Defining signature of waves!

INTERFERENCE & DIFFRACTION OF WAVES + 3 BASIC TENETS OF QUANTUM MECHANICS

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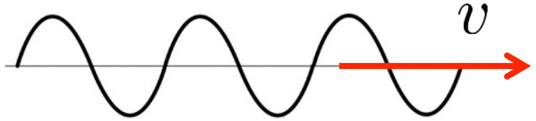
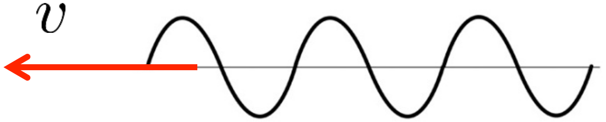
Sec. 17.2: Standing Waves

We can form a **Standing Wave** by superposing (i.e. adding together) two travelling waves of the same amplitude, wavelength, and frequency, but travelling in the opposite directions. 1D case: Watch a video.

The name Standing Wave comes from the fact that the resultant wave has an amplitude that oscillates in place.

Right Travelling Wave | **Left Travelling Wave**





$$D_R(x,t) = a\sin(kx - \omega t) \qquad D_L(x,t) = a\sin(kx + \omega t)$$

The Resultant Wave: $$D(x,t) = D_R(x,t) + D_L(x,t)$$

$$D(x,t) = \underbrace{2a\sin kx}_{A(x),\ \text{amplitude function}}\cos\omega t$$

What does this look like In 2D and 3D?!

Sec. 17.2: Standing Waves

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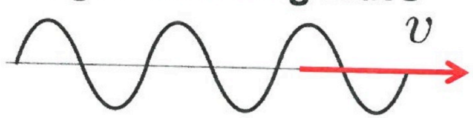
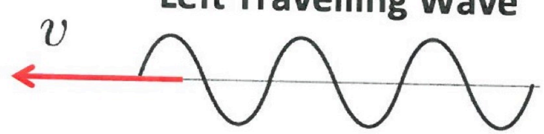


$$D_R(x,t) = a\sin(kx - \omega t)$$

Left Travelling Wave



$$D_L(x,t) = a\sin(kx + \omega t)$$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

The Resultant Wave: $D(x,t) = D_R(x,t) + D_L(x,t)$

$= a[\sin(kx-\omega t)+\sin(kx+\omega t)]$

$= 2a\sin kx\cos(-\omega t)$

$\cos(-\theta) = \cos\theta$

$$D(x,t) = \underbrace{2a\sin kx}_{A(x),\ \text{amplitude function}}\cos\omega t$$ ✓

NOTE!
NO MATTER WHAT TIME, STANDING WAVE IS ZERO AT LOCATIONS WHERE $\sin kx = 0$

What does this look like?

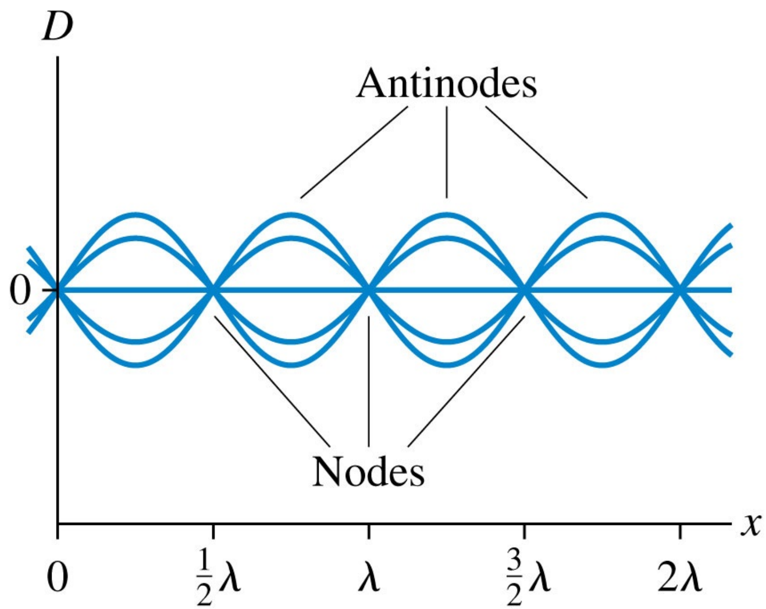
In 2D and 3D?!

(Back to 1D) Standing Wave Terminology

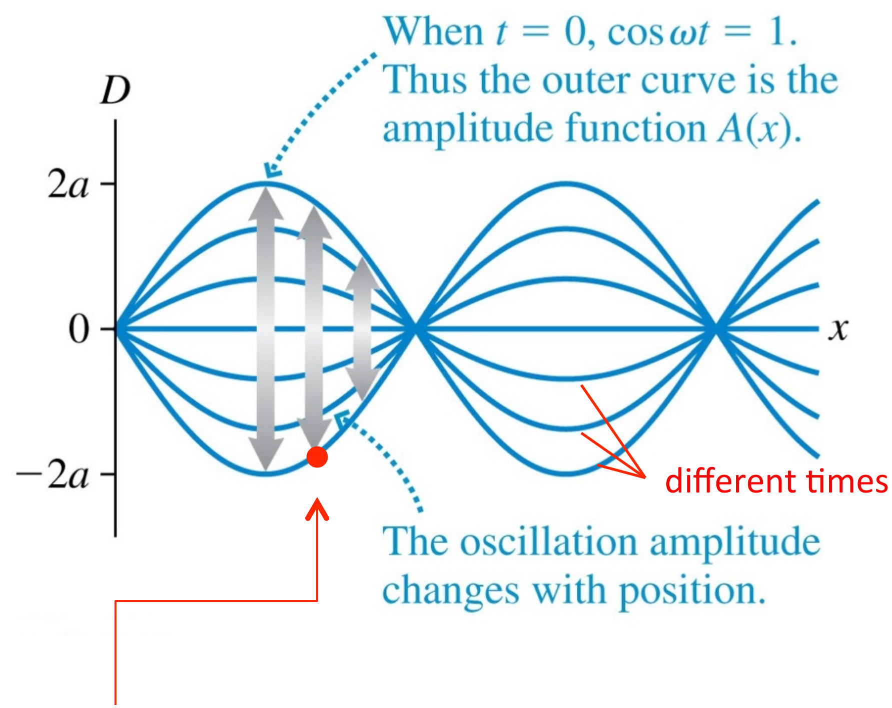
$$D(x,t) = 2a\sin kx\cos\omega t = 2a\sin\frac{2\pi x}{\lambda}\cos\omega t = A(x)\cos\omega t$$

When $\frac{x}{\lambda} = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \Rightarrow$ No displacement, a **Node**

When $\frac{x}{\lambda} = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots \Rightarrow$ Displacement is maximum, an **Antinode**



The nodes and antinodes are spaced $\lambda/2$ apart.



A point here has SHM with Amplitude A(x)

Sec. 17.3: Standing Waves on a String

For a string clamped at both ends, what are the wavelengths and frequencies of the allowed standing waves?

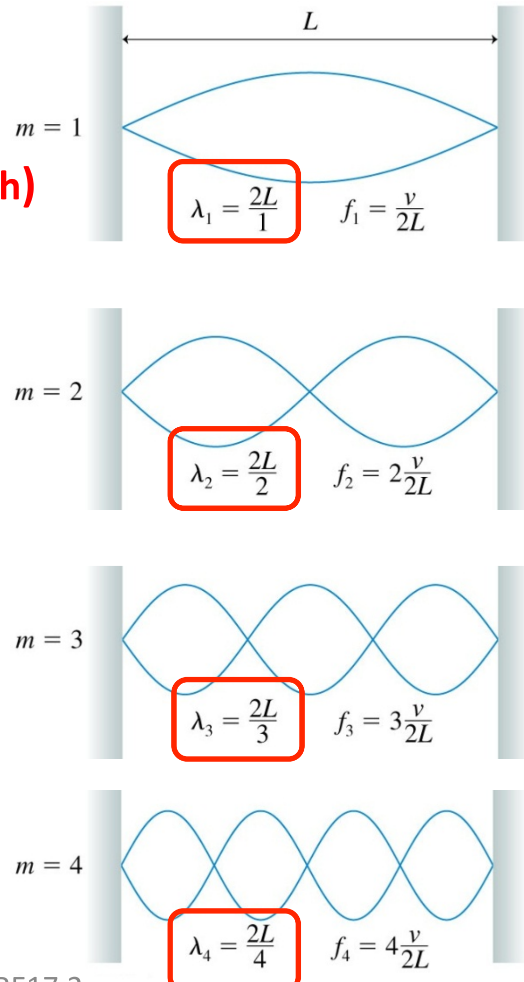
Standing wave condition: Both ends must be a node. So an integer number of half wavelengths can fit on the string.

