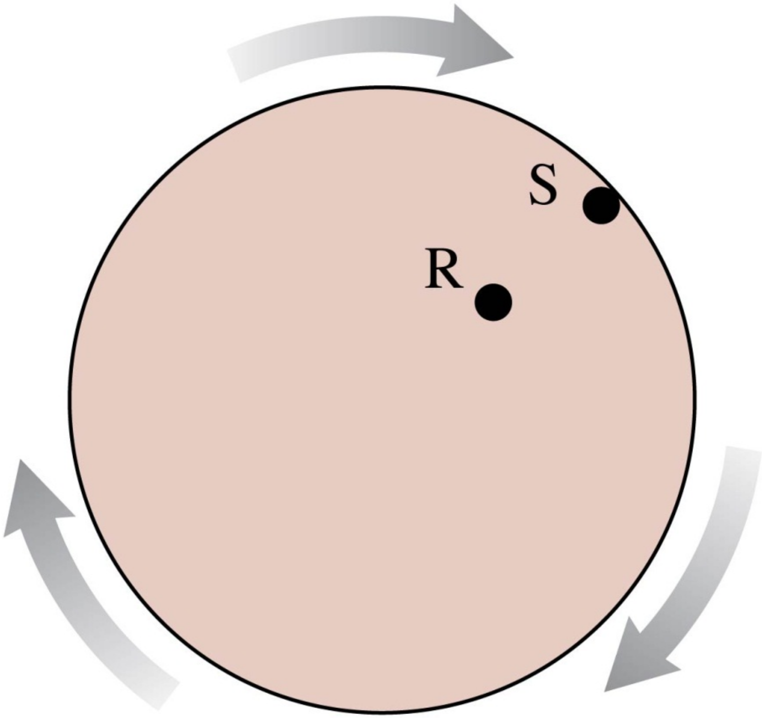


A. half

B. the same as

C. twice

D. four times

E. We can't say without knowing their radii.

Whiteboard Problem 4.5

An old-fashioned single-play vinyl record rotates on a turn-table at 45 rpm. What are:

(A) The angular velocity in rad/s?

(B) The period of the motion?

Whiteboard Problem 4.6

We don't notice how fast we are moving due to the Earth's rotation because the atmosphere moves with us.

Let's calculate our speed in m/s at this moment as we hurtle through space.

You already know the time period for the Earth's rotation. You also need the Earth's radius: R_e = 6400 km

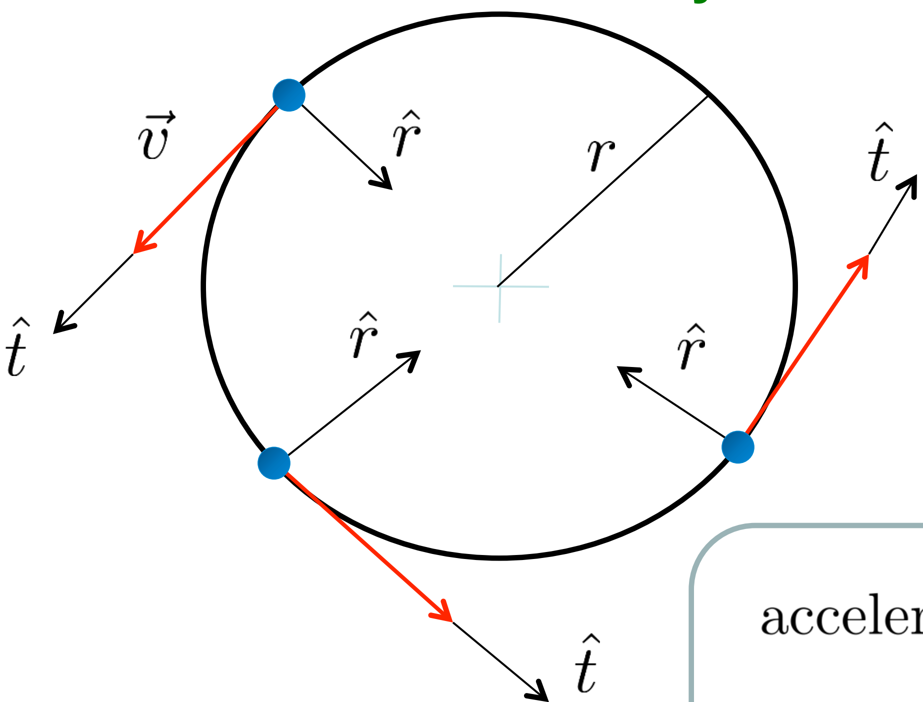
Acceleration for Uniform Circular Motion

…this is *a totally new concept arising from* 2D Circular Motion…

RADIAL ACCELERATION **(Sec 4.5)**

For an object in uniform circular motion (UCM), the speed is **constant**, but the velocity **continuously changes direction**. Thus, there must be an acceleration!

