Chapter 12: Rotation of a Rigid Body

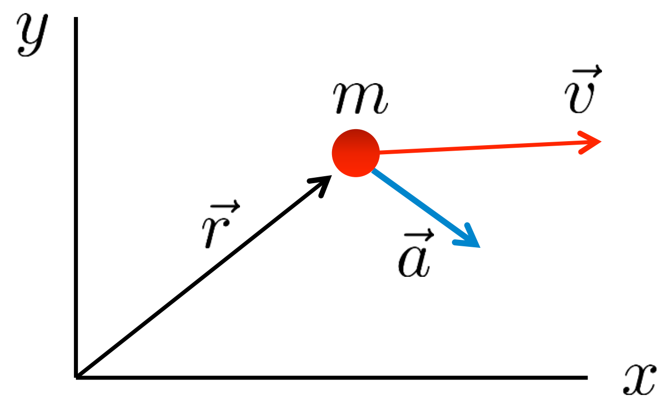
At the beginning of PHY191, we made the central observation that:

"Everything Moves!"

We have spent the entire time since then, analyzing this motion, but we have only considered one type of motion:

Translational Motion
Of a Point Particle

(picture an ice skater going back and forth across the ice)



But what else do ice skaters do? They spin – or rotate. [Sec. 12.1]

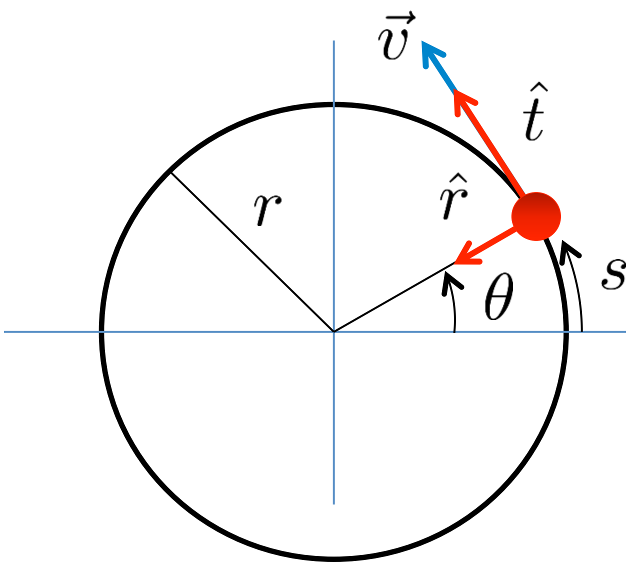
Real objects can have more motions than just translation. They can rotate (Chap 12), oscillate (i.e. vibrate) (Chap 14), and have wave motion (Chap 20). Many of the tools we have already developed can be used to treat these motions.

Warning: *Chap 12 is the longest in the book. Essentially, it repeats everything that we've done over 11 chapters for translational motion into one chapter for rotational motion*

Some Stuff from Circular Motion

Uniform and Nonuniform Circular Motion:

In Chap 4 , we first looked at circular motion.



Remember the radial ($\hat{r}$) and the tangential ($\hat{t}$) directions

For circular motion, we have the acceleration components:

$$a_r = \frac{v^2}{r}$$ (maintains the circular motion)

$$a_t = \frac{dv_t}{dt}$$ (changes the speed)

For $a_t =$ constant:

$$s_f = s_i + v_{t_i}\Delta t + \tfrac{1}{2}a_t(\Delta t)^2$$

$$v_{t_f} = v_{t_i} + a_t\Delta t$$

Or, using:

$$\theta = \frac{s}{r} \qquad \theta_f = \theta_i + \omega_i\Delta t + \tfrac{1}{2}\tfrac{a_t}{r}(\Delta t)^2$$

$$\omega = \frac{v_t}{r} \qquad \omega_f = \omega_i + \tfrac{a_t}{r}\Delta t$$

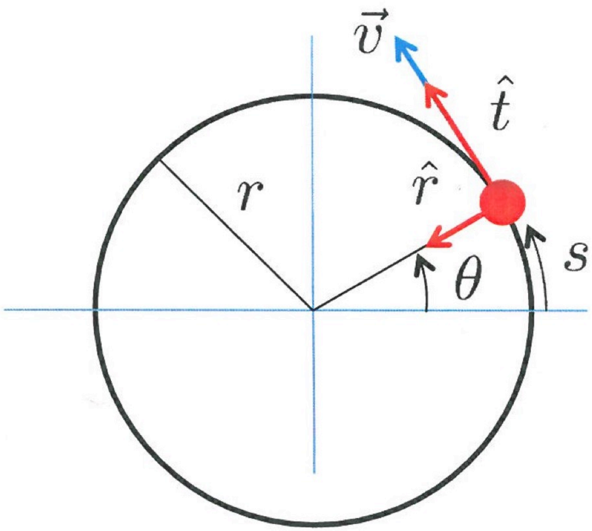
These types of equations can be used to describe the **rotation of a rigid body.**

Some Stuff from Circular Motion

Uniform and Nonuniform Circular Motion:

In Chap 4 , we first looked at circular motion.