"Fundamental" (longest wavelength)

"Second Harmonic"

"Third Harmonic"

"Fourth Harmonic"



So, following the pattern, the allowed wavelengths are:

$$\lambda_n = \frac{2L}{n}, \text{ where n = 1, 2, 3...}$$

And the allowed frequencies (v = wavespeed) are:

$$f_n = v / \lambda_n = n\, v / (2L) = n\, f_1$$

$$\text{where: } f_1 = \frac{v}{2L}$$

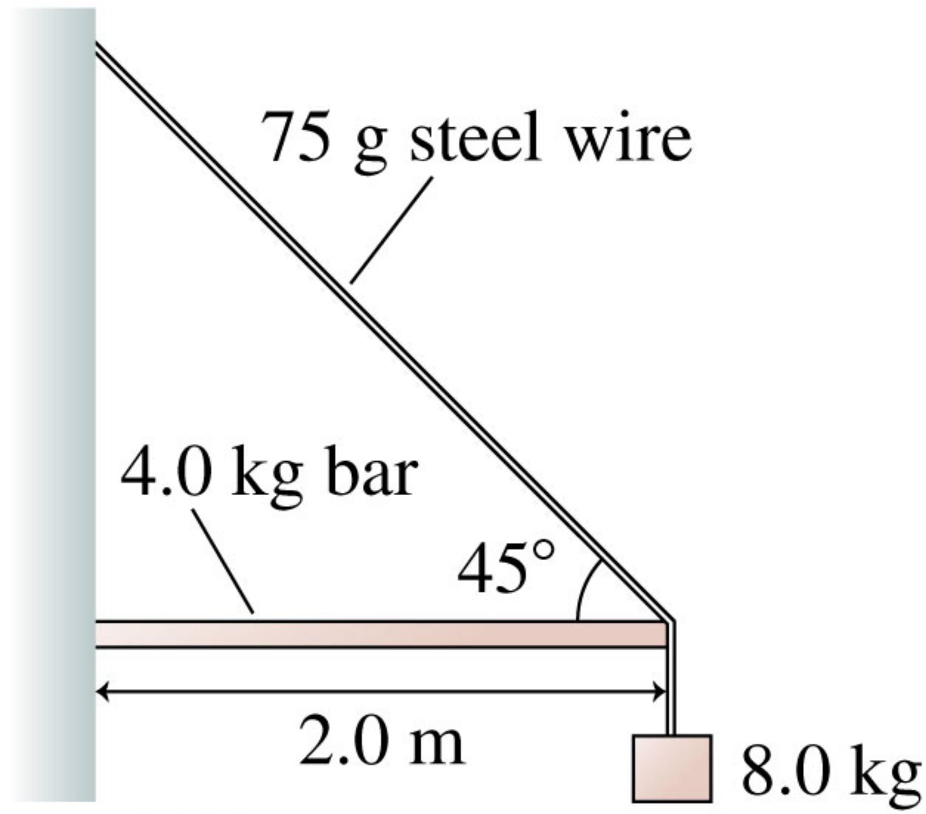
Note the important fact:

The n^{th} harmonic has n antinodes.

Whiteboard Problem 4

(A good review problem)

What is the fundamental frequency of the steel wire in the figure?



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THE FIRST BASIC TENET OF QUANTUM MECHANICS: ENERGY QUANTIZATION **(simplified description of Sections 38.4 & 38.5)**

An electron is a wave.

Q: How should I draw an atom now?

THE FIRST BASIC TENET OF QUANTUM MECHANICS: ENERGY QUANTIZATION

(simplified description of Sections 38.4 & 38.5)

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A: The concept of ENERGY QUANTIZATION

(where do the Bohr levels of an atom come from?)

THE FIRST BASIC TENET OF QUANTUM MECHANICS: ENERGY QUANTIZATION

(simplified description of Sections 40.2 – 40.4)

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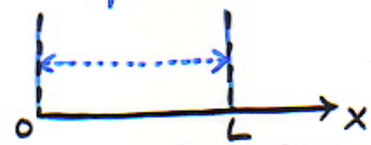
Replace

by a simpler 1-D atom



Electron going back & forth like a "particle in a box".

See Fig. 38.14

THE FIRST BASIC TENET OF QUANTUM MECHANICS: ENERGY QUANTIZATION
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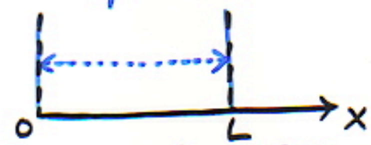

Electron going back & forth like a "particle in a box".

• Electron wavefunction $\Psi(x,t) = A \sin(kx - \omega t + \phi)$

See Fig. 38.14

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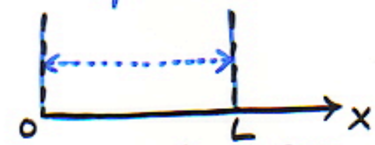

See Fig. 38.14

Electron going back & forth like a "particle in a box".

- Electron wavefunction $\psi(x,t) = A\sin(kx - \omega t + \phi)$
- Electron does not exist outside walls

$\therefore\ \psi(x \leq 0) = \psi(x \geq L) =$ ____

$\Rightarrow \psi$ has ____ at both ends.

THE FIRST BASIC TENET OF QUANTUM MECHANICS: ENERGY QUANTIZATION (simplified description of Sections 38.4 & 38.5)

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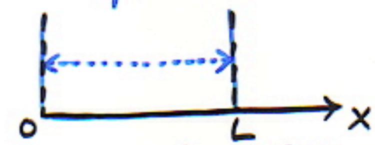

