Defining signature of waves!

INTERFERENCE & DIFFRACTION OF WAVES + 3 BASIC TENETS OF QUANTUM MECHANICS

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  - Both Transverse and Longitudinal Waves interfere

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  - Waves can behave like Particles
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- Diffraction and the *3rd basic tenet of QM:* Heisenberg Uncertainty Principle (Sec. 39.6)
  - Waves diffract, i.e., bend around obstacles

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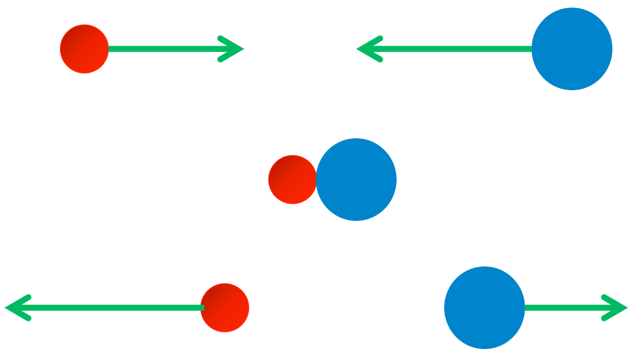
Sec. 17.1: The Superposition of Waves

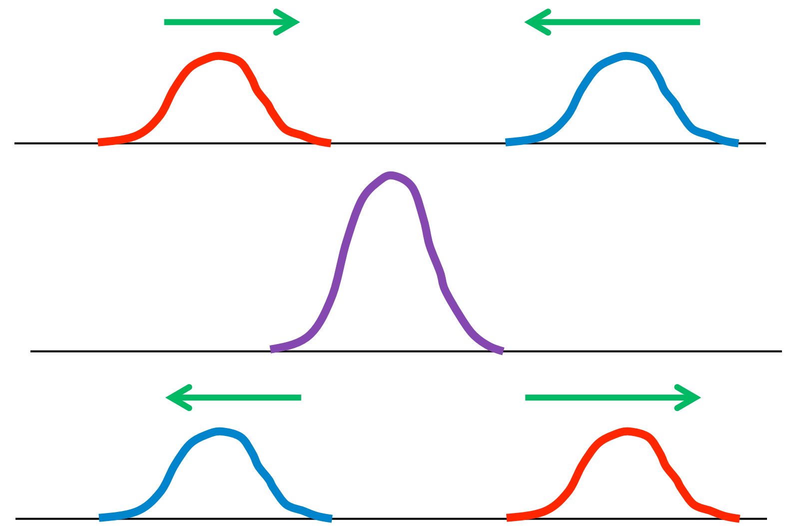
Perhaps the most important difference between particle behavior and wave behavior is what happens during collisions:

Waves:

Particles:

time





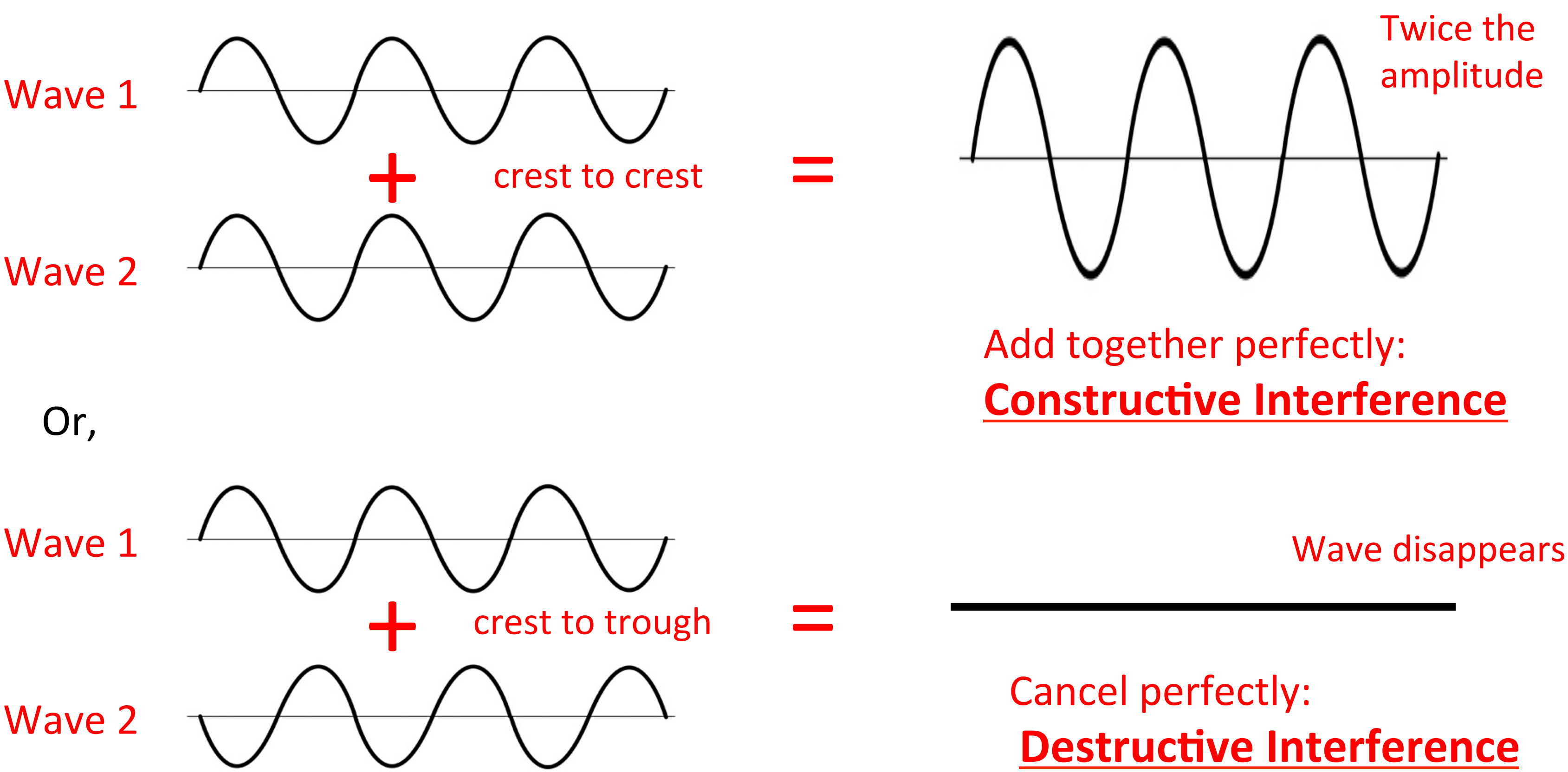
Particles can't occupy the same space; so they bounce off each other

Waves overlap and pass right through each other.

Principle of superposition When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

Sec. 17.5: 1D Wave Interference in Space w/ Sinusoidal waves

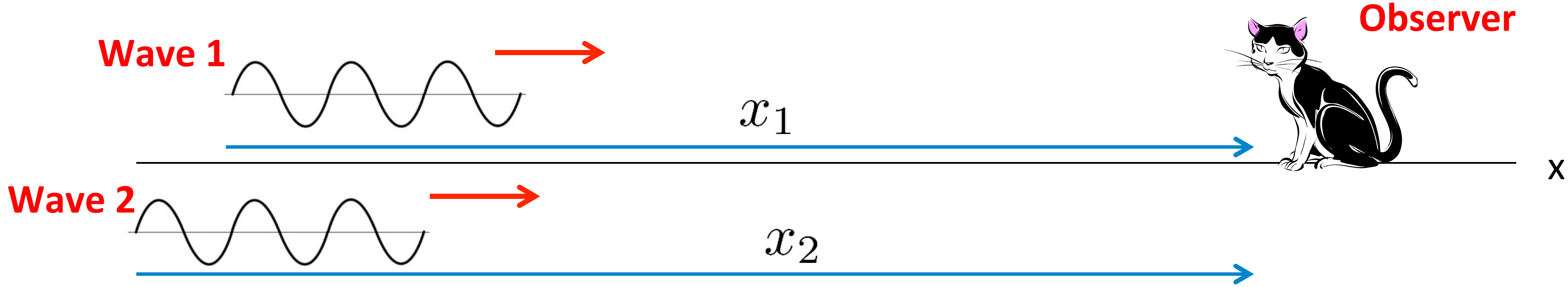
We've seen that when two waves add together (superpose), there is a possibility of:



In general, when two waves add together, you don't get anything interesting, **but these two conditions are worth a closer look.**

Sec. 17.5: 1D Wave Interference in Space w/ Sinusoidal waves

Consider two waves with the same amplitude, frequency, and wavelength that both travel in the +x direction.



At the location of the cat:

$$\text{wave 1:}\; D_1(x_1,t) = a\sin(kx_1 - \omega t + \phi_{10}) \;\; = a\sin\phi_1$$

$$\text{wave 2:}\; D_2(x_2,t) = a\sin(kx_2 - \omega t + \phi_{20}) \;\; = a\sin\phi_2$$

The cat observes the combined wave, and the way the waves combine at his location is totally determined by the **phase difference**.

$$\Delta\phi = \phi_2 - \phi_1$$

Perfect **Constructive Interference** for: $\Delta\phi = 0, 2\pi, 4\pi, \cdots = m\cdot(2\pi)$

Perfect **Destructive Interference** for: $\Delta\phi = \pi, 3\pi, 5\pi, \cdots = (m+\frac{1}{2})\cdot(2\pi)$

where: $m = 0, 1, 2, 3, \cdots$

Sec. 17.5: 1D Wave Interference in Space w/ Sinusoidal waves

Now, from the expressions for the two waves detected by the cat:

$$\begin{aligned}\Delta\phi &= (kx_2 - \omega t + \phi_{20}) - (kx_1 - \omega t + \phi_{10}) \\ &= k(x_2 - x_1) + (\phi_{20} - \phi_{10}) \\ &= \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0\end{aligned}$$

So, the two waves can have a phase difference for two reasons: a **path difference**; and an **initial (inherent) phase difference**.

So, we have the two **Interference Conditions**:

Maximum Constructive Interference: $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 = m \cdot 2\pi \text{ rad}$$

Perfect Destructive Interference: $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 = (m + \frac{1}{2}) \cdot 2\pi \text{ rad}$$

where for both: $m = 0, 1, 2, 3, \cdots$

Note: for two sources **initially in phase**, these conditions say that there's **constructive interference when the path difference is an integral number of wavelengths**, and **destructive interference when the path difference is a half integral number of wavelengths.**

Here's a practical example (noise reducing headphones) using this principle.

Whiteboard Problem 1

Two loudspeakers in a 20°C room emit 686 Hz sound waves along the x-axis.

a. If the speakers are in phase, what is the smallest distance between the speakers for which the interference of the sound waves is perfectly destructive?

b. If the speakers are out of phase, what is the smallest distance between the speakers for which the interference of the sound waves is maximum constructive?

Note: "In Phase" $\Rightarrow \Delta\phi_0 = 0$

"Out of Phase" $\Rightarrow \Delta\phi_0 = \pi$

Whiteboard Problem 1

Two loudspeakers in a 20°C room emit 686 Hz sound waves along the x-axis. $v = f\lambda \Rightarrow \lambda = \frac{343}{686} = 0.5\,m$

a. If the speakers are in phase, what is the smallest distance between the speakers for which the interference of the sound waves is perfectly destructive?

b. If the speakers are out of phase, what is the smallest distance between the speakers for which the interference of the sound waves is maximum constructive? $\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0$