θ : rad
ω : rad/s
α : rad/s^2



Remember the radial ($\hat{r}$) and the tangential ($\hat{t}$) directions

For circular motion, we have the acceleration components:

$$a_r = \frac{v^2}{r}$$ (maintains the circular motion)

$a_t = r\alpha$ ← angular acceleration

$$a_t = \frac{dv_t}{dt}$$ (changes the speed)

For $a_t =$ constant:

$$\Delta s_f = s_i + v_{t_i}\Delta t + \frac{1}{2}a_t(\Delta t)^2$$

$$v_{t_f} = v_{t_i} + a_t\Delta t$$

$$v_f^2 = v_i^2 + 2a_t\Delta s$$

Or, using:

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v_t}{r}$$

angular displacement

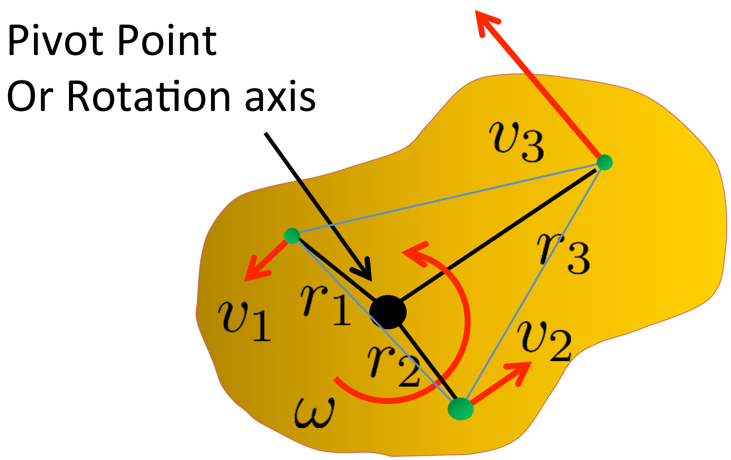
$$\Delta\theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\frac{a_t}{r}(\Delta t)^2 \quad \frac{a_t}{r} \to \alpha$$

$$\omega_f = \omega_i + \frac{a_t}{r}\Delta t \quad (\alpha)$$

These types of equations can be used to describe the **rotation of a rigid body.**

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$$

Rigid Body Rotation (Sec. 12.1)



What is a rigid body?

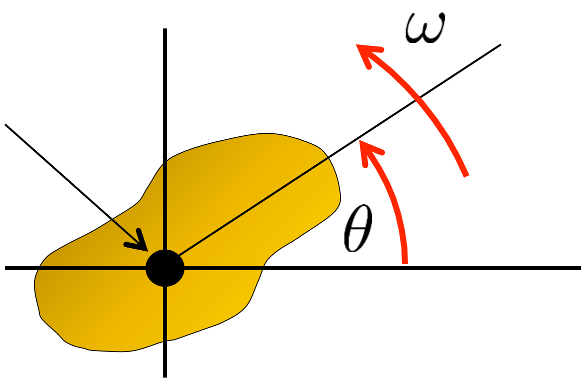
A rigid body is an extended object whose components don't move relative to each other.

Can be a finite collection of particles at fixed position or an infinite collection – i.e. a continuous body.

If the body is pivoted at some point, it is free to rotate around that point or about an axis through that point.

All points in the body have the same $\omega = \frac{v}{r}$, but different v

Rotational Kinematics (Sec. 12.1)



Basic Definitions:

$$\text{Angular Position} = \theta$$

$$\text{Angular Velocity} = \omega = \frac{d\theta}{dt}$$

$$\text{Angular Acceleration} = \alpha = \frac{d\omega}{dt}$$

Sign Convention:

CounterClockWise (CCW) is positive.

Rotational Kinematics for Constant Acceleration:

$$\text{For } \alpha = \frac{a_t}{r} = \text{constant between times } t_i \text{ and } t_f:$$

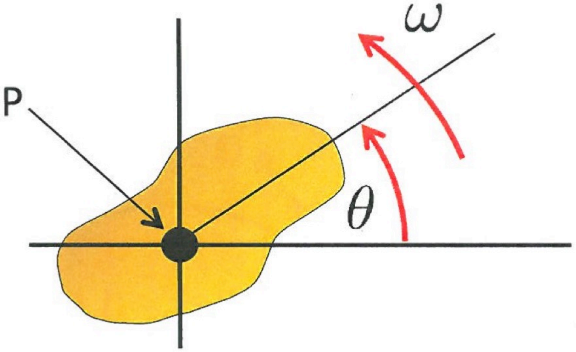
$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f = \omega_i + \alpha \Delta t \qquad \Delta t = t_f - t_i$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \qquad \Delta\theta = \theta_f - \theta_i$$

These equations should look familiar. They're the same that we used for 1D constant acceleration kinematics way back in Chap 2!

Rotational Kinematics



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Sign Convention:

CounterClockWise (CCW) is positive.

BUT...REALLY...USE RHR (see next page)

Rotational Kinematics for Constant Acceleration:

$$\text{For } \alpha = \frac{a_t}{r} = \text{constant between times } t_i \text{ and } t_f\text{:}$$

$$\theta_f = \theta_i + \omega_i \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

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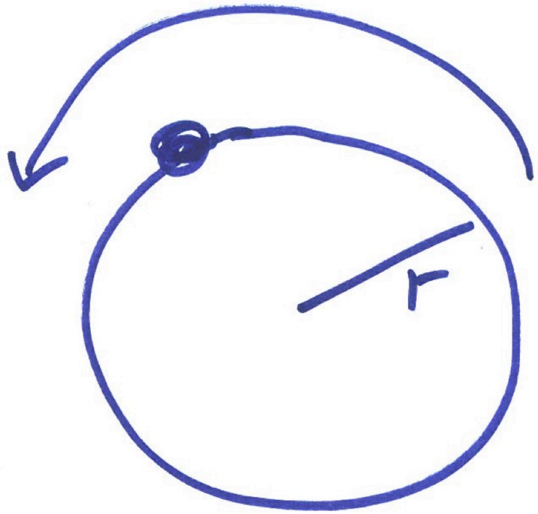
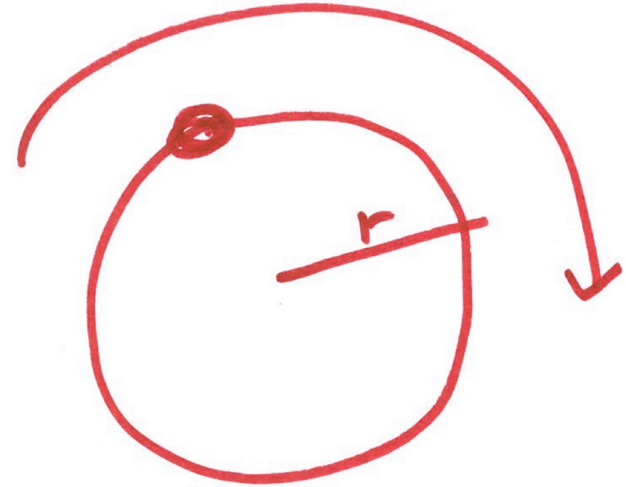
RHR ≡ RIGHT HAND RULE





