Chapter 2 Lecture

physics

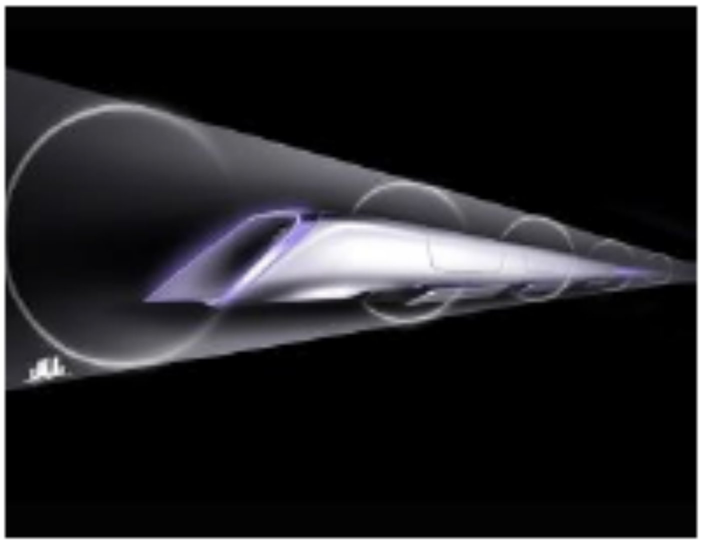
FOR SCIENTISTS AND ENGINEERS

a strategic approach

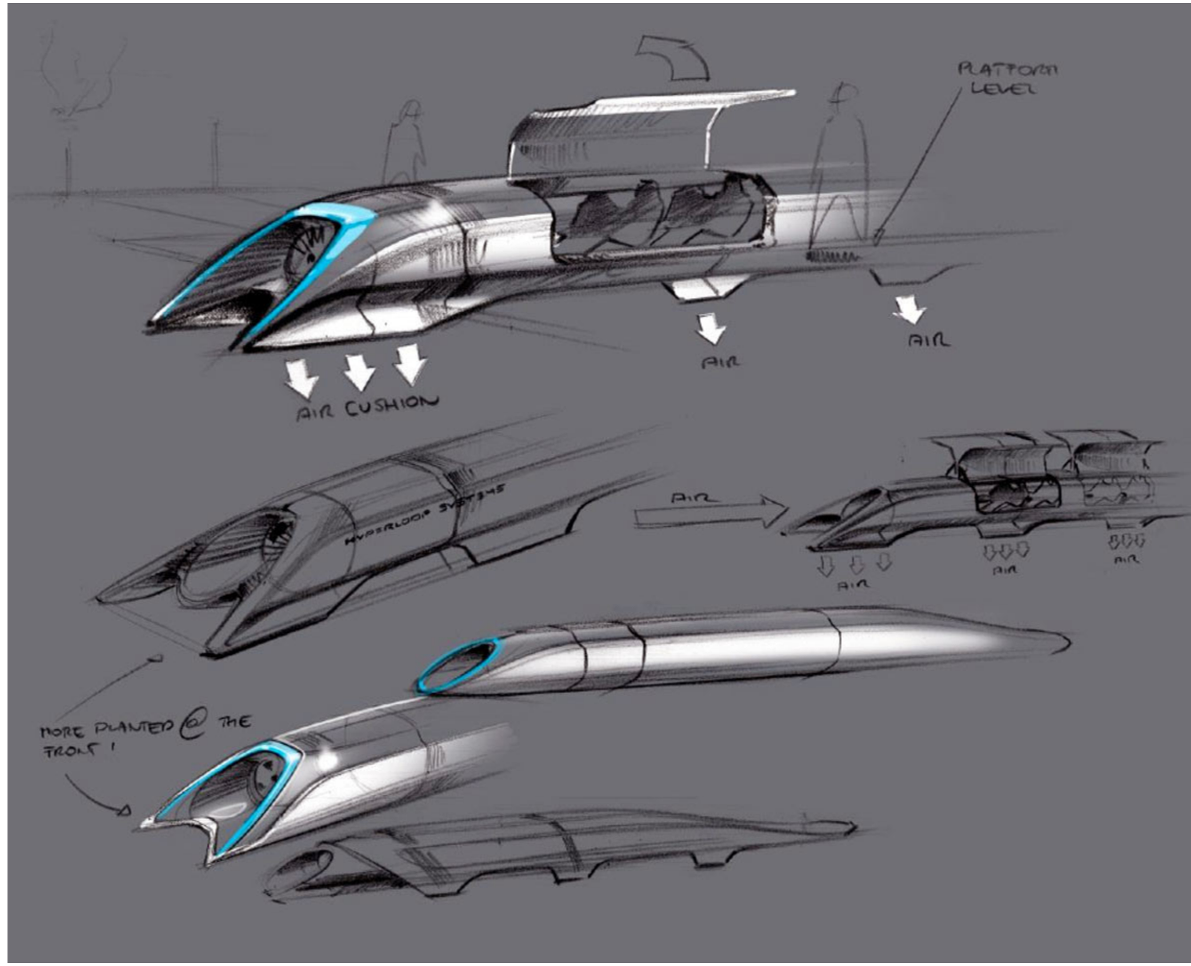
THIRD EDITION

randall d. knight

Chapter 2 Kinematics in One Dimension



Look, no rails (Image: Tesla Motors/AP Photo)



Read “Application – HYPERLOOP” posted on course website!

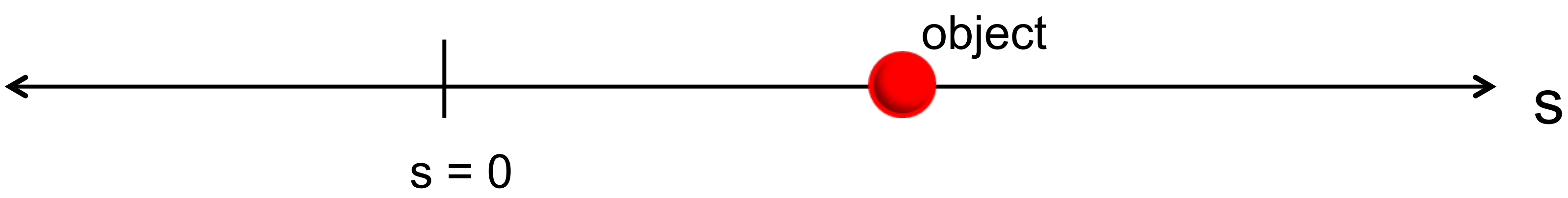
Chapter Goal: To learn how to solve problems about motion in a straight line. Example: HYPERLOOP

Objectives for Friday's (09/02) lecture

- Concept of instantaneous velocity and acceleration
- Given a Position-vs-time graph, determine the Velocity-vs-time graph
- Given a Velocity-vs-time graph, determine the Acceleration-vs-time graph

Motion in One Dimension* (Sec 1.6)

Motion along the s-axis (Q: why s?)



s = object's position coordinate along the s-axis (can be +, -, or 0)

v_s = object's velocity along the s-axis (can be +, -, or 0)

a_s = object's acceleration along the s-axis (can be +, -, or 0)

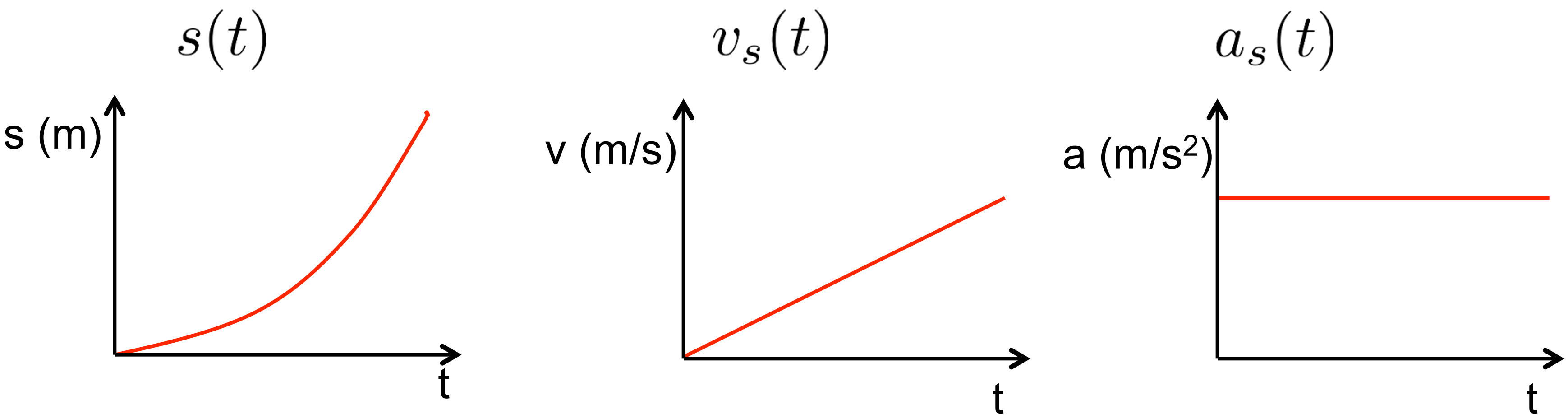
Think of "*s*" as either *x* or *y*.