Note: "In Phase" $\Rightarrow \Delta\phi_0 = 0$

"Out of Phase" $\Rightarrow \Delta\phi_0 = \pi$

a) $\Delta\phi_0 = 0$; $\Delta\phi = \pi, 3\pi, \ldots$

π, ~~3π~~, ~~$\ldots$~~ $= \frac{2\pi}{0.5}\Delta x + 0 \Rightarrow \Delta x = 0.25\,m$

b) $\Delta\phi_0 = \pi$; $\Delta\phi = 0, 2\pi, 4\pi, \ldots = \frac{2\pi}{0.5}\Delta x + \pi$

Can do either $0 = 2\pi\frac{\Delta x}{0.5} + \pi \Rightarrow \Delta x = -0.25\,m$

or $2\pi = 2\pi\,\Delta x / 0.5 + \pi \Rightarrow \Delta x = 0.25\,m$

Whiteboard Problem 2

Two loudspeakers emit sound waves along the x-axis. A listener in front of both speakers hears a maximum sound intensity when speaker 2 is at the origin and speaker 1 is at $x = 0.50$ m. If speaker 1 is slowly moved forward, the sound intensity decreases and then increases, reaching another maximum when speaker 1 is at $x = 0.90$ m.

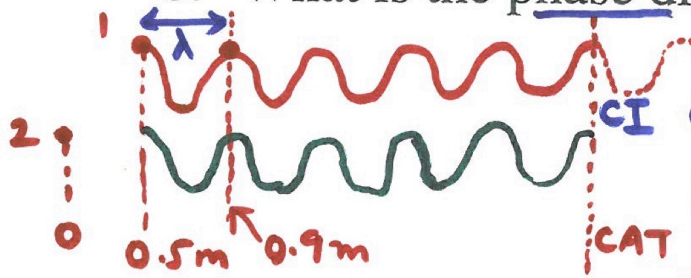
a. What is the frequency of the sound? Assume $v_{sound} = 340$ m/s.
b. What is the phase difference between the speakers?

Whiteboard Problem 2

Two loudspeakers emit sound waves along the x-axis. A listener in front of both speakers hears a maximum sound intensity when speaker 2 is at the origin and speaker 1 is at $x = 0.50$ m. If speaker 1 is slowly moved forward, the sound intensity decreases and then increases, reaching another maximum when speaker 1 is at $x = 0.90$ m.

a. What is the frequency of the sound? Assume $v_{sound} = 340$ m/s.

b. What is the phase difference between the speakers? ($\Delta\phi_0$)



CI when $\Delta x = 0.5$ m AND when $\Delta x = 0.9$ m

a) $\lambda = 0.4\text{m} \Rightarrow f = \frac{v}{\lambda} = \frac{340}{0.4} = 850\text{Hz}$

b) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0$

COULD USE

$\rightarrow \Delta x = 0.5\text{m}$; SP2 @ 0.5m

$0, 2\pi, \ldots = \frac{2\pi}{0.4}(0.5) + \Delta\phi_0$

$\Rightarrow \Delta\phi_0 = -2.5\pi$

$= -2\pi - 0.5\pi$

OR.... COULD USE

$\rightarrow \Delta x = 0.9\text{m}$; SP2 @ 0.9m

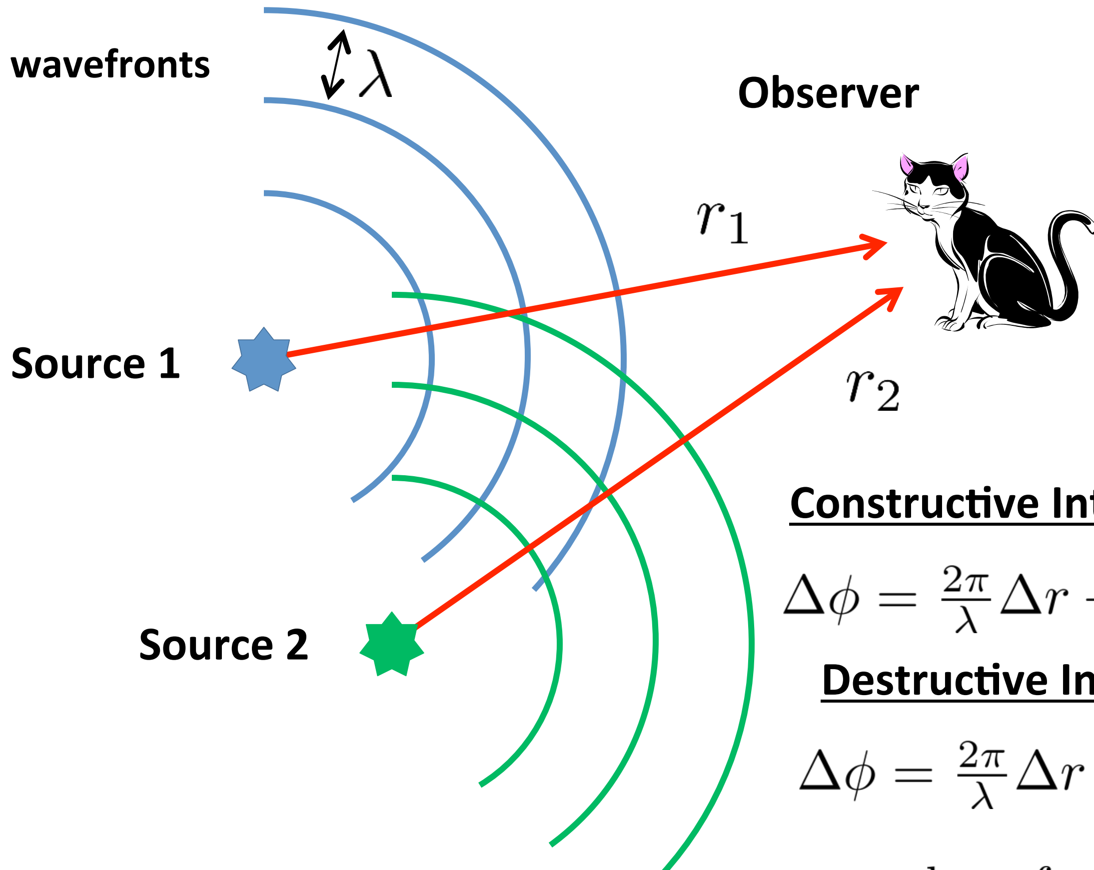
$0, 2\pi, \ldots = \frac{2\pi}{0.4}(0.9) + \Delta\phi_0$

$\Rightarrow \Delta\phi_0 = 2\pi - 2.5\pi = -0.5\pi$

SAME! GOOD !!

Sec. 17.7 Interference in Space in 2D and 3D

As we did for 1D, consider two waves of the same amplitude and frequency in 2D:



Again, the cat observes the combined wave and, the way the waves combine depends on their phase. The phase difference depends on the path difference and the initial phase.

Constructive Interference:

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta r + \Delta\phi_0 = m \cdot 2\pi \text{ rad}$$

Destructive Interference:

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta r + \Delta\phi_0 = (m + \tfrac{1}{2}) \cdot 2\pi \text{ rad}$$

