Chapter 3: Vectors and Coordinate Systems

Objectives:

- Start Chapter 3 on Vectors – the math of vector components
- Vector addition and subtraction - recall CHAPTER1_LECTURE Slide #30
- Go back and cover Sec. 2.6 which we had skipped earlier

Vectors (Chapter 3)

In Chapter 1, we conceptually introduced the **Vector**:

A Vector is a quantity that has both magnitude and direction

Some examples of vector quantities that we'll see in PHY191 are position $\vec{r}$, velocity $\vec{v}$, acceleration $\vec{a}$, force $\vec{F}$, momentum $\vec{p}$, and others.

Quantities that have only a magnitude, like mass, m, Temperature, T, are called scalars.

In Chapter 3, we want to develop and learn how to work with vectors analytically. In what we're going to do in PHY191 and PHY192, **how important Is this**?

$$(\textbf{Extremely})^{n} \quad \textbf{where } n >> 1$$

Or even $$(\textbf{Extremely})^{n!} \quad \textbf{where } n >> 1$$

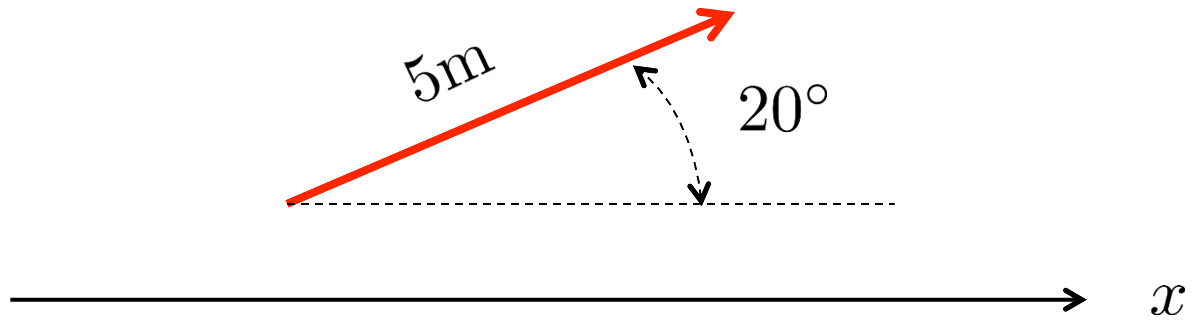
Vectors: The Very Basics . . . Arrows (Sec 3.1)

One way that Knight denotes a vector:

$$\vec{A} = \text{(length or } \underline{\text{magnitude}}\text{, direction)}$$

Note: the magnitude of a vector is always greater than or equal to zero. It is never negative!

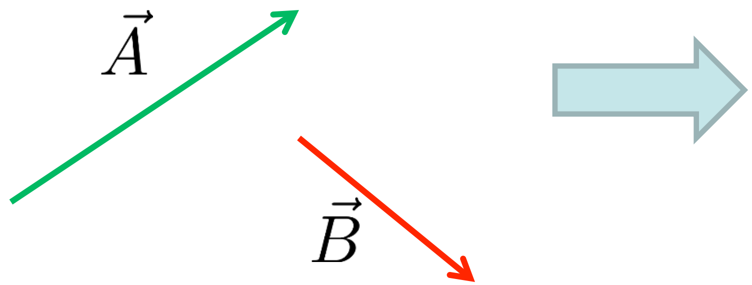
$$\text{e.g. } \vec{A} = (5\text{m, } 20^\circ \text{ above } +x \text{ axis})$$

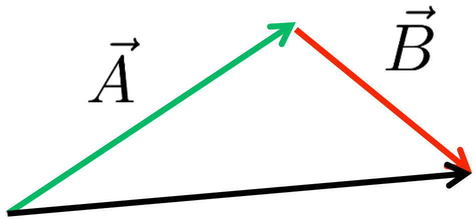


RECAP FROM CHAPTER 1

Adding or subtracting vectors graphically is useful, but not very accurate.

Vector Addition: place tails on heads.