*Note, in one dimension, we can drop the vector notation.
Just the sign (+ or -) is adequate to inform us of direction.

Motion in One Dimension (Sec 1.6 + 2.2 + 2.4)

In Chapter 1, we introduced motion diagrams to describe motion. Now, we want to represent the motion as functions of time $s(t)$, $v_s(t)$, and $a(t)$

e.g. we may have something like this:



Note, these are graphs – not pictures. <u>The object is still moving in one dimension along the s-axis</u>. Our goal in Chapter 2 is to develop the mathematical relations between $s(t)$, $v_s(t)$, and $a(t)$

Recap! $\vec{v}_{\text{avg}} = \dfrac{\Delta \vec{r}}{\Delta t}$ $\qquad$ $\vec{a}_{\text{avg}} = \dfrac{\Delta \vec{v}}{\Delta t}$

Motion in One Dimension (Sec 1.6 + 2.2 + 2.4)

In Chapter 1, we introduced motion diagrams to describe motion. Now, we want to represent the motion as functions of time $s(t)$, $v_s(t)$, and $a(t)$

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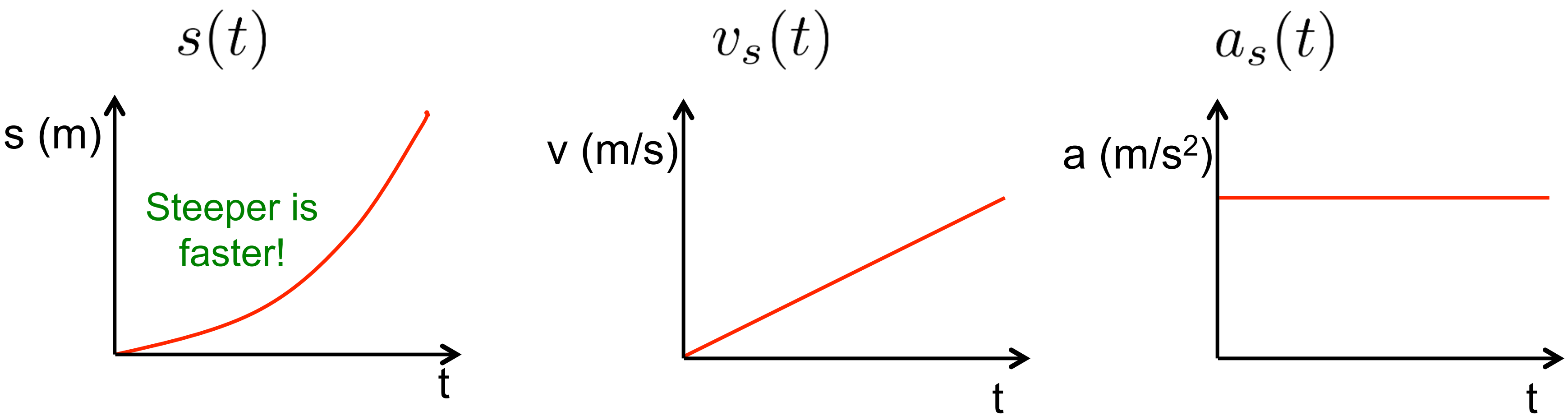

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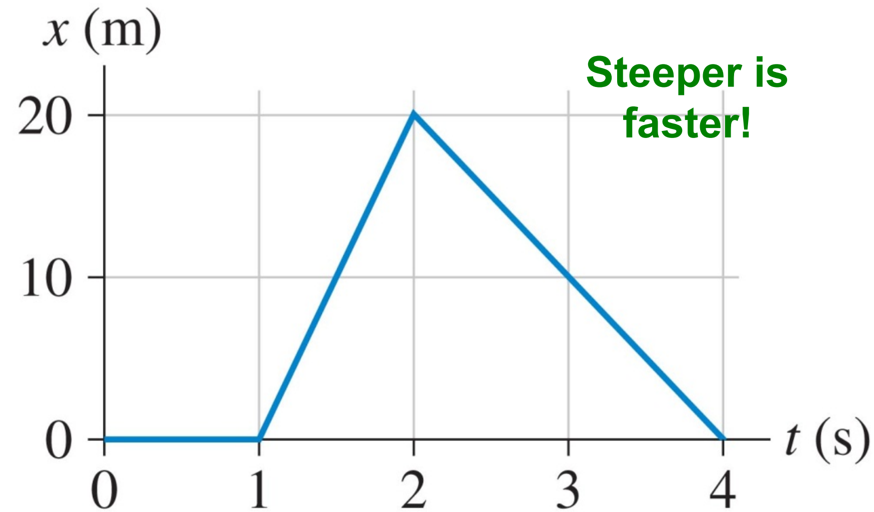
Recap! $\vec{v}_{s,\,\text{avg}} = \dfrac{\Delta \vec{s}}{\Delta t}$ $\qquad$ $\vec{a}_{s,\,\text{avg}} = \dfrac{\Delta \vec{v}_s}{\Delta t}$

Whiteboard Problem 2.1: Find v_s given *s* (simple case)

Simple case: Position-time graph is a straight line, a.k.a. Uniform Motion

(SEC 2.1)

Here is a position graph of an object:



What is the object's velocity from t = 1s to t = 2s?

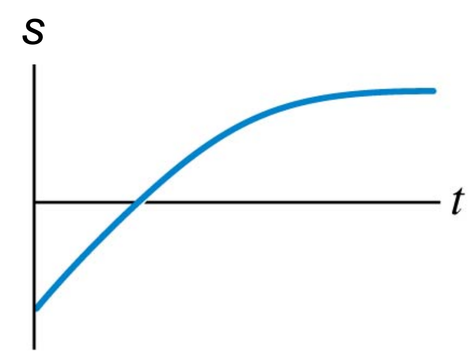
What is the object's velocity from t = 2s to t = 4s?

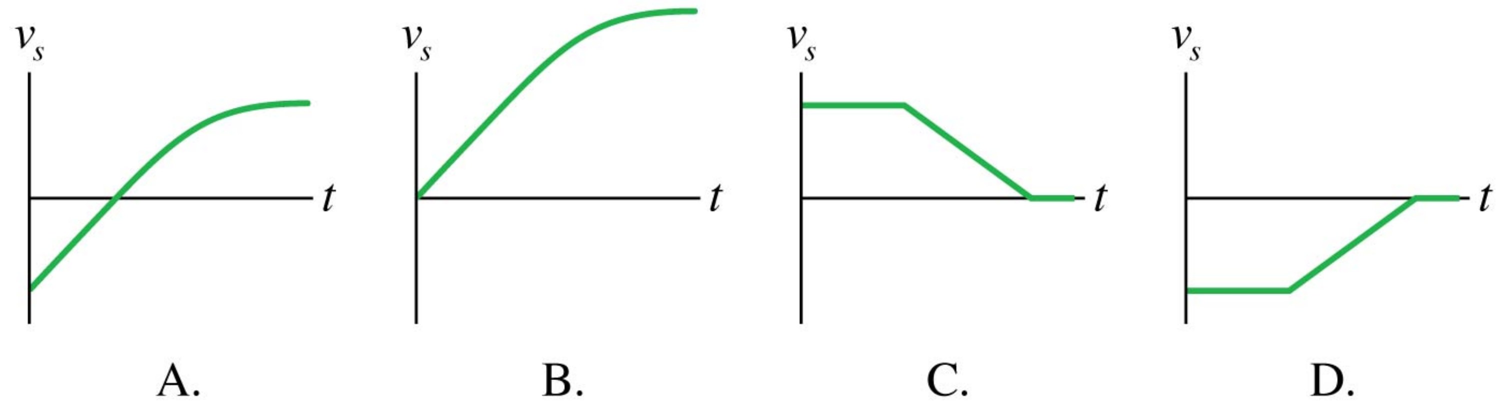
Apply $$\vec{v}_{s,\,avg} = \frac{\Delta \vec{s}}{\Delta t}$$

Stop to Think 2.2 in book: What if *s(t)* vs *t* is not a straight line?