$$\text{where for both: } m = 0, 1, 2, 3, \cdots$$

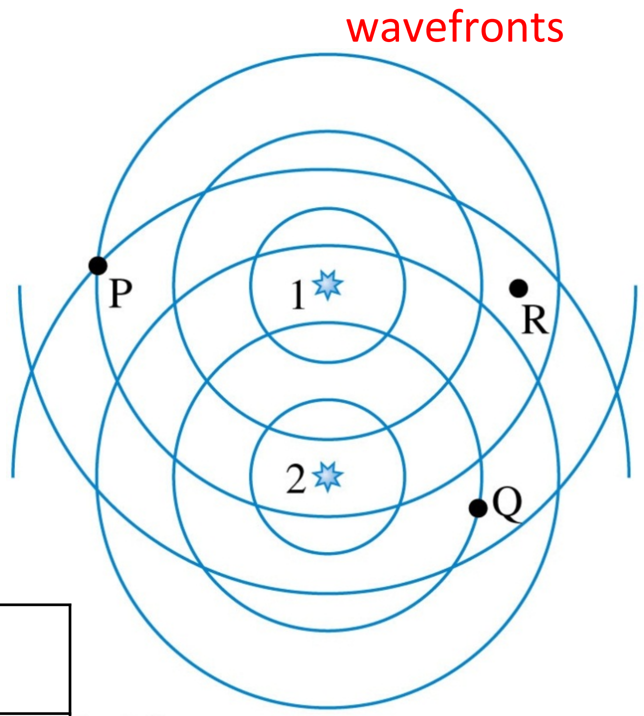
Note: these are the same conditions that we had for 1D, but x is replaced with r. The big difference here is in the geometry.

Sec. 17.7 Whiteboard Problem 3

FIGURE shows the circular wave fronts emitted by two wave sources.

a. Are these sources in phase or out of phase? Explain.

b. Make a table with rows labeled P, Q, and R and columns labeled r_1, r_2, Δr, and C/D. Fill in the table for points P, Q, and R, giving the distances as multiples of λ and indicating, with a C or a D, whether the interference at that point is constructive or destructive.



	r_1	r_2	Δr	C / D
P				
Q				
R				

Sec. 17.7 Whiteboard Problem 3

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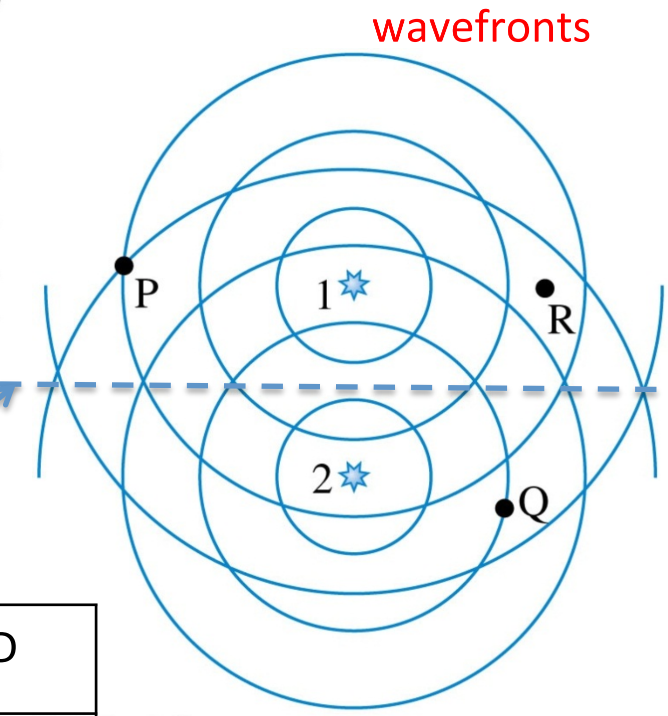
$\Delta r = 0$ along this line