B/c by definition, $\vec{a} = d\vec{v}/dt$



$\hat{r} =$ The instantaneous radial direction, Always toward the center of the circle

$\hat{t} =$ The instantaneous tangential direction, Always tangent to the circle.

For Uniform Circular Motion:

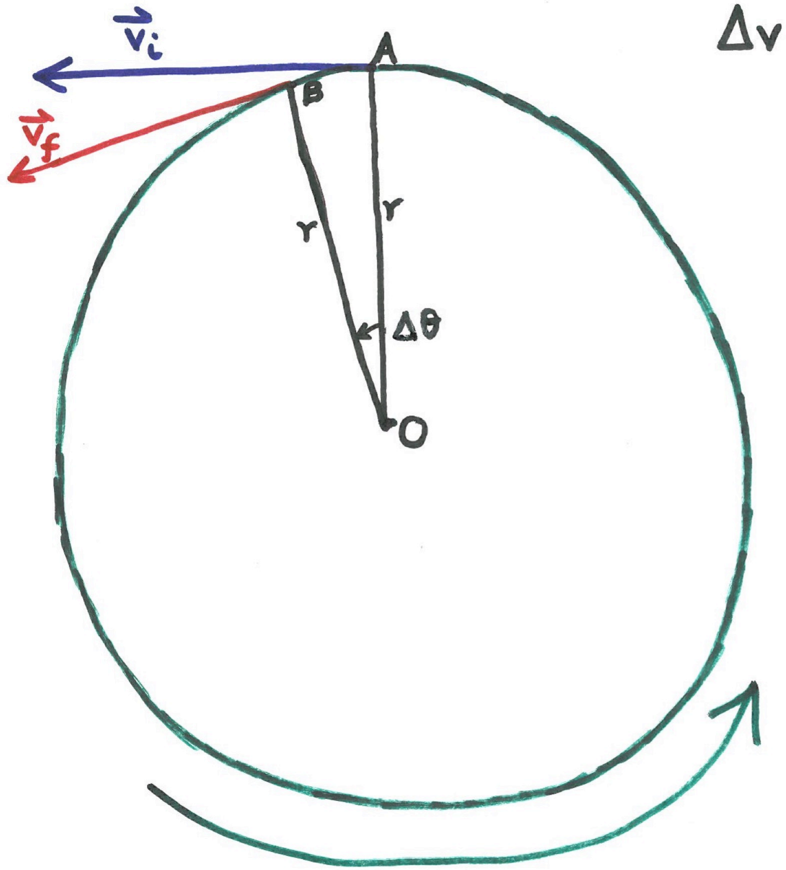
$\vec{v}$ is in the tangential direction

$\vec{a}$ is in the radial direction

$$\text{acceleration, } \vec{a} = \left(\frac{v^2}{r}, \text{ towards the center}\right)$$

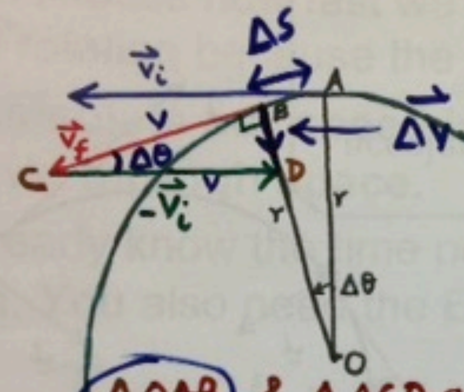
$$\underbrace{\text{or radially inward}}_{\text{“Centripetal”}}$$

Why is centripetal acceleration = v^2 / r ? (Sec 4.5)



$\vec{\Delta v} = \vec{v}_f - \vec{v}_i$

Why is centripetal acceleration = v^2 / r?