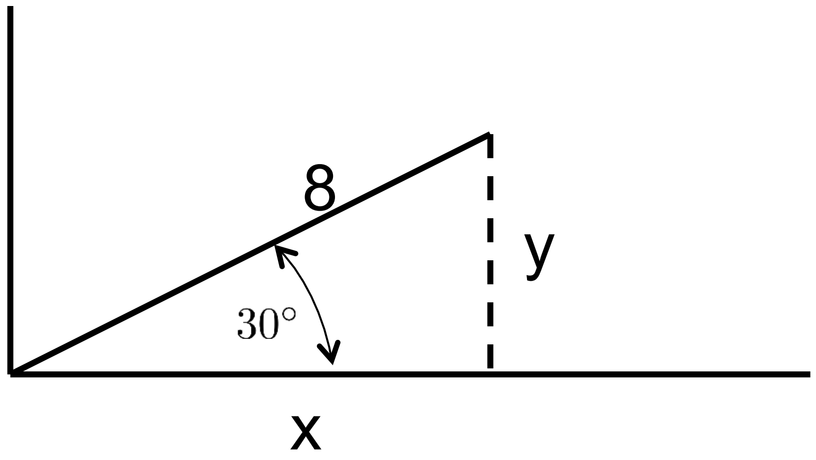
Remember…Steeper is faster! **(SEC 2.1)**

Which velocity-versus-time graph goes with this position graph?





Math Review Activity

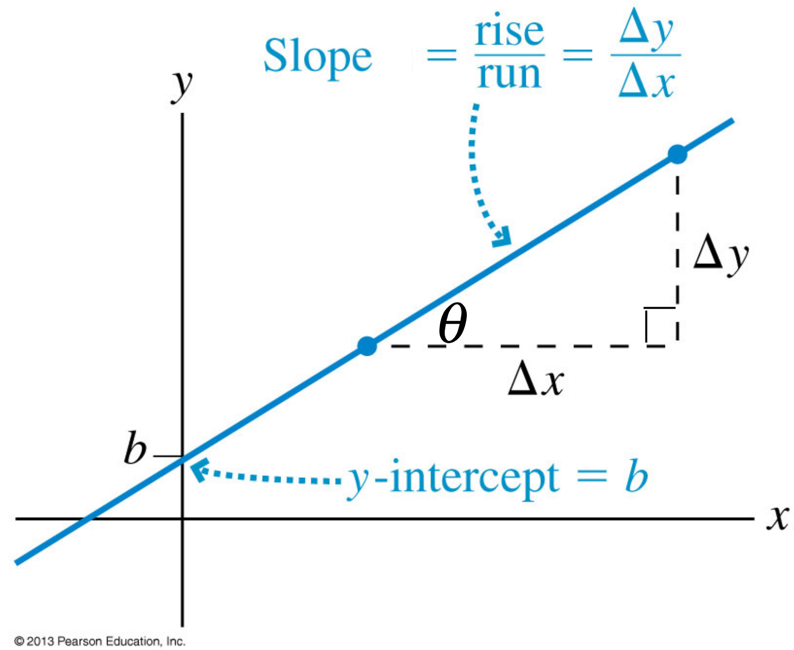


For the triangle shown above, calculate x and y:

"Steeper is Faster" – let's make this **quantitatively precise**

(SEC 2.1)

"Steepness" of line = its "slope"



Basic Trig Review

In any right-angle triangle:

$\tan\theta$ = opp / adj = slope

$\sin\theta$ = opp / hyp

$\cos\theta$ = adj / hyp

$$\tan\theta = \sin\theta / \cos\theta$$

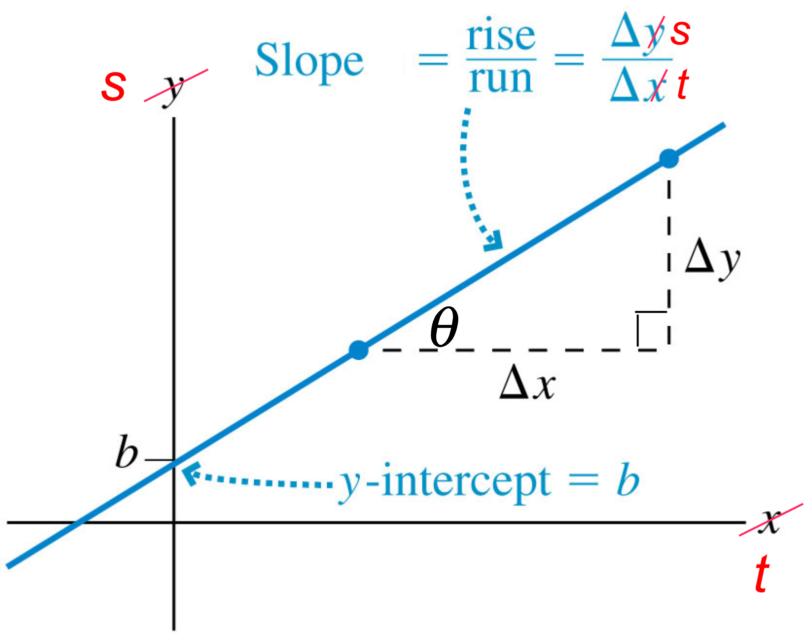
"Steeper is Faster" – let's make this quantitatively precise

(SEC 2.1 + 2.2)

"Steepness" of line = its "slope"

$$\vec{v}_{s,\,avg} = \frac{\Delta \vec{s}}{\Delta t}$$

Slope of position-time graph gives the velocity!



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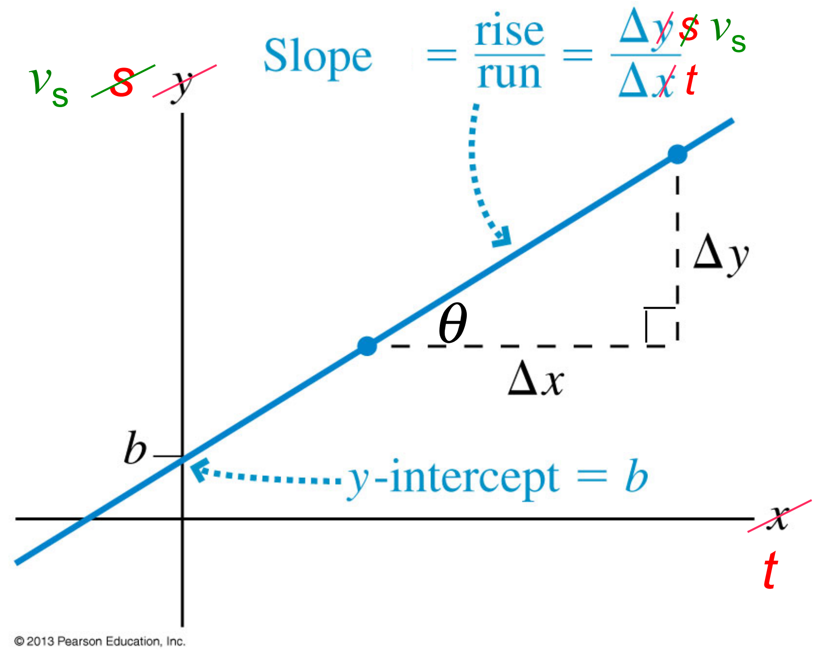
(SEC 2.1 + 2.7)

“Steepness” of line = its “slope”

$$\vec{v}_{s,\,avg} = \frac{\Delta \vec{s}}{\Delta t}$$



Slope of position-time graph gives the velocity!