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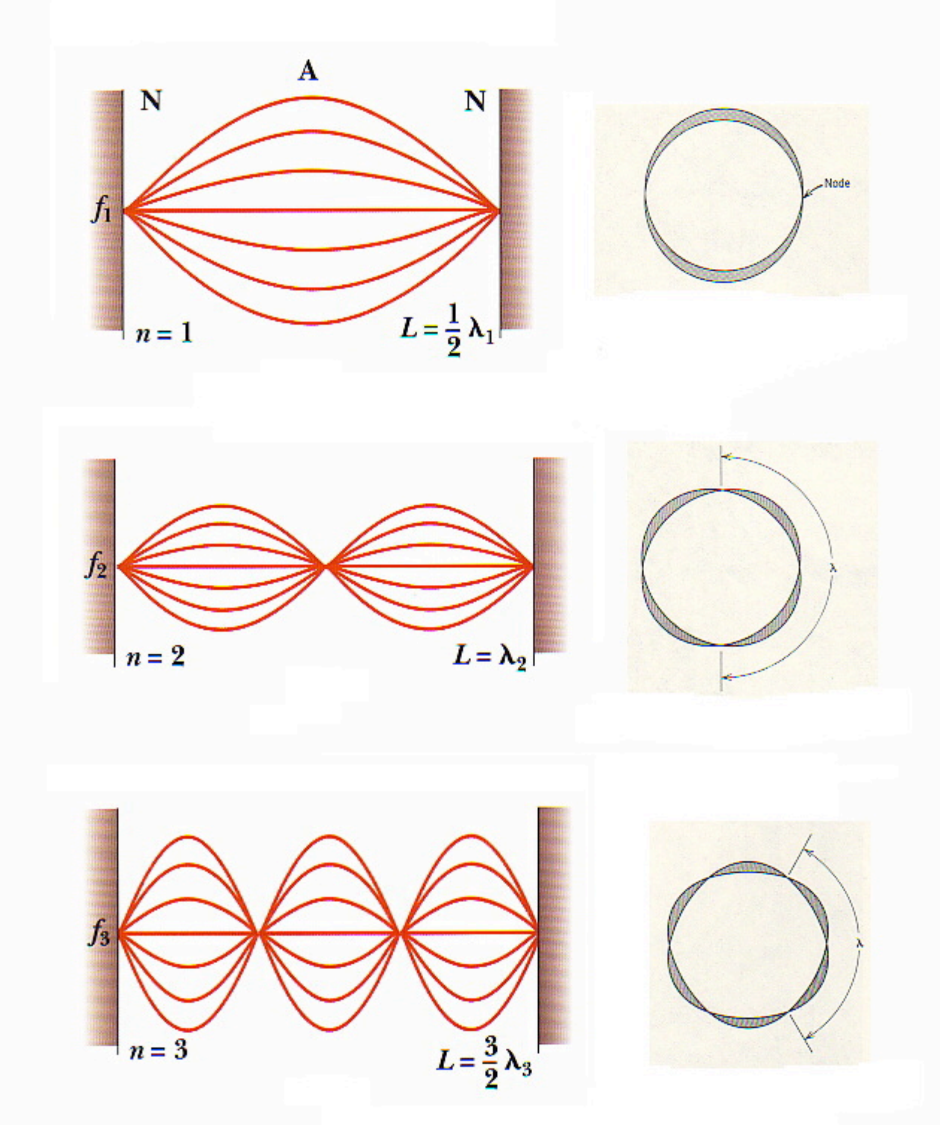
$\therefore \; \psi(x \leq 0) = \psi(x \geq L) =$ ______

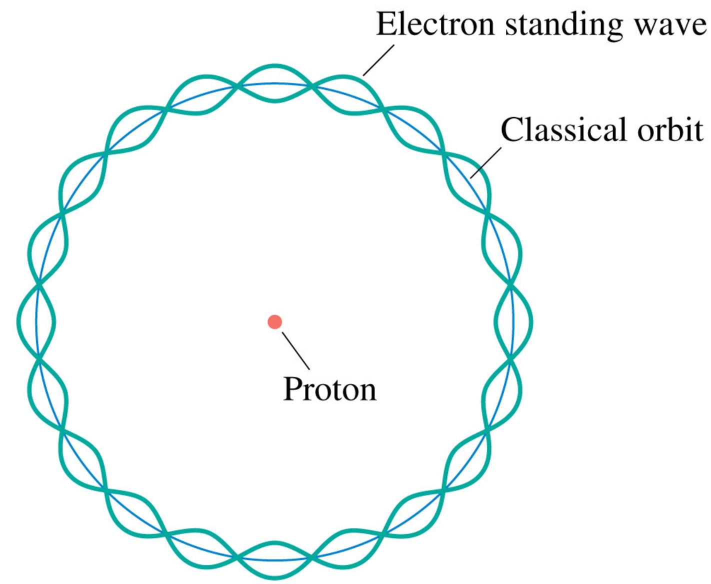
$\Rightarrow \psi$ has ______ at both ends.

- ψ is beginning to remind us of ______ ______ !!

Sec. 38.4: BOHR MODEL - THE ENERGY-LEVELS OF AN ATOM ARE QUANTIZED! (see Fig. 38.18)



$$\lambda_n = 2L / n, \text{ where } n = 1, 2, 3\ldots$$



An $n = 10$ electron standing wave around the orbit's circumference.

- In a Bohr hydrogen atom, the wave-like electron sets up a circular standing wave.
- The orbital circumference is an integer number (10 in this case) of the electron's wavelength

Defining signature of waves!

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Use Videos to prove: “Particles can behave like Waves”

- Recall from the pbs-nova video: Brian Greene’s “Fabric of the Cosmos” that you watched just after Thanksgiving that the **double-slit interference experiment** with *electrons* (not just water waves) yields an interference pattern!

  This means electrons must be waves, not particles!

- Let’s re-watch a ~7 minute segment from 14:07 to 20.33 from Brian Greene’s “Fabric of the Cosmos” located at https://www.youtube.com/watch?v=eCFTVdExxPA

- Next, let’s watch another video on the double-slit experiment, with electrons. http://www.youtube.com/watch?v=DfPeprQ7oGc

BUT WAVES CAN BEHAVE LIKE PARTICLES TOO!

BEATS &

THE 2ND BASIC TENET OF QUANTUM MECHANICS:

WAVE-PARTICLE DUALITY

Defining signature of waves!

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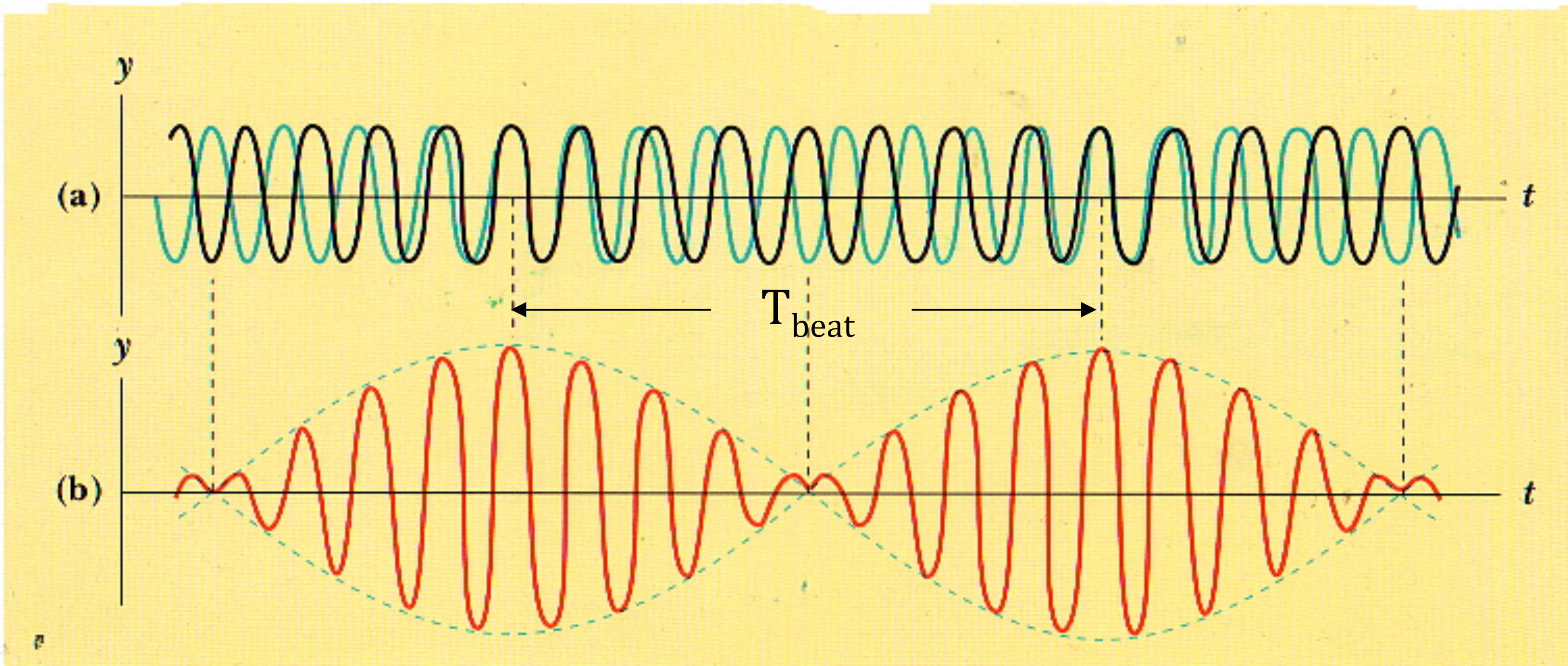
Sec. 17.8: Beats - Wave Interference in Time

Back to 1D waves. What do we get if we add two waves together that are travelling in the same direction, *but have slightly different frequencies*?