$\vec{\Delta v} = \vec{v}_f - \vec{v}_i = \vec{v}_f + (-\vec{v}_i)$

$|\vec{v}_f| = |\vec{v}_i| = v$

$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{\Delta v}}{\Delta t}$

$\vec{\Delta v} = \vec{v}_f - \vec{v}_i$ points toward center

"RADIALLY INWARD" [CENTRIPETAL]

ΔOAB & ΔACD are similar

$$\frac{\Delta s}{r} = \frac{\Delta v}{v}$$

$$\Rightarrow \frac{\Delta s}{\Delta t} = \frac{r}{v}\frac{\Delta v}{\Delta t}$$

TAKE $\lim_{\Delta t \to 0}$

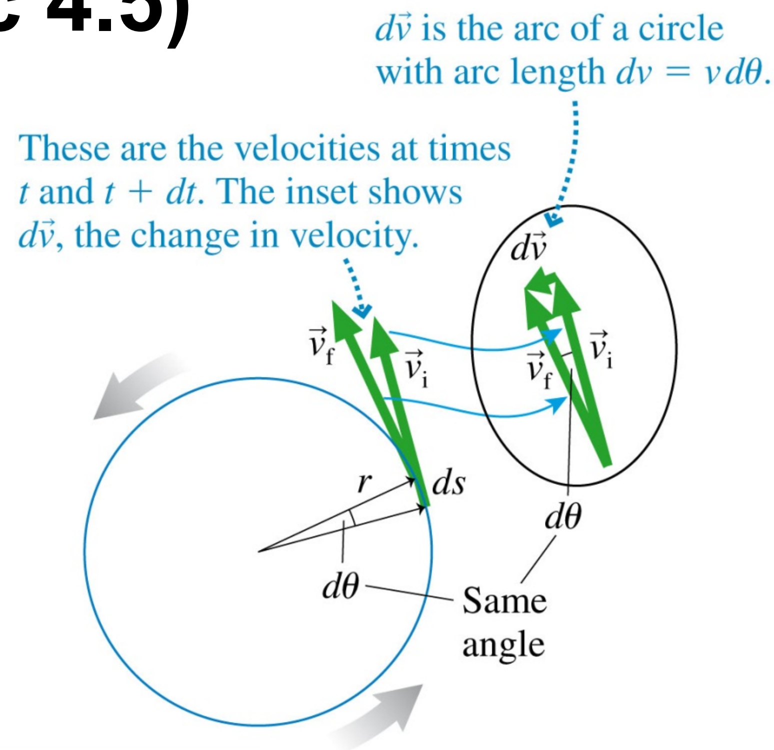
$$v = \frac{r}{v} a_r \Rightarrow \boxed{a_r = \frac{v^2}{r}}$$

Centripetal (or Radial) **Acceleration** in **Uniform** Circular Motion!

Q: Why? What's the origin?
A: Use Vector Subtraction!

(Sec 4.5)

- The figure shows the velocity $\vec{v}_i$ at one instant and the velocity $\vec{v}_f$ an infinitesimal amount of time *dt* later.
- By definition, $\vec{a} = d\vec{v}/dt$.
- By analyzing the isosceles triangle of velocity vectors, we can show that:



$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right) \qquad \text{(centripetal acceleration)}$$

which can be written in terms of angular velocity as:

$$a_{radial} = \omega^2 r = v^2 / r$$

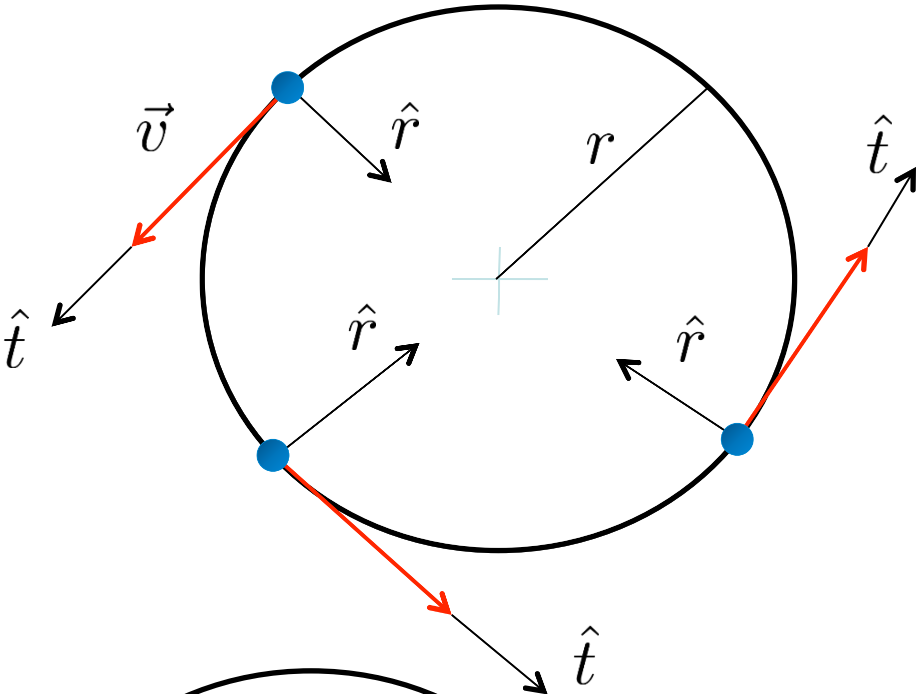
Whiteboard Problem 4.7 (Sec 4.5)

Remember the fastest electric car (Buckeye Silver Bullet) can exceed 300 mph. Why can't it go faster, say, at 400 mph? Answer: B/c it's tires explode!

Let's calculate the radial, or centripetal, acceleration required to make a tire of radius 0.25m rotate at 400 mph. Compare it to g! How many g's of acceleration are we talking??

What about your car at 60 mph – how many g's do your tires need?

Non-Uniform Circular Motion (Sec 4.6)



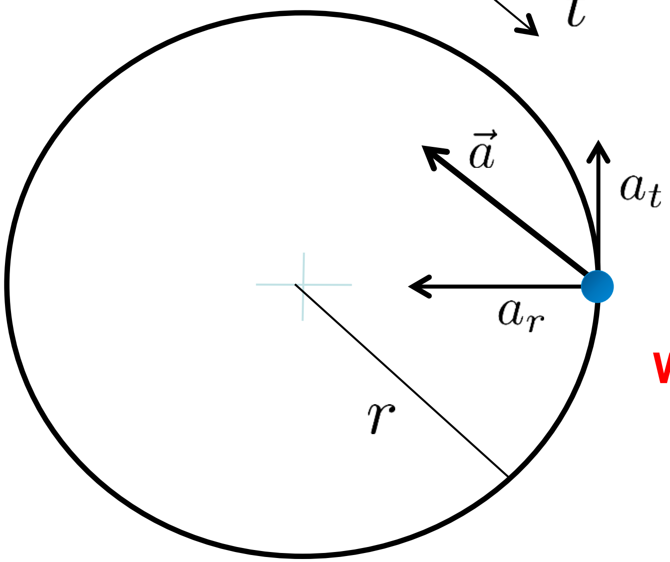
For UCM – a *centripetal acceleration* causes a directional change but no change in speed

$$a_r = \frac{v_t^2}{r}$$

For **non**-UCM – a *tangential acceleration* causes a tangential velocity change, i.e., a change in speed

$v_t = \omega r$ By analogy: $a_t = \alpha r$

$$\alpha \equiv \frac{d\omega}{dt}$$



Total acceleration

$$\vec{a} = \sqrt{a_r^2 + a_t^2}$$

When a_t = 0 we get UCM!

$$\omega_f = \omega_i + \alpha \Delta t$$

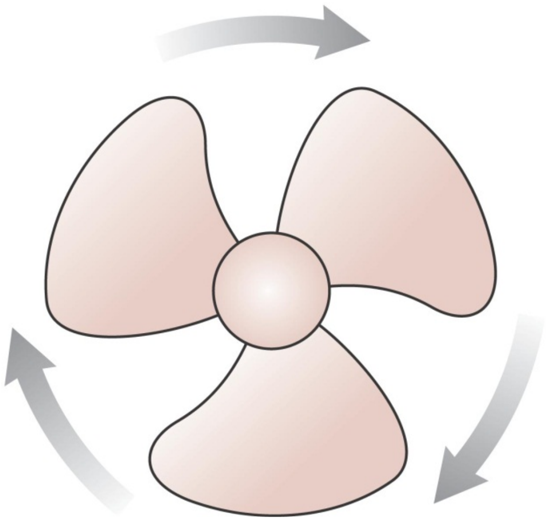
$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$\boxed{\textbf{Be careful, } \vec{a} \neq \alpha}$

QuickCheck (Sec 4.6)

The fan blade is slowing down.
What are the signs of ω and α?