…and, because $a_{avg} = \frac{\Delta \vec{v}_s}{\Delta t}$

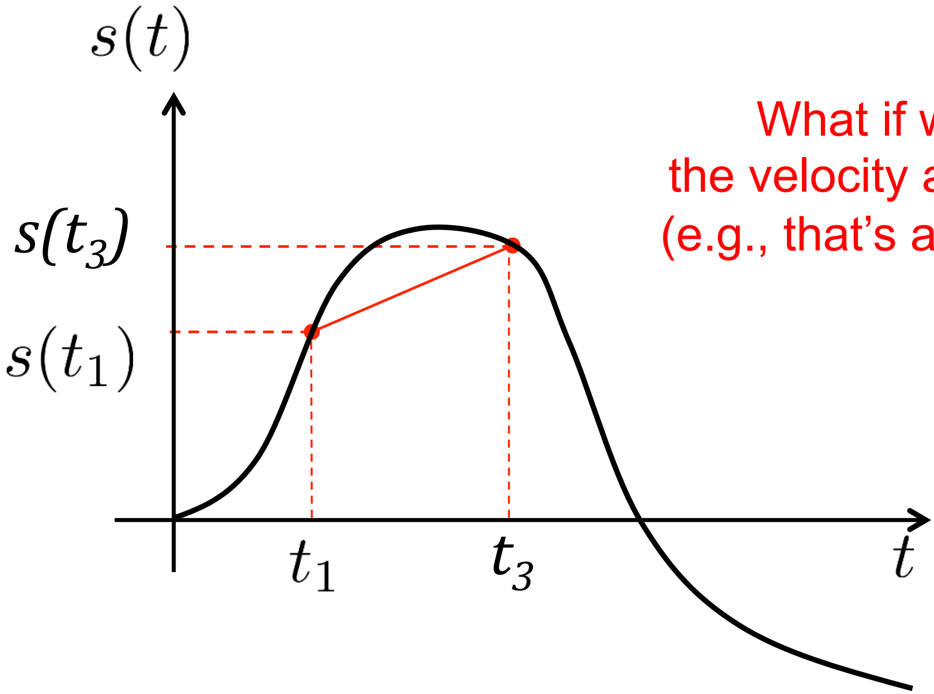
Slope of velocity-time graph gives the acceleration!

BUT…so far, we’ve only discussed *straight-line s – t* and *v – t* graphs.

General 1D Motion: Average velocity NOT useful!

Position vs time graph is NOT a straight line, a.k.a. Non-uniform Motion

(SEC 2.2)



What if we want to know the velocity at a specific time, t_1? (e.g., that's all a cop cares about!)

Average Velocity between t_1 & t_3 $= v_{s\ avg} = \frac{\Delta s}{\Delta t} = \frac{s(t_3) - s(t_1)}{t_3 - t_1}$

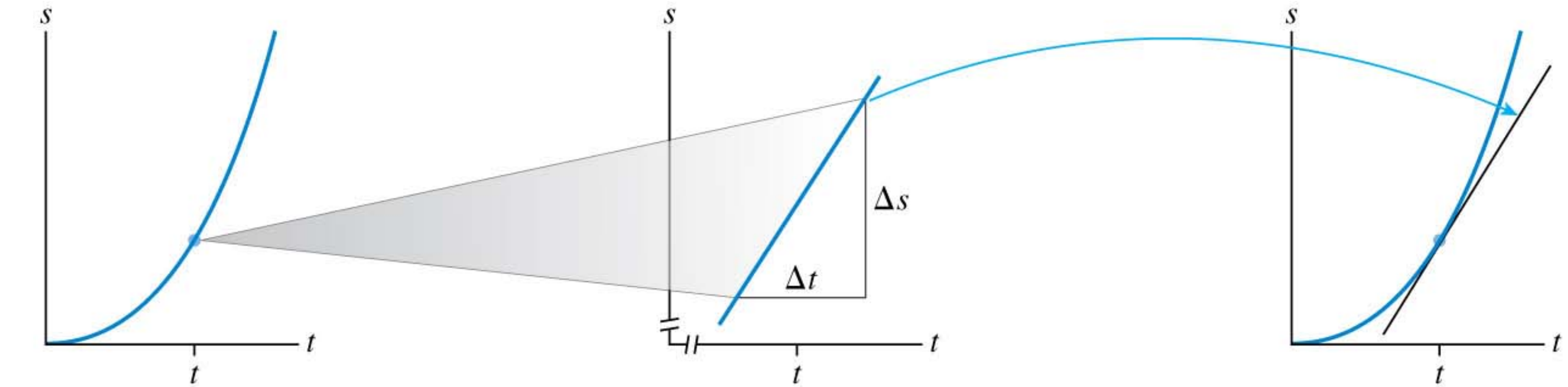
= slope of the line connecting points 1 & 3

To find the instantaneous velocity at t_1
we make Δt infinitesimally small (make t_1 and t_3 closer)

(SEC 2.2)

Motion diagrams and position graphs of an accelerating car:

Q: What is the instantaneous velocity at time *t* ?



What is the velocity at time *t*?

Zoom in on a *very* small segment of the curve centered on the point of interest. This little piece of the curve is essentially a straight line. Its slope $\Delta s/\Delta t$ is the average velocity during the interval Δt.

The little segment of straight line, when extended, is the tangent to the curve at time *t*. Its slope is the instantaneous velocity at time *t*.

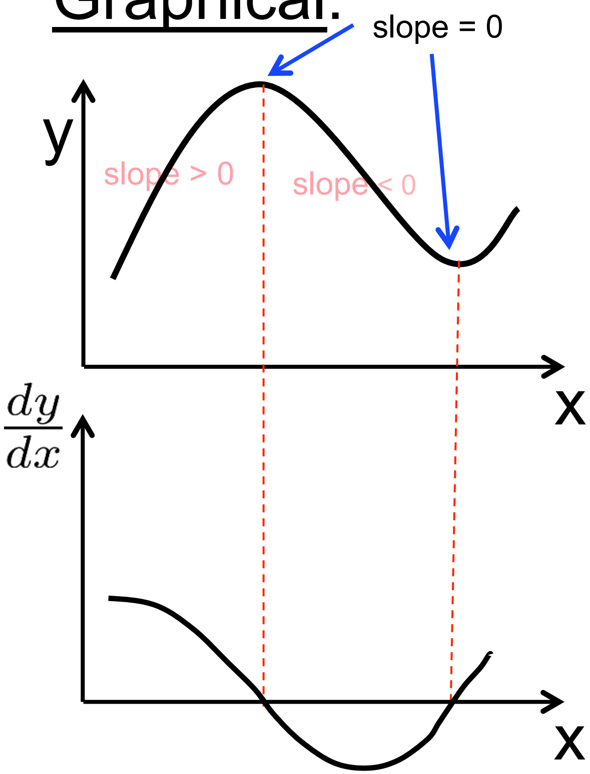
$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \quad \text{(instantaneous velocity)}$$

i.e., Slope of *s(t)* at point *t*
= Derivative of *s(t)* at *t*

What kinds of derivatives will we need to do?