ω ⊙ OUT-OF-PAGE ⊗ INTO PAGE

SPEEDING UP ⊙ " ⊗ INTO PAGE

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

SLOWING DOWN ⊗ INTO PAGE ⊙

Whiteboard Problem 1: Problem 12-1, A refresher

2. || A high-speed drill reaches 2000 rpm in 0.50 s.
   a. What is the drill's angular acceleration?
   b. Through how many revolutions does it turn during this first 0.50 s?

You may assume that the angular acceleration is constant.

In rotational problems, you do have to be careful with your units. Angular position, velocity, and acceleration can be used in a variety of units; <u>however, when you use an angular quantity to calculate a linear quantity, the angular quantity must be in radians.</u>

e.g. in $v = r\omega$, ω must be in radians/sec.

Whiteboard Problem 1: Problem 12-1, A refresher

$\omega_f = \frac{2000 \times 2\pi}{60}$ rad/s, $\omega_i = 0$

2. || A high-speed drill reaches 2000 rpm in 0.50 s.
   a. What is the drill's angular acceleration?
   b. Through how many revolutions does it turn during this first 0.50 s?

a) $\omega_f = \omega_i + \alpha \Delta t \Rightarrow \alpha = \frac{\frac{2000 \cdot 2\pi}{60} - 0}{0.5} = 419\,\text{rad/s}^2$

You may assume that the angular acceleration is constant.

b) $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2 = \frac{1}{2}(419)(0.5)^2 = 52$ rads

$\Rightarrow$ revs $= \frac{52}{2\pi} = 8.3$ revs.

In rotational problems, you do have to be careful with your units. Angular position, velocity, and acceleration can be used in a variety of units; however, when you use an angular quantity to calculate a linear quantity, the angular quantity must be in radians.

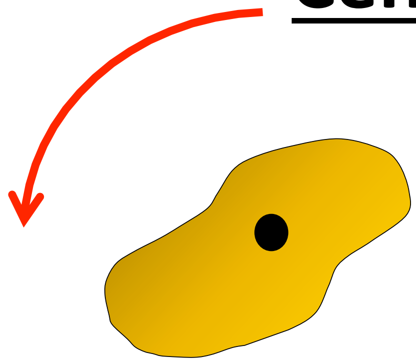
e.g. in $v = r\omega$, ω must be in radians/sec.

Two new concepts for "Rotation of a Rigid Body"

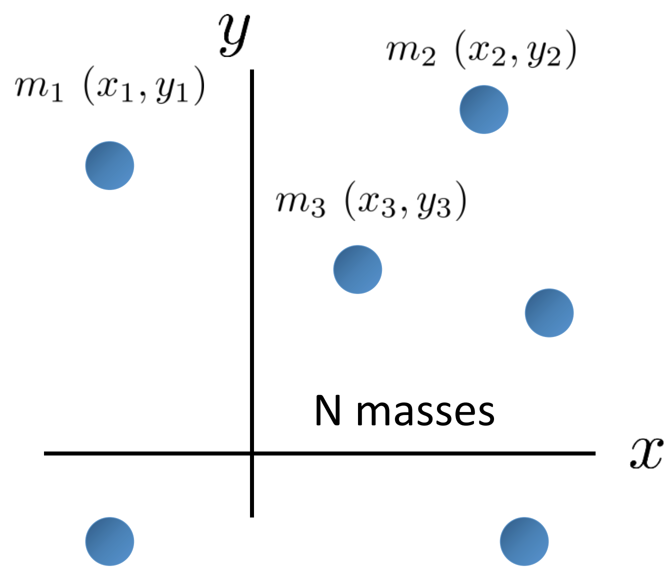
1. Center of Mass [Sec 12.2]

2. Moment of Inertia (this is the rotational analog of mass) [Sec 12.4]

Center of Mass (Sec. 12.2)



Center of Mass for a Discrete Collection of Point Masses:



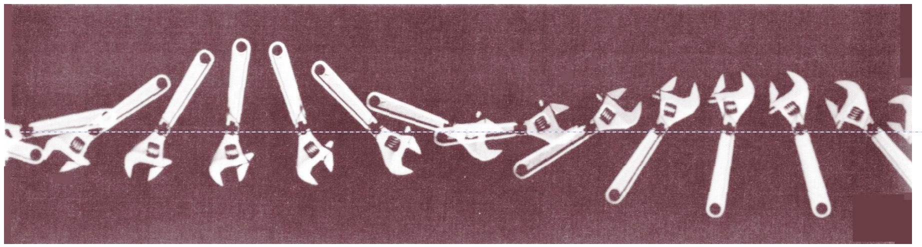
Center of Mass Coordinates:

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

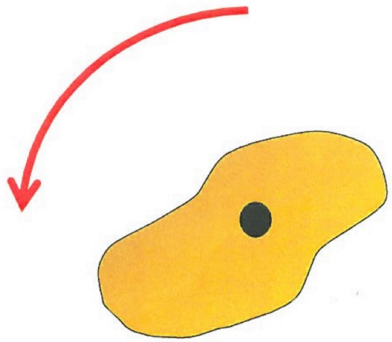
$$y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$\text{where}: M = \text{Total Mass} = \sum_{i=1}^{N} m_i$$

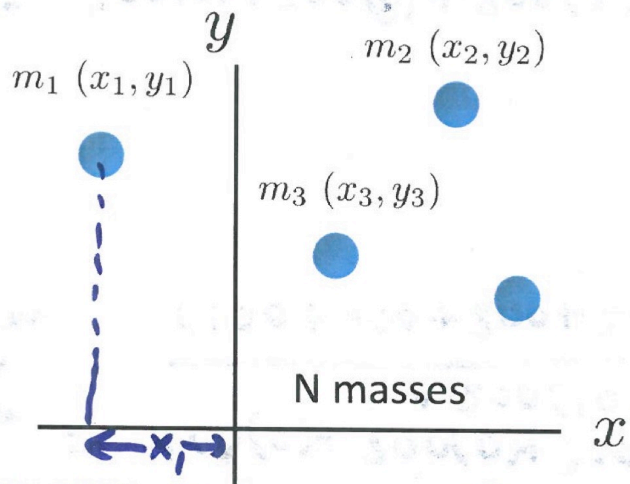
The center of mass is the point about which an unconstrained rigid body will rotate.



Center of Mass



Center of Mass for a Discrete Collection of Point Masses:



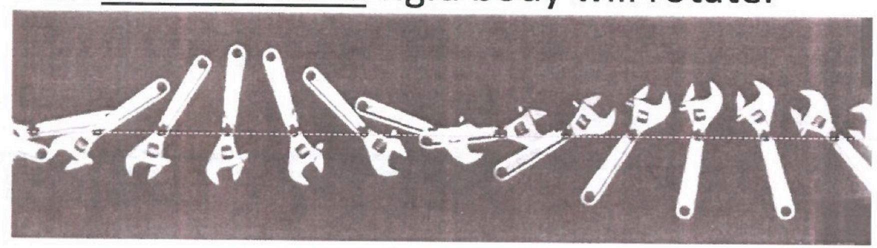
Center of Mass Coordinates:

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_6x_6}{(m_1 + m_2 + m_3 + \ldots + m_6)}$$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$\text{where}: M = \text{Total Mass} = \sum_{i=1}^{N} m_i$$

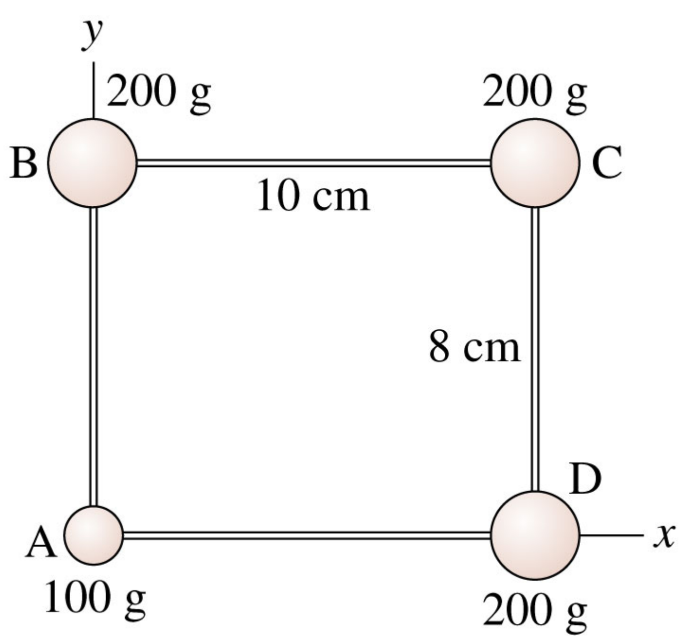
The center of mass is the point about which an <u>unconstrained</u> rigid body will rotate.



Whiteboard Problem 2: Problem 12-13a

13. || The four masses shown in FIGURE EX12.13 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.



Does it matter what units you work in for this problem?

Whiteboard Problem 12-13a

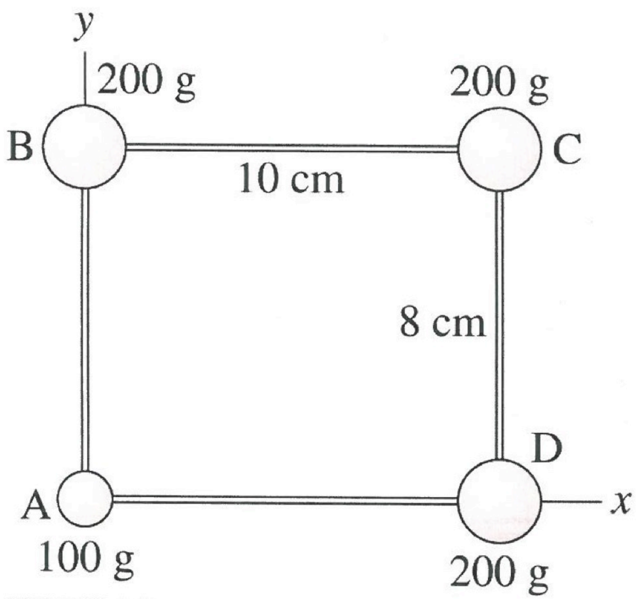
13. || The four masses shown in FIGURE EX12.13 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.