A. ω is positive and α is positive.

B. ω is positive and α is negative.

C. ω is negative and α is positive.

D. ω is negative and α is negative.

E. ω is positive and α is zero.

(Sign Convention: CCW = positive and CW = negative)

How Do We Solve Problems in Rotational Kinematics?

Same steps as for 1D kinematics, but use slightly different set of equations (under Ch. 12 of your equation sheet)

(Sec 4.4 – 4.6)

TABLE 4.1 Rotational and linear kinematics for constant acceleration

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}{}^2 = v_{is}{}^2 + 2a_s\,\Delta s$

Graphs can be interpreted in similar fashion to linear motion

Just as slope of the *Position-time* / *Velocity-time* curve is *Velocity v* / *Acceleration a*

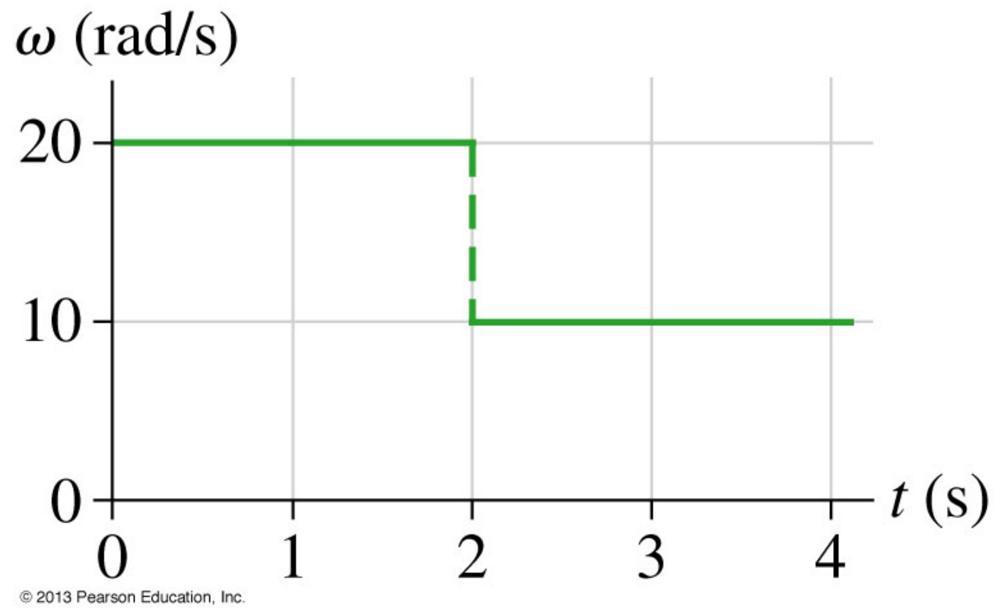
we find slope of the *Angular position-time* / *Angular velocity-time* curve is *Angular Velocity* ω / *Angular or Tangential Acceleration a*

And just as area under the *Velocity-time* / *Acceleration-time* curve is *Displacement* Δs / *Velocity v*

we find area under the *Angular velocity-time* / *Angular acceleration-time* curve is *Angular Displacement* $\Delta\theta$ / *Angular Velocity* ω

Whiteboard Problem 4.8 (Sec 4.6)

The graph below shows the angular-velocity vs time graph for a particle undergoing circular motion. How many revolutions does the object make during the first 4s?



Whiteboard Problem 4.9 (#4.77)

A long string is wrapped around a 6.0-cm-diameter cylinder, initially at rest, that is free to rotate on an axle. The string is then pulled with a constant acceleration of 1.5 m/s^2 until 1.0 m of string has been unwound. If the string unwinds without slipping, what is the cylinder's angular speed, in rpm, at this time?