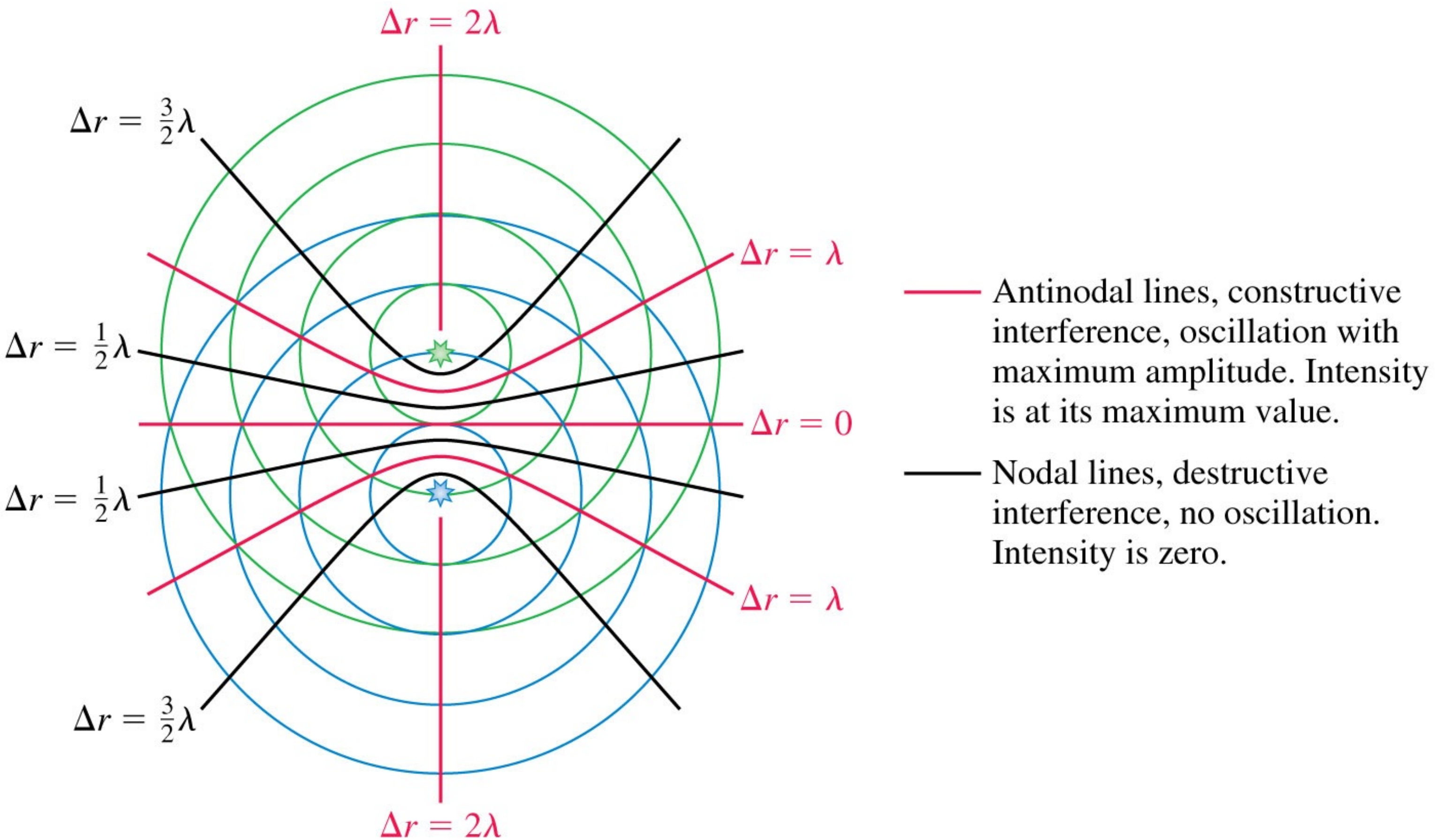
The fact that constructive interference is seen along this line indicates the 2 sources are in phase.



	r_1	r_2	Δr	C / D
P	3λ	4λ	λ	C
Q	3.5λ	2λ	1.5λ	D
R	2.5λ	3.5λ	λ	C

Sec. 17.7: 2D Interference Patterns in Space

If you extend WB Problem 3 over the entire 2D plane, you get patterns of interference. Shown below are interference patterns for **Two In-Phase Sources**:



antinodes

nodes

Remember, these are waves, so [they're moving!]

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