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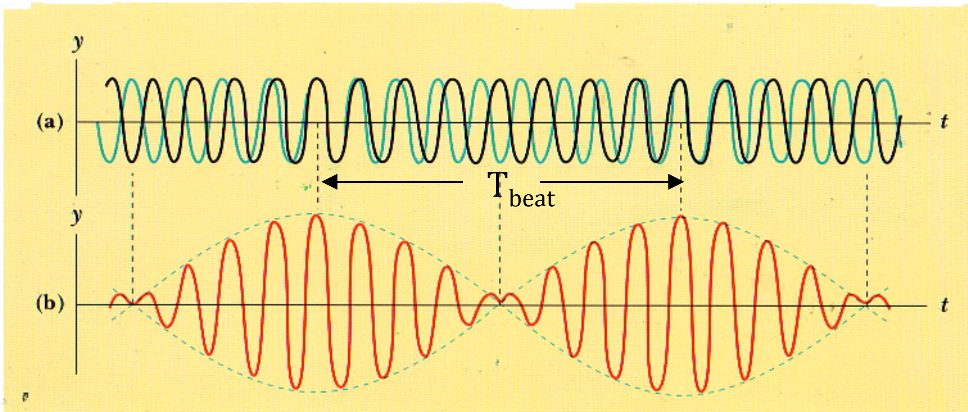


observes the sum of the two waves

Q: What is the beat frequency $f_{beat} = 1 / T_{beat}$?

Sinusoidal sound waves emanating from tuning forks = D_1 (x, t) and D_2 (x, t) respectively
Where $D_1(x, t) = A \sin(k_1 x - \omega_1 t + \phi_1)$ and $D_2(x, t) = A \sin(k_2 x - \omega_2 t + \phi_2)$

- Phase difference $\Delta\phi$ =
- The initial phases ϕ_1 , ϕ_2 are irrelevant for understanding Beats because____________
- You, located at x = __________________________ hear Beats, which means spatial location is irrelevant too for understanding Beats!
- Constructive interference if
- Destructive interference if
- BEAT PERIOD T_{beat}:

Therefore, **Beat frequency f_{beat}** =

Whiteboard 6: Problem # 17.75

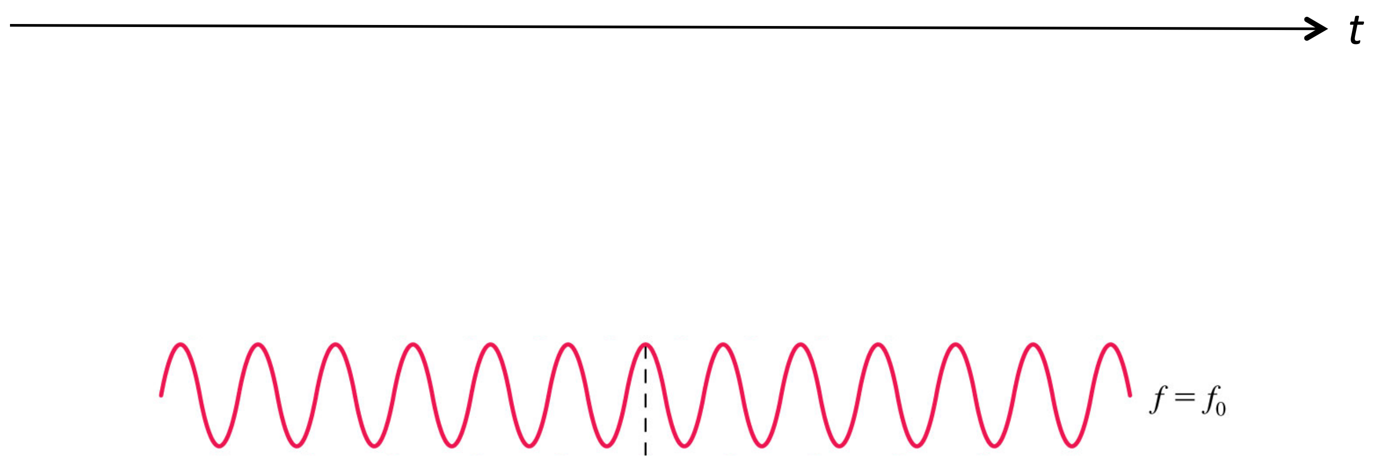
Recap of Doppler effect and application of beats

Two loudspeakers emit 400 Hz notes. One speaker sits on the ground. The other speaker is in the back of a pickup truck. You hear 8 beats per second as the truck drives away from you. What is the truck's speed?

Sec. 39.5: Waves can behave like Particles

Q: How can Waves give rise to a (fuzzy) Particle? A: "FUZZY PARTICLE" = WAVEPACKET!

A Wave Packet Is the Superposition of Many Component Waves



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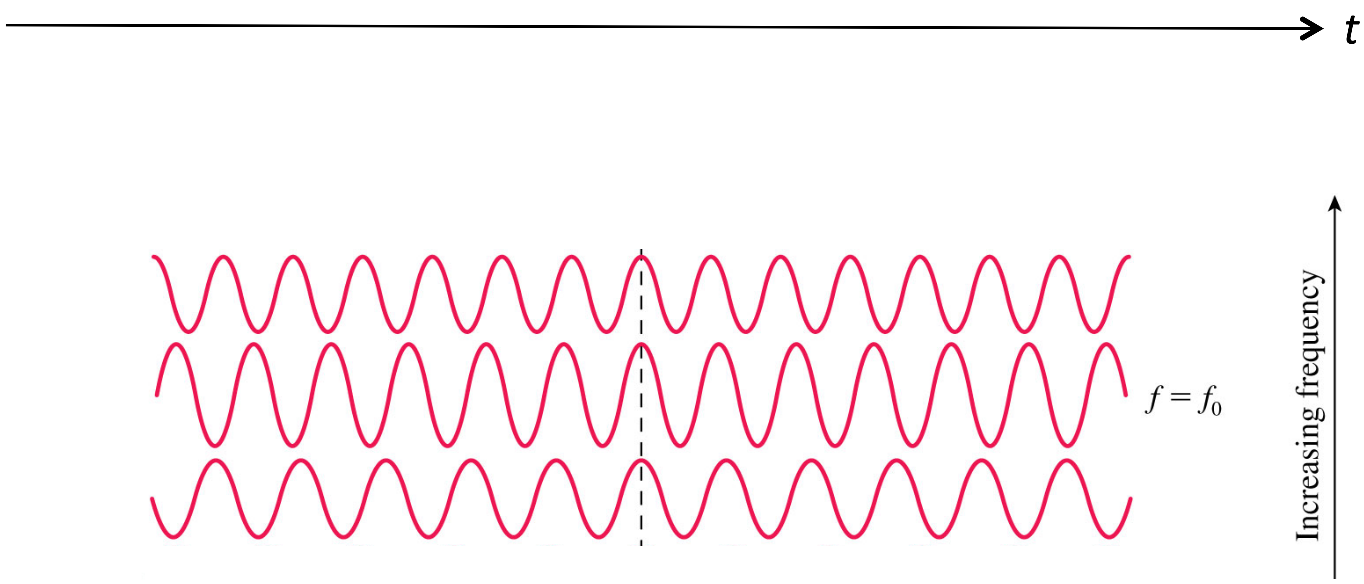