(SEC 2.2)

Graphical:



Analytical:

$$y(x) = \text{constant} \qquad \frac{dy}{dx} = 0$$

$$y(x) = cx^n \qquad \frac{dy}{dx} = cnx^{n-1}$$

(c & n are constants)

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

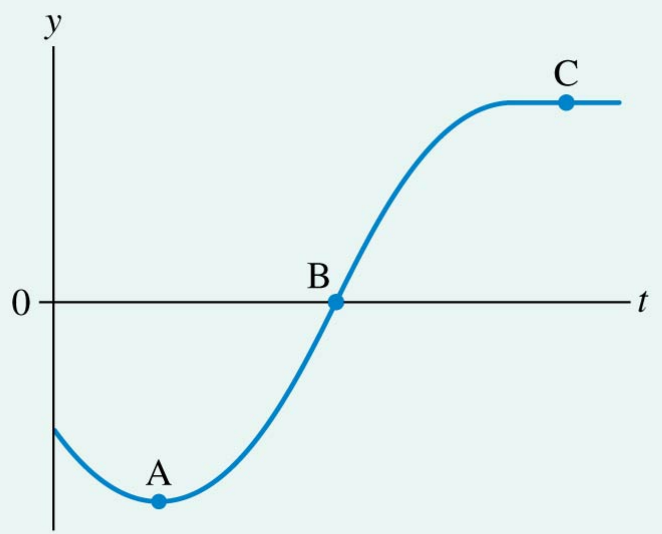
Others as we need them.

Whiteboard Problem 2.2: Finding Velocity from Position Graphically (SEC 2.2)

Example 2.3 **Finding velocity from position graphically**

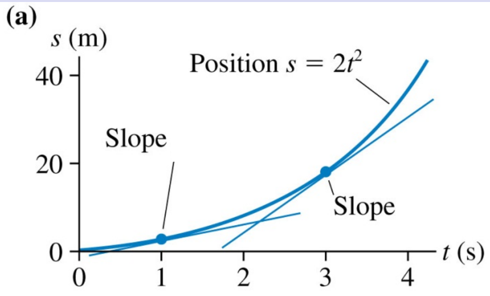
The figure shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least speed?
b. At which point or points does the elevator have maximum velocity?
c. Sketch an approximate velocity-versus-time graph for the elevator.



Whiteboard Problem 2.3: A little calculus (SEC 2.2)

Suppose the position of a particle as a function of time is $s = 2t^2$ m where t is in s. Plot the particle's velocity as a function of time from $t = 0$ to 4 s.



Calculate the derivatives $\left(\frac{d}{dx}\right)$ of the following functions:

$A)\ f(x) = 4x^1$

$B)\ f(x) = 2x^7 + x^2$

$C)\ f(x) = 1,000,000$

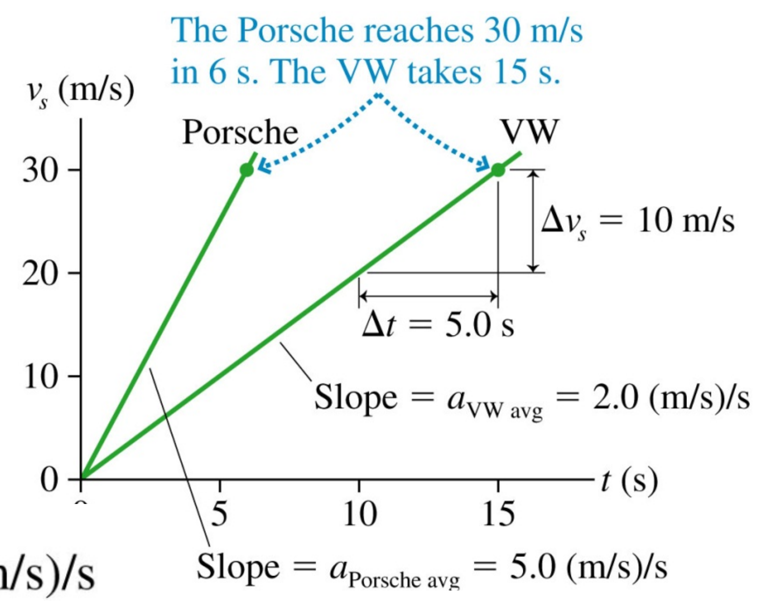
$D)\ f(x) = 5x + 67$

Acceleration – the final ingredient in Kinematics

(SEC 2.4 + 2.7)

- Imagine a competition between a Volkswagen Beetle and a Porsche to see which can achieve a velocity of 30 m/s *in the shortest time*.
- The table shows the velocity of each car, and the figure shows the velocity-versus-time graphs.
- Both cars achieved every velocity between 0 and 30 m/s, so neither is faster.
- But for the Porsche, **the rate at which the velocity changed** was:

t(s)	$v_{Porsche}$(m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
0.4	2.0	0.8
⋮	⋮	⋮

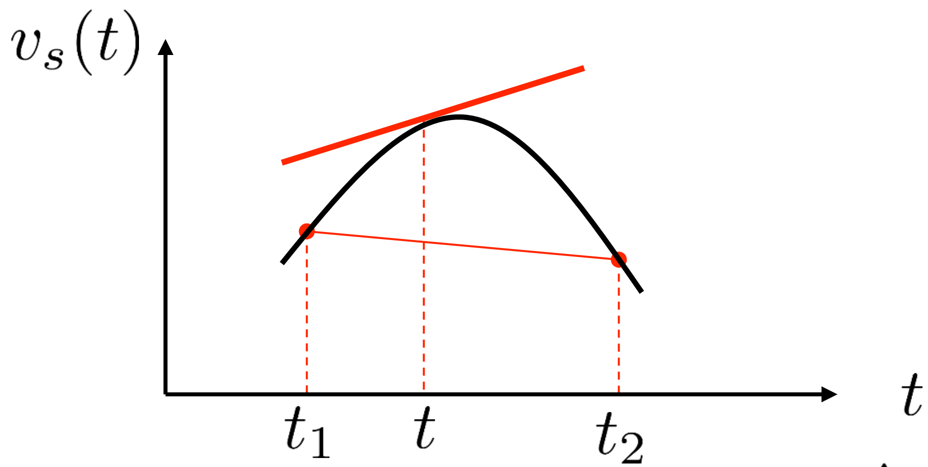


$$\text{rate of velocity change} = \frac{\Delta v_s}{\Delta t} = \frac{30 \text{ m/s}}{6.0 \text{ s}} = 5.0 \text{ (m/s)/s}$$

Acceleration is the Rate of Change of Velocity

(SEC 1.5 + 2.4 + 2.7)

Just as $v_s(t)$ is the slope of the $s(t)$-curve,
the acceleration $a(t)$ is the slope of the $v_s(t)$-curve!



Average Acceleration between t_1 & $t_2 = a_{s_{avg}} = \frac{\Delta v_s}{\Delta t} = \frac{v_s(t_2) - v_s(t_1)}{t_2 - t_1}$

Instantaneous Acceleration at t $= a_s(t) = \frac{dv_s}{dt}$

= slope of the tangent line to $v_s(t)$ at point *t*

= instantaneous rate of change of $v_s(t)$

Acceleration, like velocity (and displacement), is a vector quantity and has both magnitude and direction.

Whiteboard Problem 2.4

(SEC 1.5 + 2.2 + 2.7)

A particle moving along the x-axis has its position described by the function:

$$x = (2t^2 - t + 1)\ \mathrm{m}$$

where t is in seconds.

At t = 2 s what are the particle's

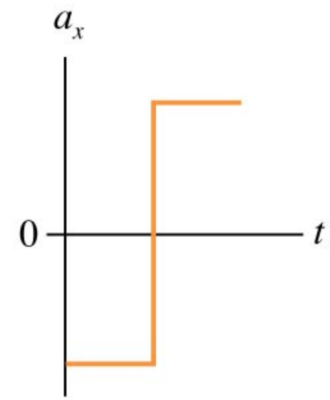
(A) position?