$X_{cm} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i$

$X_{cm} = \frac{100(0) + 200(0) + 200(10) + 200(10)}{(100+200+200+200)}$

$= \frac{40}{7}$ cm



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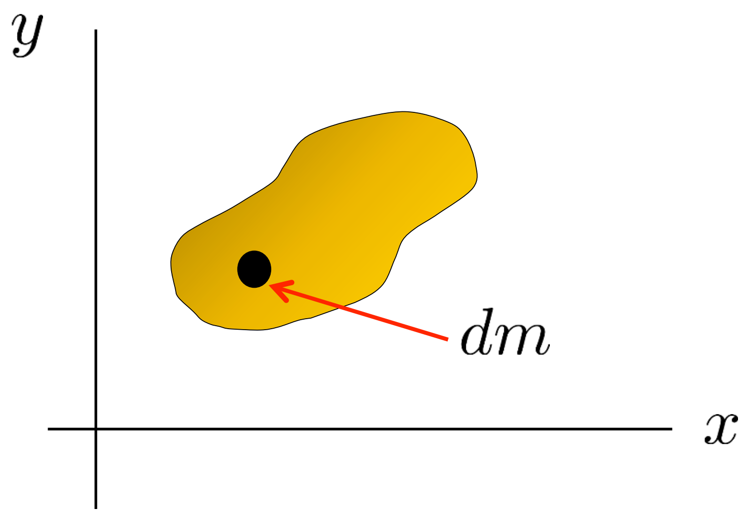
Does it matter what units you work in for this problem?

$Y_{cm} = \frac{100(0) + 200(8) + 200(8) + 200(0)}{(700)}$

$= \frac{32}{7}$ cm

Quick Note about Continuous Bodies



The center of mass coordinates are at:

$$x_{cm} = \frac{1}{M}\int x\,dm$$

$$y_{cm} = \frac{1}{M}\int y\,dm$$

These can become nasty integrals - **we won't do any!** But if you take Engineering Mechanics, you'll do a lot.

Your author makes the important point that for **continuous bodies that are symmetric and have uniform density, the center of mass is at the geometric center.**

Note that in many cases the center of mass may lie entirely outside the body!

Rotational Motion – Part 2

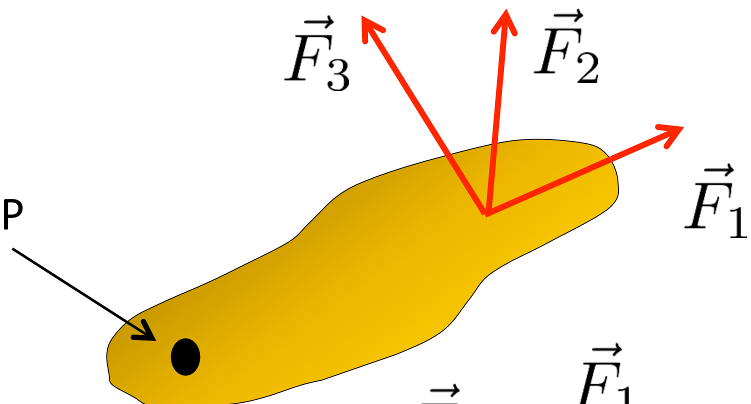
Torque (Sec. 12.5)

Torque (also called "moment" in engineering) is the **rotational analog of force**.

An applied force on an object causes an acceleration: $\vec{a} = \dfrac{\vec{F}}{m}$

An applied force on a rigid body can create a torque that causes an angular acceleration.
But what does torque depend on? Consider:

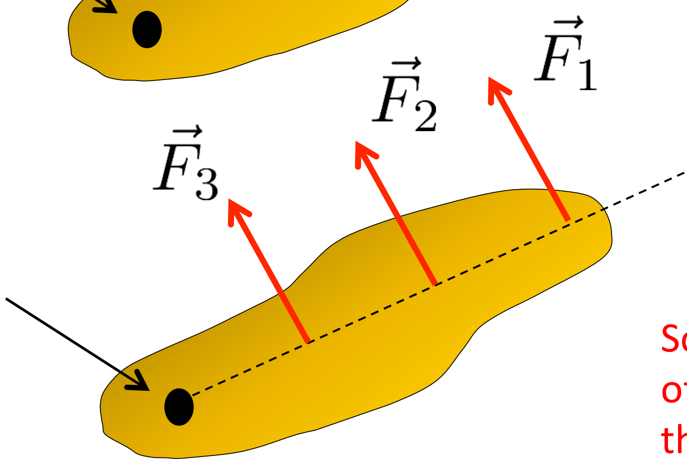
Pivot Point, P



The forces have equal magnitudes, which force causes the most rotation?
(answer: $\vec{F}_3$)

Or,

Pivot Point,P



The forces have equal magnitudes, which force causes the most rotation?
(answer: $\vec{F}_1$)

So, the greater the distance from P to the point of application of the force and the more perpendicular the force is to the line between P and the force, the more rotation.