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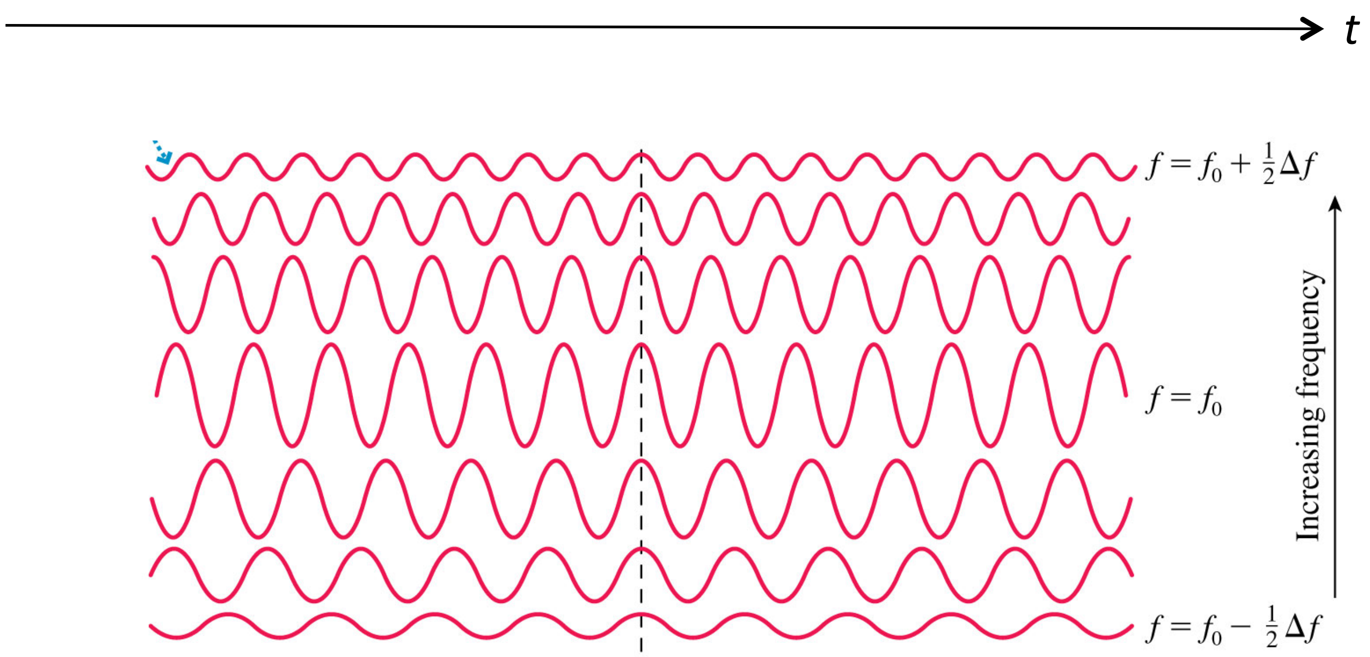

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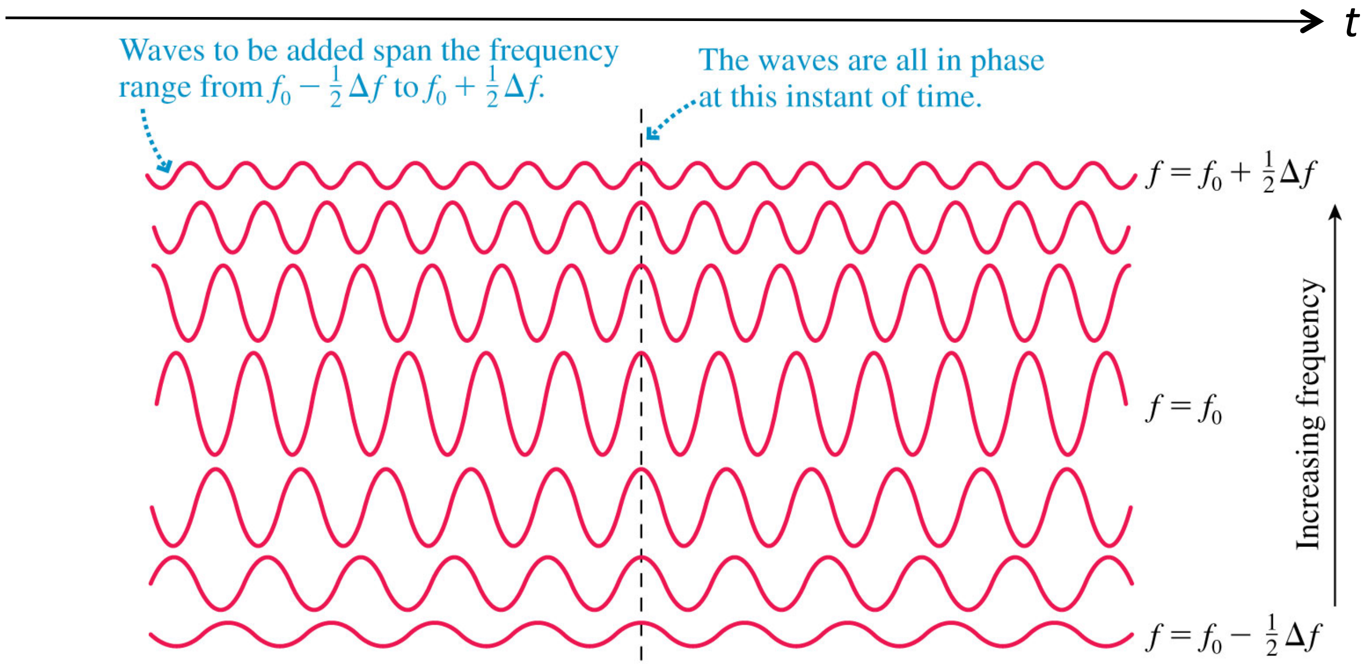

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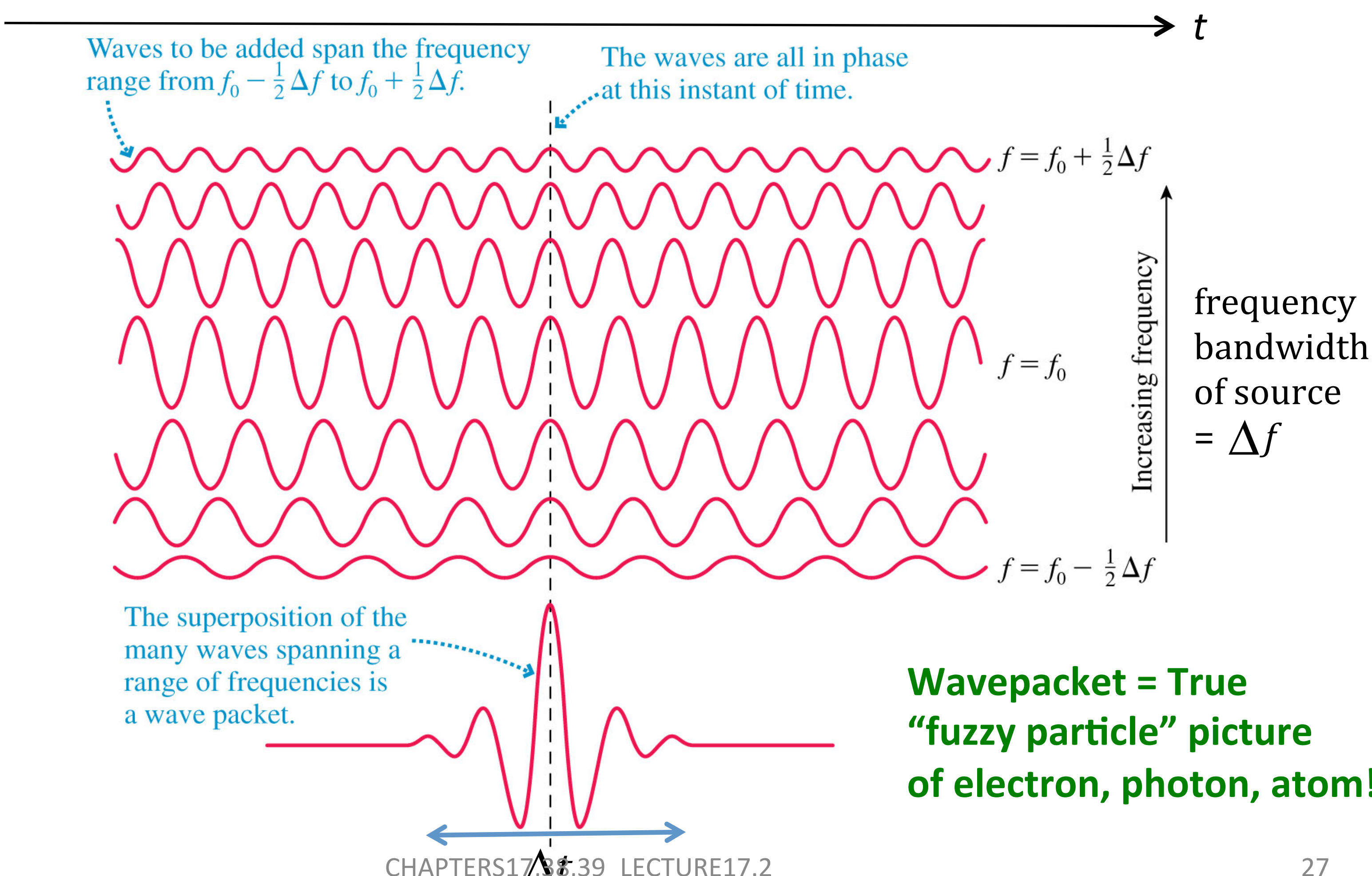