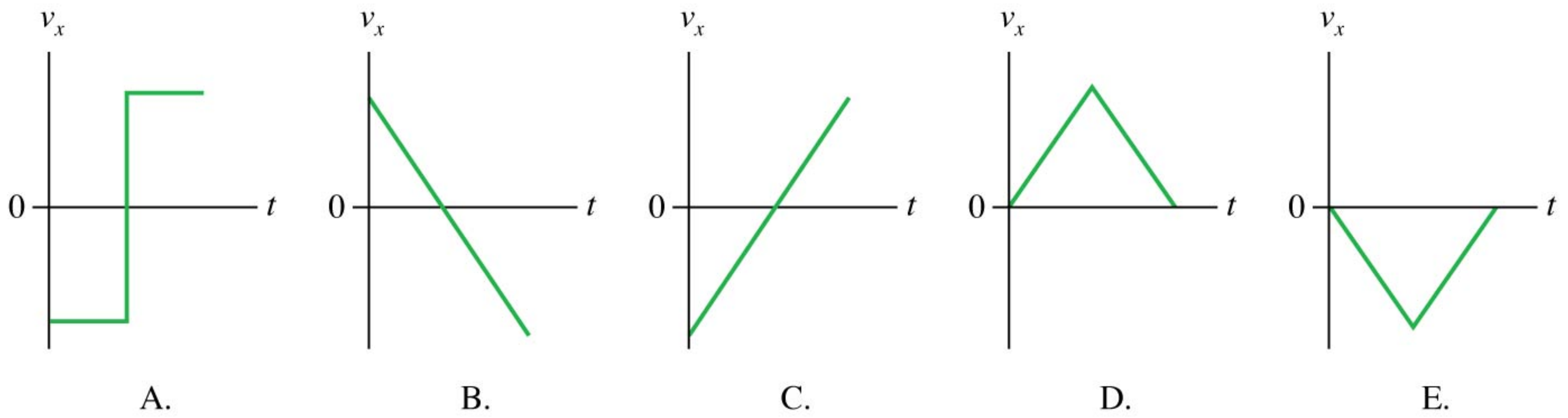
(B) velocity?

(C) acceleration?

Similar to Stop-to-think 2.4 in book **(SEC 2.4 + 2.7)**

Which velocity-versus-time graph goes with this acceleration graph?





What we've learnt so far…

(SEC 1.5 + 2.2 + 2.7)

Velocity v_s at time t is the slope (or derivative) of the s-t curve at time t

$$v_s \equiv \frac{ds}{dt}$$

Acceleration a_s at time t is the slope (or derivative) of the v_s-t curve at time t

$$a_s \equiv \frac{dv_s}{dt}$$

So, given the displacement as a function of time, i.e., the s-t curve, we can find the velocity and acceleration at any instant.

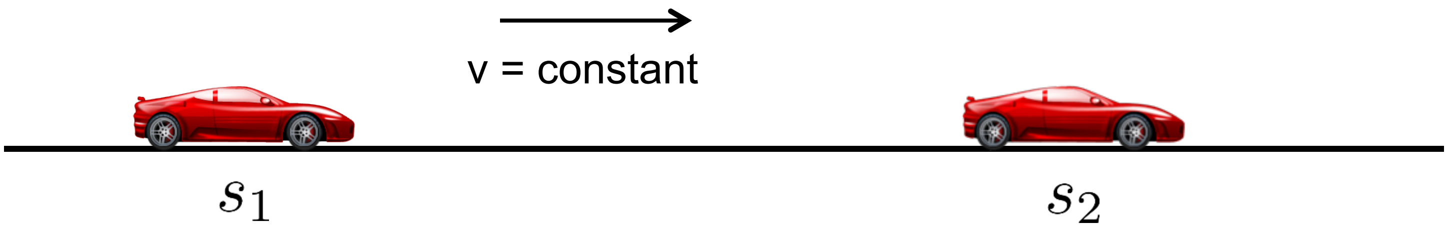
How about the reverse? Given the acceleration and velocity curves, can we determine the displacement? **(SEC 2.3)**

How to get Position from Velocity (Simple case)

(SEC 2.3)

We've seen how to get velocity from position: $v_s(t) = \frac{ds}{dt}$

How do we get position from velocity? This is something you do everyday:



At time t_1, you're at $s = s_1$; you drive until $t = t_2$; how far did you go?

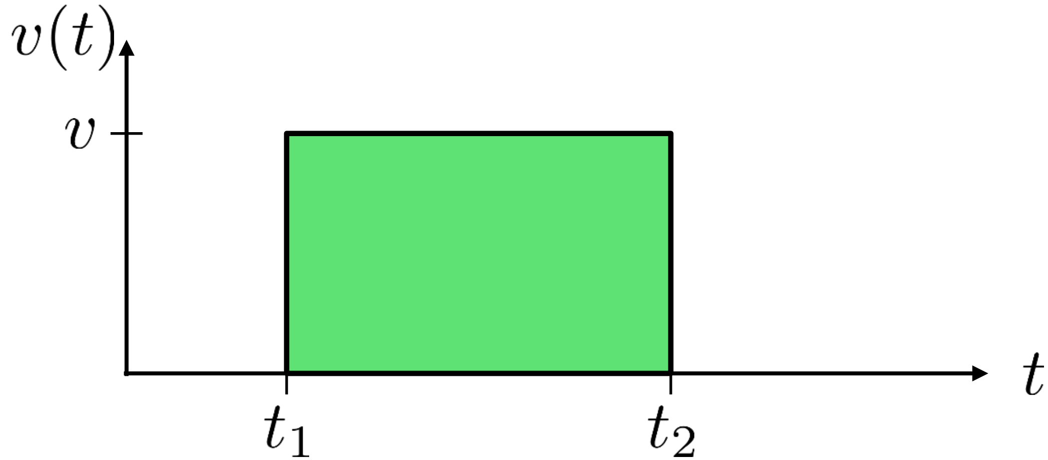
$$s_2 = s_1 + v \times (\text{time travelled})$$

$$\text{Or, } s_2 = s_1 + v\Delta t \quad \text{where} \quad \Delta t = t_2 - t_1$$

What does this look like on a graph?

How to get Position from Velocity (Simple case)

(SEC 2.3)



$$s_2 = s_1 + v\Delta t \quad \text{where} \quad \Delta t = t_2 - t_1$$

$$\text{Or, } s_2 = s_1 + \text{ (area under the } v(t) \text{ from } t_1 \text{ to } t_2)$$

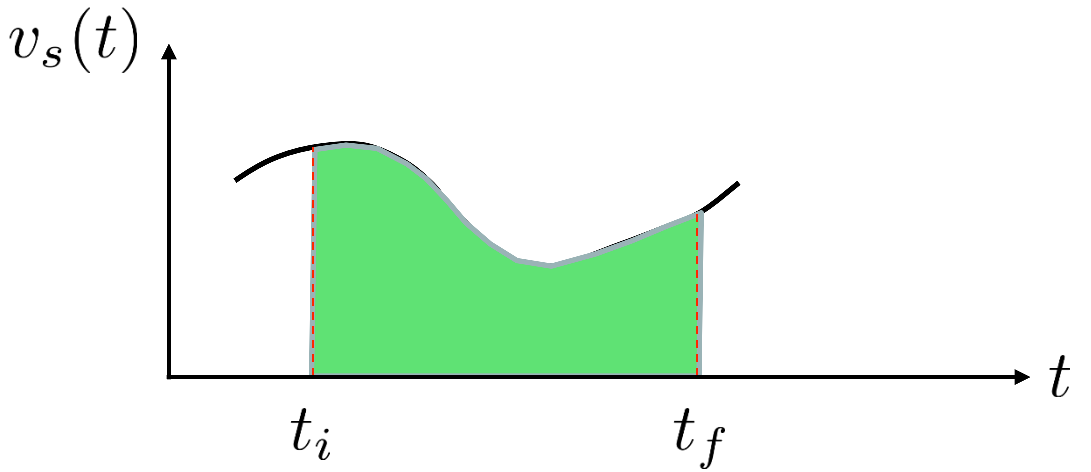
If you have velocity for some time, you ***accumulate*** position. In calculus, we call this accumulation, **integration**.

$$\text{(area under the } v(t) \text{ from } t_1 \text{ to } t_2) = \int_{t_1}^{t_2} v(t)dt$$

How to get Position from Velocity – General case

(SEC 2.3)

Easy for regular shapes, what about more complex velocity graphs?