Rotational Motion – Part 2

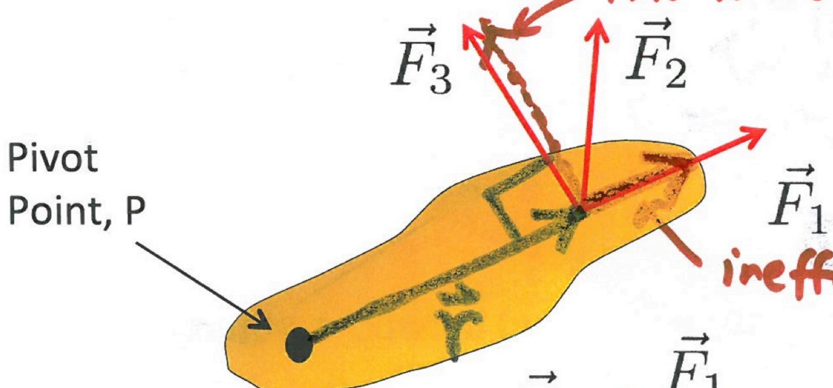
Torque

Torque (also called “moment” in engineering) is the **rotational analog of force**.

An applied force on an object causes an acceleration: $$\vec{a} = \frac{\vec{F}}{m}$$

An applied force on a rigid body can create a torque that causes an angular acceleration. But what does torque depend on? Consider:

This is the component of F_2 that yields torque

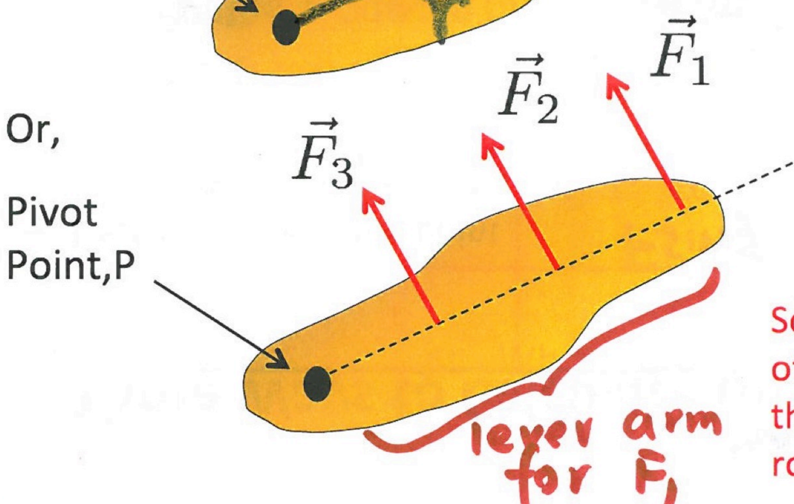


The forces have equal magnitudes, which force causes the most rotation?

(answer: $\vec{F}_3$)

ineffective in producing rotation

Or,



The forces have equal magnitudes, which force causes the most rotation?

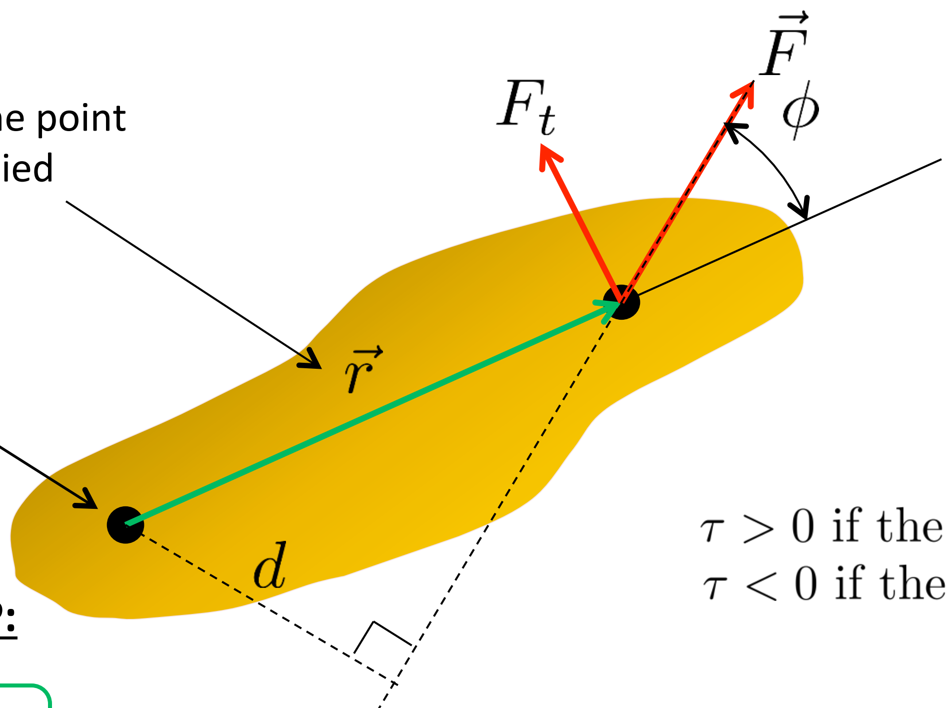
(answer: $\vec{F}_1$)

So, the greater the distance from P to the point of application of the force and the more perpendicular the force is to the line between P and the force, the more rotation.

Three Ways to Calculate the Torque Produced by a Force (Sec. 12.5)

Vector from P to the point where force is applied

Pivot Point, P



$\tau > 0$ if the rotation is CCW
$\tau < 0$ if the rotation is CW

Torque of F about P:

1.) $\tau = rF\sin\phi$ [Units: Nm]

2.) $\tau = rF_t$; where: F_t = tangential component of $\vec{F}$ ($\perp \vec{r}$) = $F\sin\phi$

So, $\tau = rF\sin\phi$

3.) $\tau = dF$ where d = the perpendicular distance from P to the line of $\vec{F}$ ("torque arm" or "moment arm")