Wavepacket = True “fuzzy particle” picture of electron, photon, atom!

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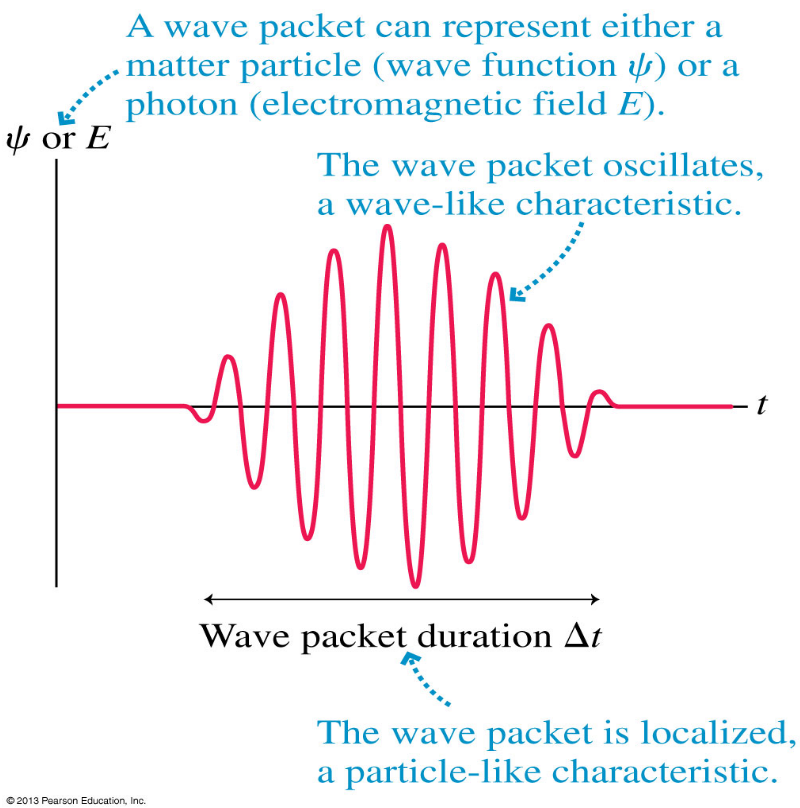
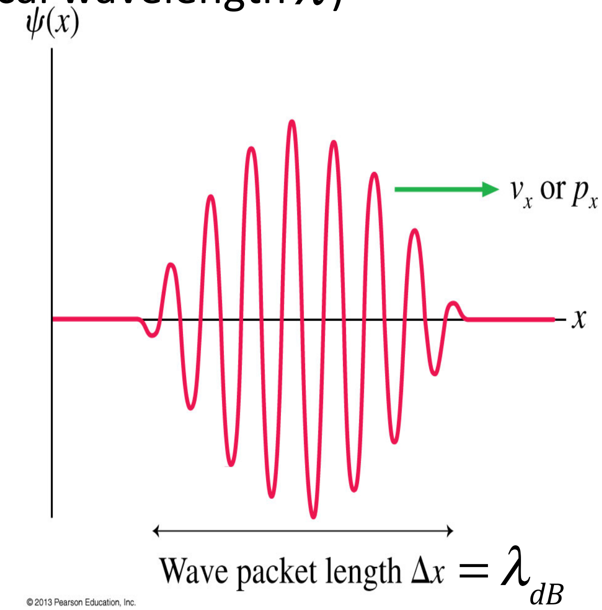
Sec. 39.5: Fuzzy spatial extent of wavepacket = DeBroglie Wavelength

History graph of wavepacket



Relation between time-duration Δt of wavepacket and frequency bandwidth Δf of source:
$\Delta t = 1 / \Delta f$.

Snapshot of wavepacket
(Note: For a photon, λ_{dB} is just the optical wavelength λ)



The spatial extent of the wavepacket is given by the deBroglie wavelength: $\lambda_{dB} = h / p$
where $p = mv$ is the momentum of a particle of mass m moving with velocity v.
λ_{dB} *is the "wavelength of the (fuzzy) particle".*