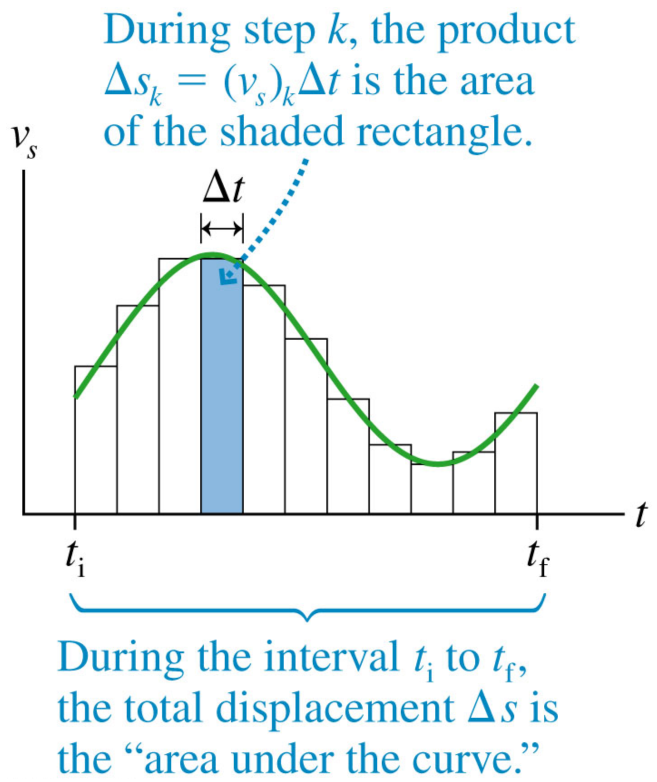
$$\text{The position at time } t_f = x_f = x_i + \int_{t_i}^{t_f} v_s(t)dt$$

$$= x_i + \text{ Area under the } v_s(t) \text{ curve from } t_i \text{ to } t_f$$

How do we compute integrals?

Finding Position From Velocity (SEC 2.3)

- The integral may be interpreted graphically as the total area enclosed between the *t*-axis and the velocity curve.
- The total displacement Δs is called the "area under the curve."



$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f$$

Finding Position from Velocity (SEC 2.3)

- Suppose we know an object's position to be s_i at an initial time t_i.
- We also know the velocity as a function of time between t_i and some later time t_f.
- Even if the velocity is not constant, we can divide the motion into N steps in which it is approximately constant, and compute the final position as:

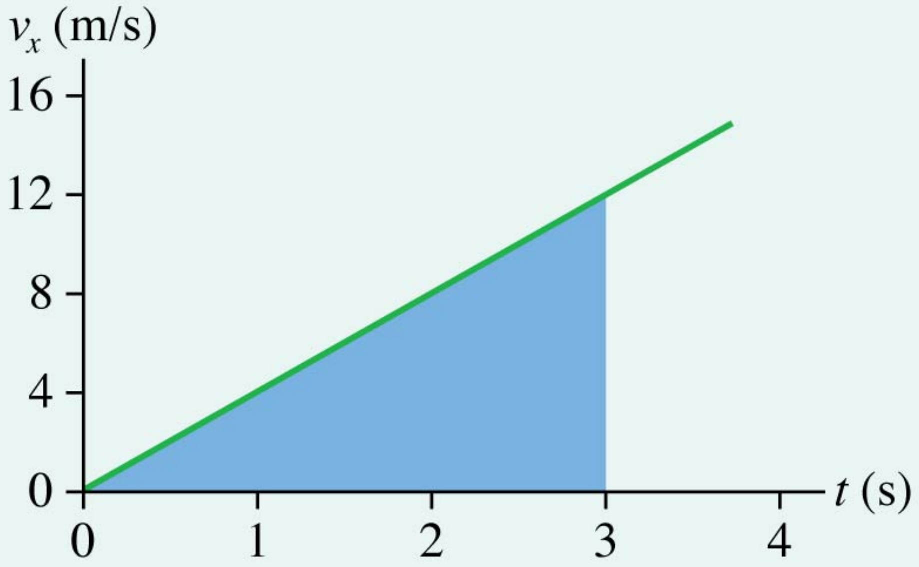
$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \, \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt$$

- The curlicue symbol is called an *integral*.
- The expression on the right is read, "the integral of $v_s\,dt$ from t_i to t_f."

Example 2.5 Finding Displacement from Velocity (SEC 2.3)

Example 2.5 The displacement during a drag race

The figure below shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?



Example 2.5 The Displacement During a Drag Race (SEC 2.3)

Example 2.5 The displacement during a drag race

MODEL Represent the drag racer as a particle with a well-defined position at all times.

SOLVE The question "How far?" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.12, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

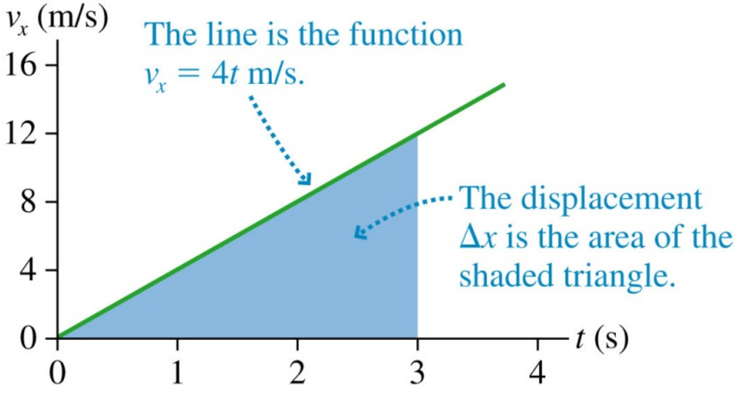
$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$

$$= \tfrac{1}{2} \times \text{base} \times \text{height}$$

$$= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The drag racer moves 18 m during the first 3 seconds.

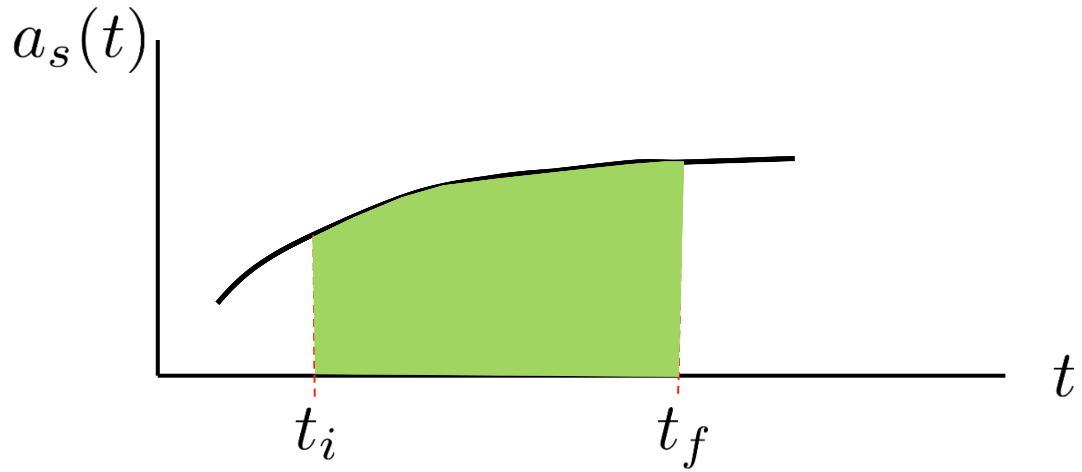
ASSESS The "area" is a product of s with m/s, so Δx has the proper units of m.