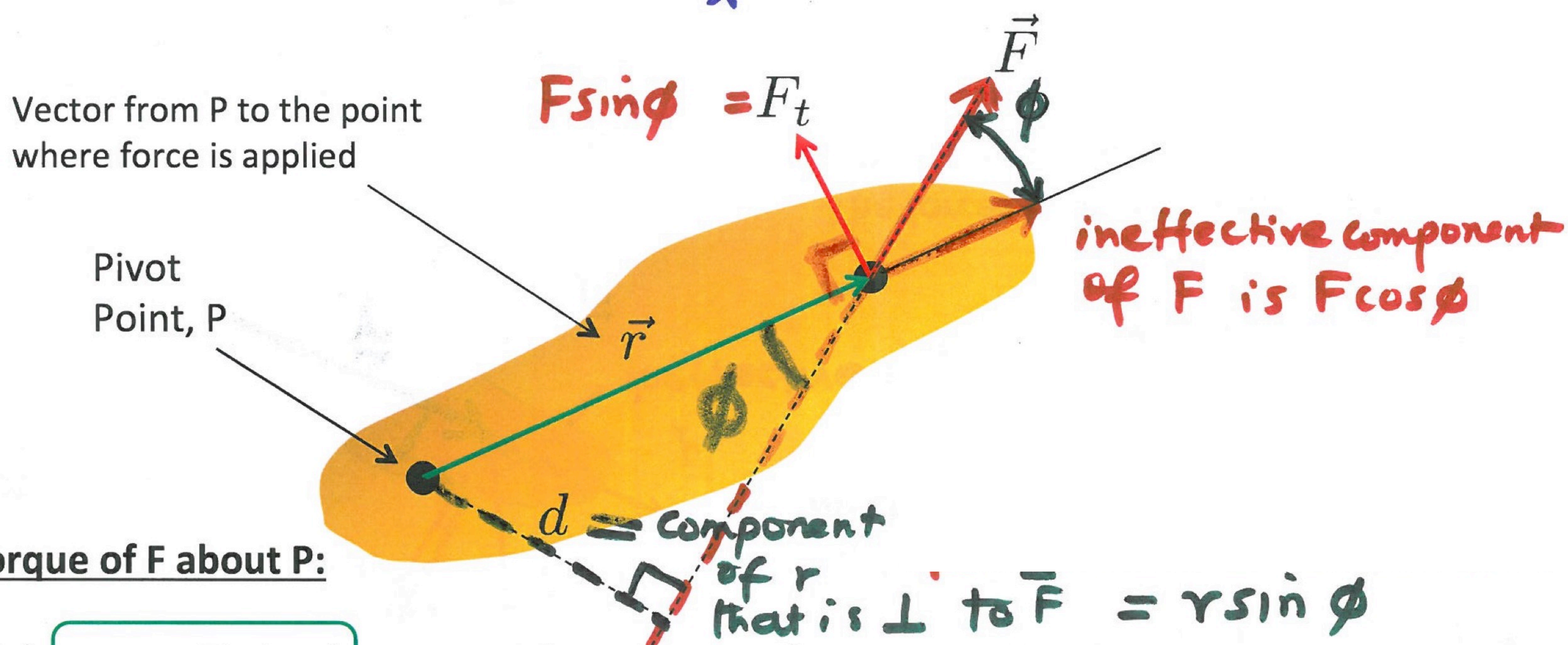
$d = r\sin\phi$

So, $\tau = rF\sin\phi$

There is a more general way to calculate the torque produced by a force

Let's do that now.

Three Ways to Calculate the MAGNITUDE OF THE Torque Produced by a Force



Torque of F about P:

1.) $\boxed{\tau = rF\sin\phi}$ ✓ [Units: Nm]

2.) $\tau = rF_t;$ where: F_t = tangential component of $\vec{F}$ ($\perp \vec{r}$) $= F\sin\phi$

So, $\boxed{\tau = rF\sin\phi}$ ✓

3.) $\tau = dF$ where d = the perpendicular distance from P to the line of $\vec{F}$ ("torque arm" or "moment arm")

$d = r\sin\phi$

So, $\boxed{\tau = rF\sin\phi}$ ✓

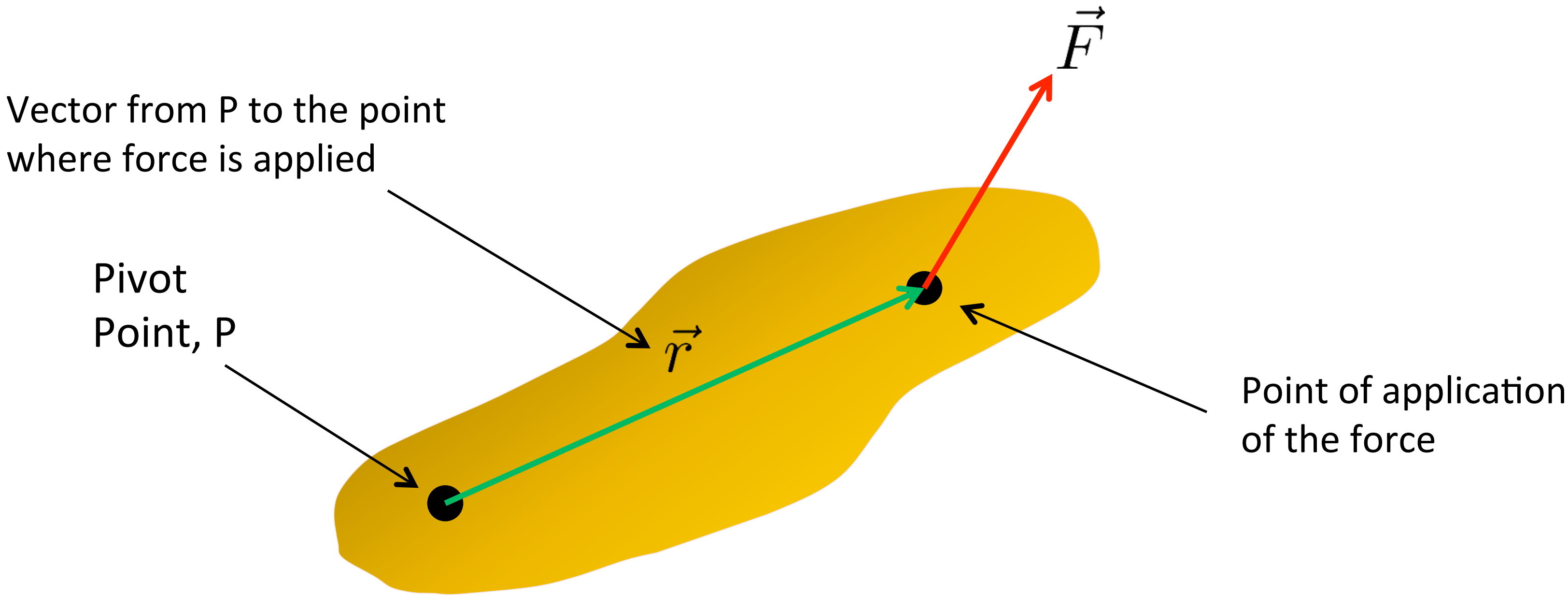
BUT... WHAT ABOUT DIRECTION?