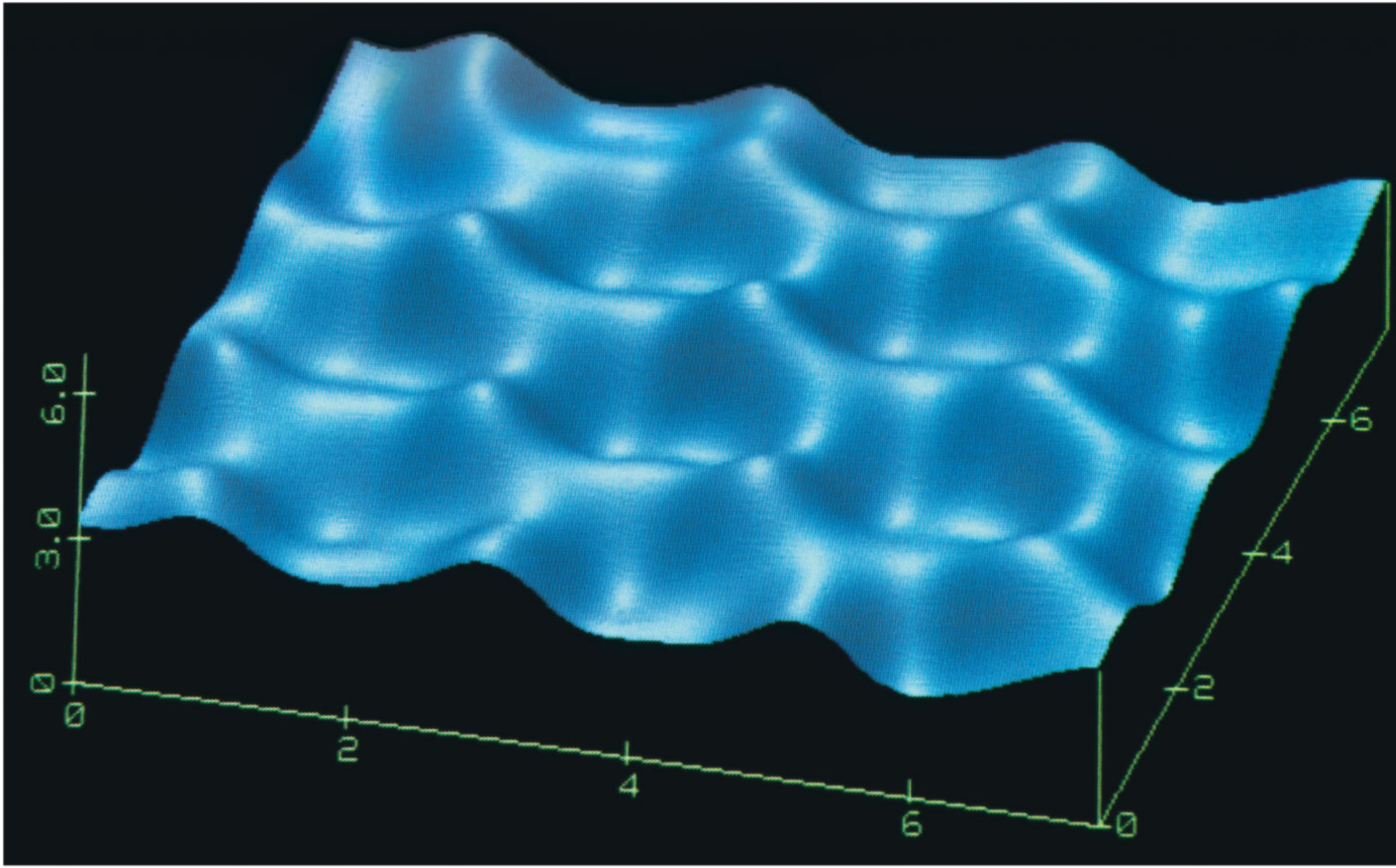
Whiteboard 7

The deBroglie wavelength of a photon is simply the wavelength λ. For yellow light λ = 0.5 μm. In Samir's's lab downstairs, at what speed should an ultra cold Rubidium atom be moving such that the atom's deBroglie wavelength equals the wavelength of yellow light? [A Rubidium atom has an atomic weight of 85, meaning it has a total of 85 protons and neutrons. Mass of proton ~ mass of neutron = 1.67 x 10^{-27} kg]

Nobel prize 2001, 2005, 2012: New state of matter forms when the *deBroglie wavelength for each atom* **exceeds** *the interatomic separation* in the sample…this is called a **Bose-Einstein Condensate**.

Let's watch http://www.youtube.com/watch?v=nAGPAb4obs8
And, if time permits, https://www.youtube.com/watch?v=1RpLOKqTcSk&t=3s

Applications: Atom laser, Nanofabrication, Quantum Computer….



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Surface of graphite, imaged with atomic resolution by a scanning tunnel microscope. The hexagonal ridges show the most probable locations of the electrons (Chapter 39 cover photo).

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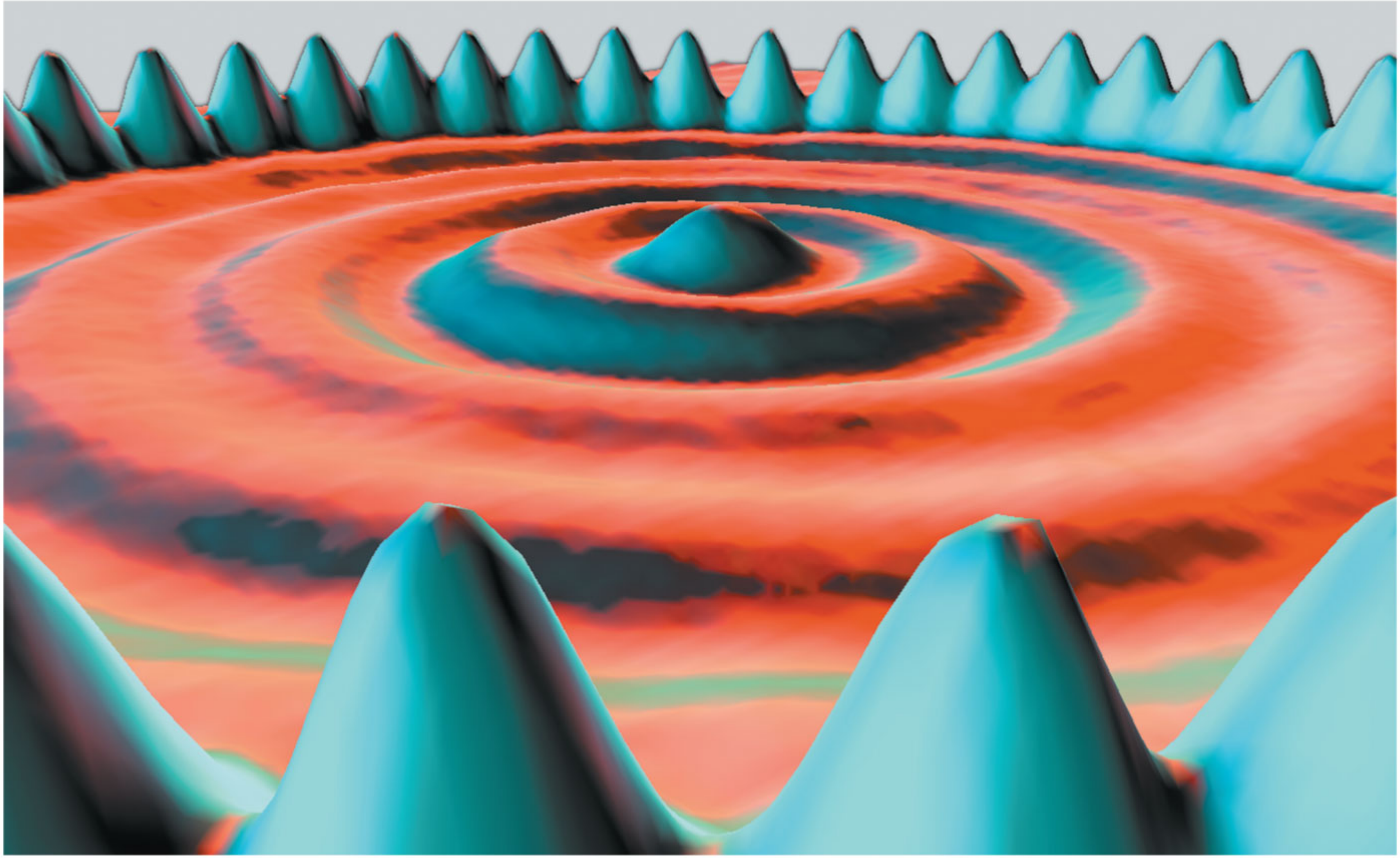
In this example of atomic engineering, 35 Xenon atoms have been manipulated with the probe tip of a scanning tunnel microscope (Chapter 40 cover photo).



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A scanning tunnel microscope image shows an electron standing wave in a “quantum corral” made from 60 iron atoms (Chapter 38 cover photo).

Defining signature of waves!

INTERFERENCE & DIFFRACTION OF WAVES + 3 BASIC TENETS OF QUANTUM MECHANICS

- What does interference mean? (Sections 17.1 & 17.5)
  - Constructive and Destructive interference
  - Both Transverse and Longitudinal Waves interfere

- Standing Waves & the *1st basic tenet of QM:* Energy Quantization
  - Standing Waves created by 2 traveling waves in opposite direction; Nodes and Antinodes (Sections 17.2, 17.3, Figs. 17.5, 17.6, 17.9)
  - The energy-levels of an atom are quantized (Bohr model) because the electron is a wave! (classnotes)