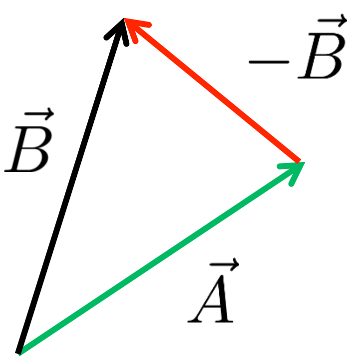


$$\vec{C} = \vec{A} + \vec{B}$$

Vector Subtraction: treat as: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$\vec{A} - \vec{B}$

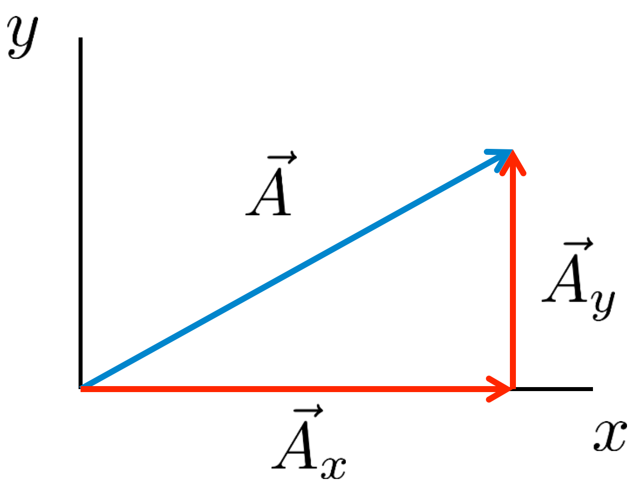
$\vec{D} = \vec{A} - \vec{B}$



Component Vectors & **Vector Components** ???? (Sec 3.3)

Your author kind of confuses things here by defining “component vectors” (which are never used again) and **“vector components”** (which will be used forever). The two are not the same thing; however they are related.

Component Vectors: (with respect to some coordinate system)



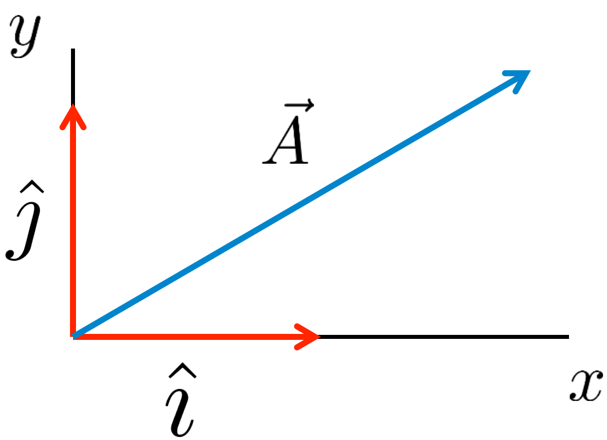
From vector addition:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$\vec{A}_x$ and $\vec{A}_y$ are called the **“Component Vectors”** of vector $\vec{A}$

Vector Components (the **useful** one) – Sec 3.3 + 3.4

Coordinate Unit Vectors:



Define the Unit Vectors:

(what does unit mean?)

$$\hat{\imath} \equiv (1, +x \text{ direction})$$

$$\hat{\jmath} \equiv (1, +y \text{ direction})$$

(Note: unit vectors wear hats)

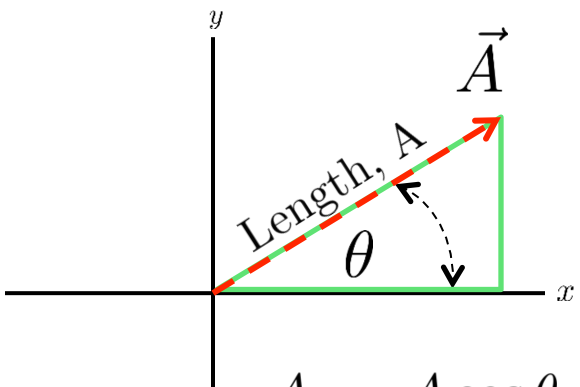
So, using the "Component Vectors" from above:

$$\vec{A} = \vec{A}_x + \vec{A}_y \equiv A_x\hat{\imath} + A_y\hat{\jmath}$$

where: A_x and A_y are called $\underline{\textbf{The Components of Vector } \vec{A}}$