How to get Velocity from Acceleration - General

(SEC 2.7)

As you might guess, we get the velocity from the acceleration in a similar way



The velocity at time $t_f = v_f = v_i + \int_{t_i}^{t_f} a_s(t)dt$

$= v_i +$ Area under the $a_s(t)$ curve from t_i to t_f

Important Concept

(SEC 2.3 + 2.7)

- Acceleration is the derivative (slope) of velocity
- Velocity is the derivative (slope) of position

- Conversely position is the integral (area under *v(t)* curve) of velocity
- Velocity is the integral (area under *a(t)* curve) of acceleration

A Little More Calculus: Integrals (SEC 2.3)

- Taking the derivative of a function is equivalent to finding the slope of a graph of the function.
- Similarly, evaluating an integral is equivalent to finding the area under a graph of the function.
- Consider a function *u* that depends on time as $u(t) = ct^n$, where *c* and *n* are constants:

$$\int_{t_i}^{t_f} u\,dt = \int_{t_i}^{t_f} ct^n\,dt = \frac{ct^{n+1}}{n+1}\Bigg|_{t_i}^{t_f} = \frac{c{t_f}^{n+1}}{n+1} - \frac{c{t_i}^{n+1}}{n+1} \qquad (n \neq -1)$$

- The vertical bar in the third step means the integral evaluated at t_f minus the integral evaluated at t_i.
- The integral of a sum is the sum of the integrals. If *u* and *w* are two separate functions of time, then:

$$\int_{t_i}^{t_f} (u + w)\,dt = \int_{t_i}^{t_f} u\,dt + \int_{t_i}^{t_f} w\,dt$$

Reminder of integral-evaluation

$$A) \int_{t_i}^{t_f} x^2 dx = \frac{x^3}{3}\Big|_{t_i}^{t_f} = \frac{(t_f)^3}{3} - \frac{(t_i)^3}{3} = \frac{1}{3}\left((t_f)^3 - (t_i)^3\right)$$

$$C) \int_4^9 11dx = \frac{11x}{1}\Big|_4^9 = 99 - 44 = 55$$

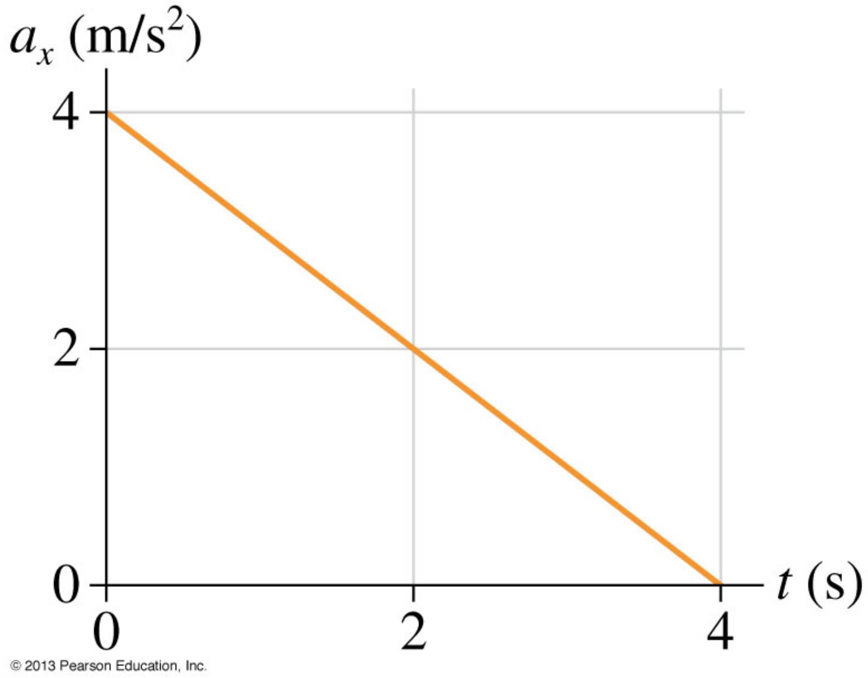
$$B) \int_0^t 4x^5 dx = \frac{4x^6}{6}\Big|_0^t = \frac{2t^6}{3} - \frac{2*0^6}{3} = \frac{2t^6}{3}$$

$$D) \int_{t_i}^{t_f} f(x)dx = \left(\frac{3x^7}{7} + \frac{2x^2}{2} + \frac{10x}{1}\right)\Big|_{t_i}^{t_f} = \left(\frac{3(t_f)^7}{7} + (t_f)^2 + 10(t_f)\right) - \left(\frac{3(t_i)^7}{7} + (t_i)^2 + 10(t_i)\right)$$

where $f(x) = 3x^6 + 2x + 10$

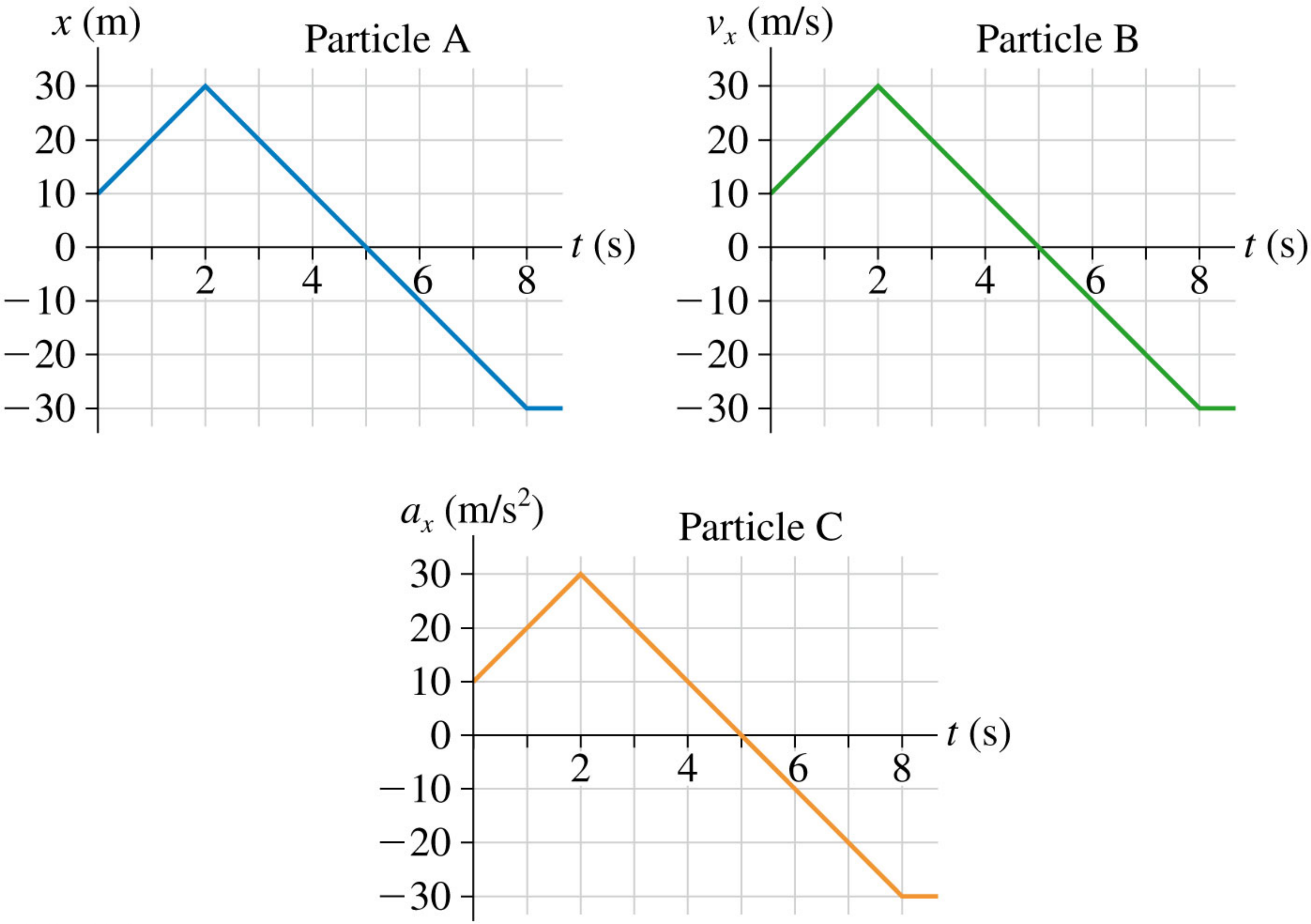
Whiteboard Problem 2.5 (Chapter 2, Problem 31)

The below graph shows the acceleration vs time graph of a particle moving long the x-axis. Its initial velocity is v_{x0} = 8.0 m/s at t_0 = 0 s. What is the particle's velocity at 4.0 s?



Whiteboard Problem 2.6

Three particles move along the x-axis, each starting with v_{0x} = 10 m/s at t_0 = 0 s. Find each particle's velocity at t = 7.0 s.



Quiz 1 (09/02)

Postponed to 09/07