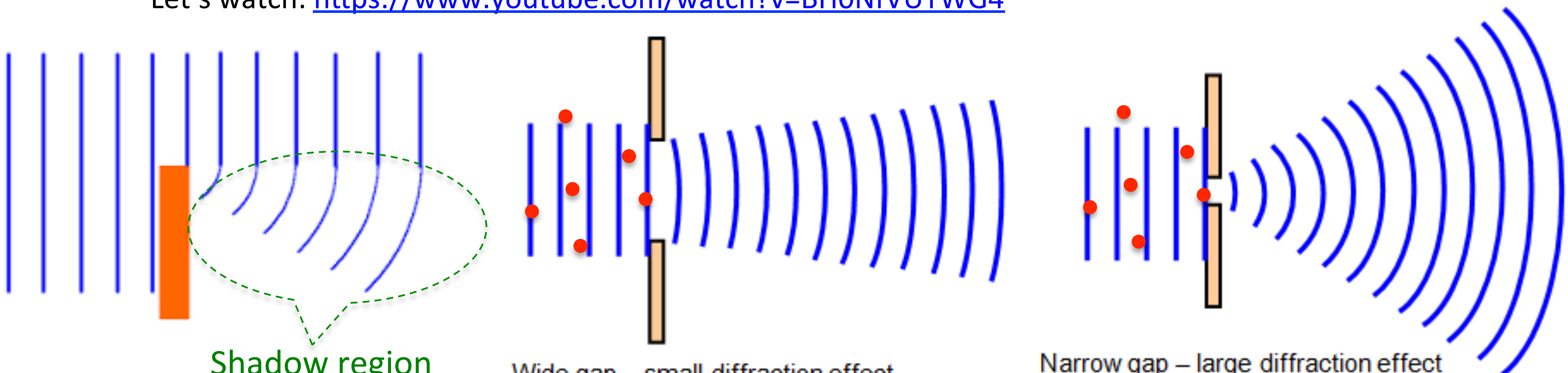
- Beats & the *2nd basic tenet of QM:* Wave-Particle Duality
  - Waves can behave like Particles
    - Consider Beats (Demo with tuning forks) (Section 17.8).
    - Many sinusoidal waves added together yield a wavepacket.
    - Time-duration of wavepacket and frequency bandwidth of the source are related.
    - Spatial extent of wavepacket – deBroglie wavelength (Section 38.4 and 39.5)
    - Can the deBroglie wavelength of an electron, or of an atom, equal the optical wavelength?
  - Particles can behave like Waves
    - Consider the two-slit interference experiment (Remember the "Fabric of Cosmos" video?)
    - Electrons interfere! Demo: Davisson-Germer expt
    - Atoms interfere! New state of Matter – Bose-Einstein condensate (1997, 2001 Physics Nobels)

- Diffraction and the *3rd basic tenet of QM:* Heisenberg Uncertainty Principle (Sec. 39.6)
  - Waves diffract, i.e., bend around obstacles

Diffraction and the *3rd basic tenet of QM:* Heisenberg Uncertainty Principle

Distinctive property of Waves: They "diffract" or "bend around" obstacles.

Let's watch: https://www.youtube.com/watch?v=BH0NfVUTWG4



Statement of Uncertainty for Waves: (Known this for centuries)
"The more precisely you try to locate a small part of the wave and give that part a specific direction, the less precisely you end up knowing where that part is headed."

Statement of Uncertainty for **Particles** (photons, electrons, atoms):
THE HEISENBERG UNCERTAINTY PRINCIPLE (1927) **[Sec. 39.6]**
"The more precisely you try to locate the position of a moving particle, the less precisely you know where that particle is headed (i.e., it's momentum)."

HUP: Uncertainty in particle's position (Δx) and uncertainty in particle's momentum (Δp) are related by $\Delta x \,.\, \Delta p \geq h/2$, where h is Planck's constant and is equal to 6.6 x 10^{-34} J-s.

Corollary of HUP: You can never precisely know either the position or the momentum of a particle (b/c neither Δx nor Δp can become zero!)

Whiteboard problem 5: # 39.23

HUP (in 1D)

Andrea, whose mass is 50 kg, thinks she is sitting at rest in her 5.0 m long dorm room as she does her physics homework. But HUP says she cannot be completely at rest. According to HUP within what range is her velocity likely to be? With this velocity how much distance would Andrea move in 14 billion years (14×10^9 years is the estimated age of the universe) ?

In the hydrogen atom the velocity of the electron as it whizzes around the atomic nucleus has an upper bound of 0.007 c = 2×10^6 m/s. Assuming this to be the velocity uncertainty Δv, calculate the position uncertainty Δx of the electron (remember: mass of electron = 9.1×10^{-31} kg).
Compare Δx with the size of the hydrogen atom (diameter = 0.1 nm).

Whiteboard problem 5: # 39.23

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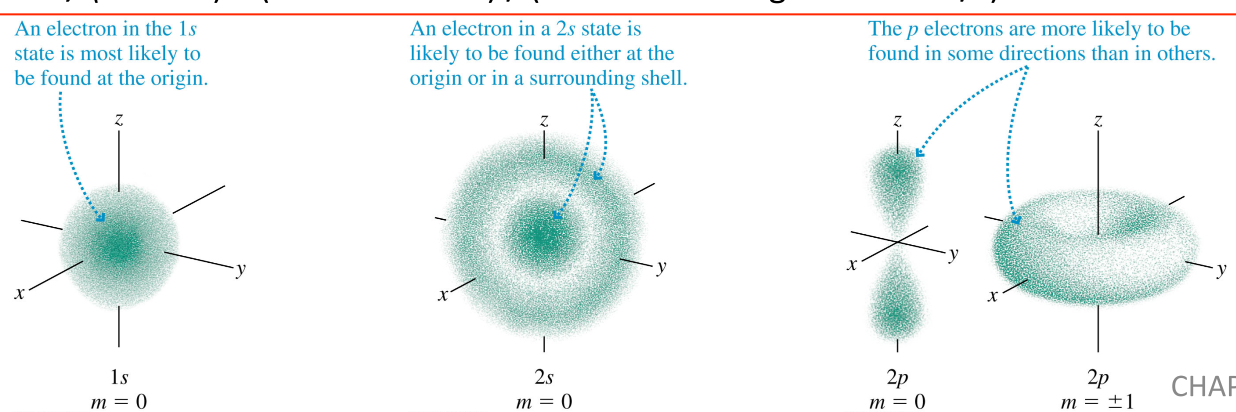
B/c HUP says she's moving, we must conclude her position uncertainty Δx could be as large as 5.0 m.
Then, according to HUP, her momentum uncertainty is $\Delta p = h / 2\ \Delta x$
and her velocity uncertainty (remember $p = mv$, so $\Delta p = m\Delta v$, and $\Delta v = \Delta p / m = h / 2m\Delta x$
Therefore, $\Delta v = (6.63 \times 10^{-34}$ J-s) / (2 x 50kg x 5m) = 10^{-36} m/s (!)
In 14 billion years, Andrea would have moved ~ 10^{-19} m, which is several orders of magnitude less than the size of an atomic nucleus! It is safe to say she *is effectively at rest*.

In the hydrogen atom the velocity of the electron as it whizzes around the atomic nucleus has an upper bound of 0.007 c = 2×10^6 m/s. Assuming this to be the velocity uncertainty Δv, calculate the position uncertainty Δx of the electron (remember: mass of electron = 9.1×10^{-31} kg).
Compare Δx with the size of the hydrogen atom (diameter = 0.1 nm).

$\Delta x = h / (2\ m\Delta v)$ = (6.63×10^{-34} J-s) / ($2 \times 9.1 \times 10^{-31}$ kg x 2×10^6 m/s) = 0.2 nm = 2 X size of atom!



Probability density cloud for electron in hydrogen atom for ground state and 1st few excited states

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What are the quantized energies for the 1D particle-in-the-box model of the Bohr atom?

From "standing waves" we learnt that $\lambda_n = 2L / n$ - - - - - - - - - - (1)
So Eqn. (1) gives the wavelength for an electron whizzing around a nucleus in an atom.

But we also know that the electron wavelength is given by the deBroglie relation:
$\lambda = h / p$ - - - - - - - - - (2)

Equations (1) and (2) are describing the same thing, and are hence equal.

Now the energy of the electron may be simply thought of as its Kinetic Energy (we're off by a factor of 2 here, but let's ignore that for now). Thus the energy is

$E = ½ mv^2 = ½ (mv)^2 / m = ½ p^2 / m$, which from Eqn. 2 is $½ (h / \lambda)^2 / m = ½ h^2 / (m\lambda^2)$.
Plonking in Eqn. 1 this becomes $E_n = ½ h^2 / (m (2L/n)^2)$, i.e.,

$$E_n = n^2 (h^2 / 8mL^2)$$ This is Eqn. 38.18 in the book

where we have snuck a subscript n under E to make clear that we have derived an expression for the energy of the n^{th} quantum orbit (or mode). This result is consistent with Bohr's model of the atom: The energy rises as n^2 as one moves away from the nucleus.