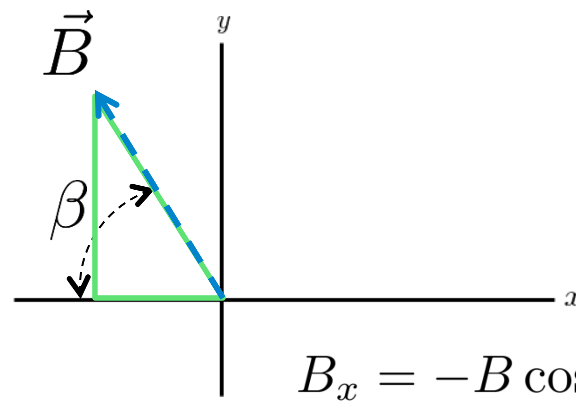
Note: the **Components of a Vector** are scalars and can be positive, negative, or zero.
In a few of the problems that we'll do, we'll need three dimensions, i.e. a z-axis too;
The unit vector in the z-direction is $\hat{k}$

Suppose that we know the magnitude and direction of the vector and we want the components: (i.e. we know: $\vec{A} = (A, \theta)$)



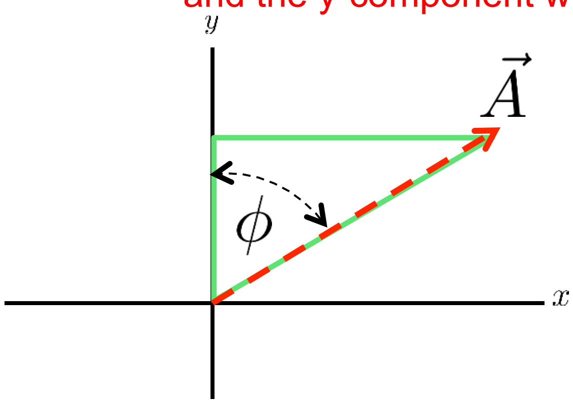
$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

Or



$$B_x = -B\cos\beta$$
$$B_y = B\sin\beta$$

Note: a common mistake is to always associate the x-component with the cosine and the y-component with the sine. What about:



Here:
$$A_x = A\sin\phi$$
$$A_y = A\cos\phi$$

You have to look at the triangle and the axes!

Whiteboard Problem 3-1

Find each vector's x- and y-components

(A) $\vec{v} = (10 \text{ m/s, negative y-direction})$

(B) $\vec{a} = (20 \text{ m/s}^2\text{, } 30^\circ \text{ below positive x-axis})$

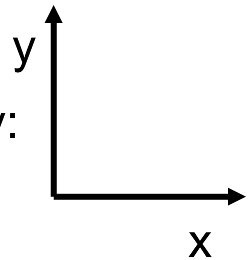
(C) $\vec{F} = (100 \text{ N, } 36.9^\circ \text{ counterclockwise from positive y-axis})$

Inclined Plane (Sec. 2.6): **Tilted Axes**

Another special case of constant acceleration

- Constant downward acceleration provided by a gravitational pull on a frictionless plane near the surface of the Earth

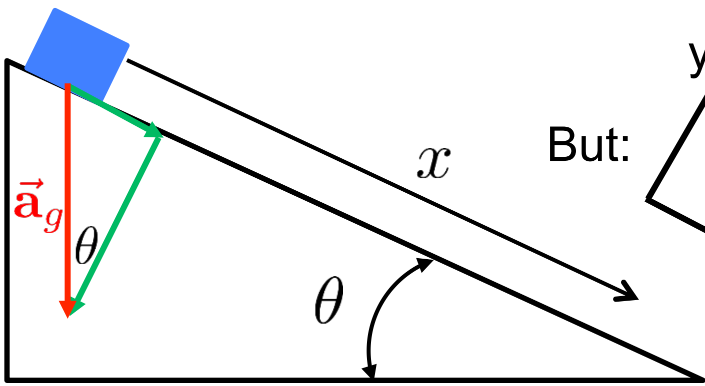
Typically:



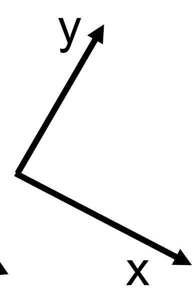
$$s \Leftrightarrow x$$

$$a_s \Leftrightarrow a_x = g\sin\theta$$

(If you choose +x as down the incline)



But:



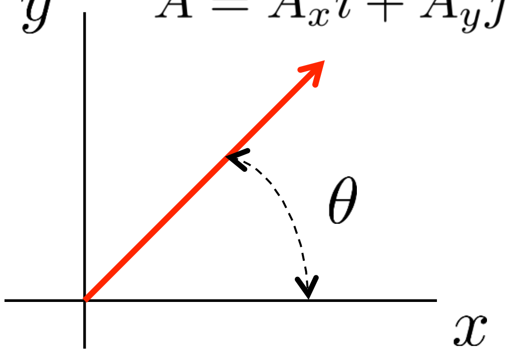
Hint: Think about what happens as $\theta \to 90°$

We can treat it as a 1D situation!

How About Going the Other Way? (Sec 3.3 + 3.4)

Suppose we know a vector's components, how do we find its magnitude and direction?

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$



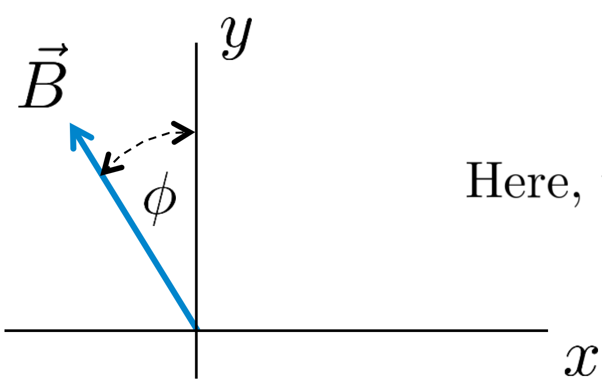
From the Pythagorean theorem:

$$\text{Magnitude of } \vec{A} = A = |\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2}$$

(Note: Both A and $|\vec{A}|$ denote the Vector Magnitude)

$$\text{Direction: } \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Be careful with the direction: you have to be smarter than your calculator; look at the triangle, e.g:



$$\text{Here, the angle from } +y \text{ axis: } \phi = \tan^{-1}\left(\frac{|B_x|}{B_y}\right)$$

Again, you have to look at the triangle.

Whiteboard Problem 3.2: Problem # 11 at end of Chapter 3

Draw each of the following vectors, label an angle that specifies the vector's direction, and then find the vector's magnitude and direction.

a) $\vec{A} = 3.0\hat{i} + 7.0\hat{j}$

b) $\vec{a} = (-2.0\hat{i} + 4.5\hat{j})$ m/s^2

c) $\vec{v} = (14\hat{i} - 11\hat{j})$ m/s

d) $\vec{r} = (-2.2\hat{i} - 3.3\hat{j})$ m

Adding and Subtracting Vectors with Components

To add two or more vectors, you just add up their components:

e.g.
$$\vec{C} = \vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})$$
$$= \underbrace{(A_x + B_x)}_{C_x}\hat{\imath} + \underbrace{(A_y + B_y)}_{C_y}\hat{\jmath}$$

Likewise, to subtract two vectors, you just subtract the components:

$$\vec{D} = \vec{A} - \vec{B} = \underbrace{(A_x - B_x)}_{D_x}\hat{\imath} + \underbrace{(A_y - B_y)}_{D_y}\hat{\jmath}$$

Working with vectors in terms of their components is incredibly easy.
Later we'll also develop the dot product and the cross product.

Whiteboard Problem 3.3: Ch3, #16

16. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{F} = \vec{A} - 4\vec{B}$.

   a. Write vector $\vec{F}$ in component form.

   b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$.

   c. What are the magnitude and direction of vector $\vec{F}$?