There is a more general way to calculate the torque produced by a force

Let's do that now.

General Definition of Torque Produced by a Force

Sections 12.5 and 12.10



The Torque produced by the force, F, about the point P is:

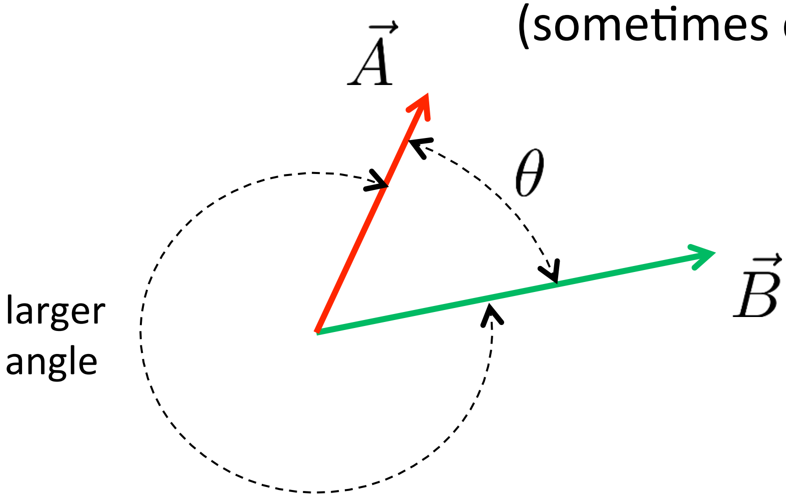
$$\text{Torque of } \vec{F} \text{ about P:} \quad \vec{\tau} = \vec{r} \times \vec{F}$$

What is this?

It's called a Vector Cross Product.

Vector Cross Product (Sec. 12.10)

(sometimes called a Vector Product)



$$\text{Vector, } \vec{C} = \vec{A} \times \vec{B}$$

$$\text{Where: } |\vec{C}| = AB\sin\theta$$

Direction of $\vec{C}$ is from the Right Hand Rule (RHR)

Another use of RHR !

Where: A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$

θ is the smaller of the two angles between $\vec{A}$ and $\vec{B}$

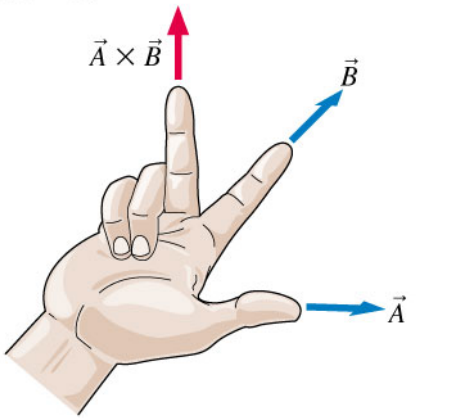
$$\text{So, } \vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta, \text{direction by RHR})$$

What is the Right Hand Rule (RHR)?

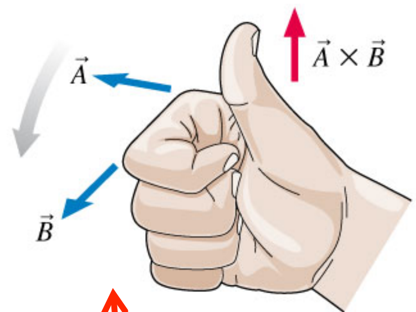
The Right Hand Rule (From your text, Sec. 12.10)

Using the right-hand rule

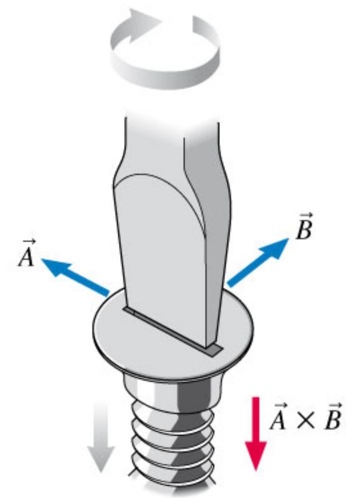
Spread your *right* thumb and index finger apart by angle α. Bend your middle finger so that it is *perpendicular* to your thumb and index finger. Orient your hand so that your thumb points in the direction of $\vec{A}$ and your index finger in the direction of $\vec{B}$. Your middle finger now points in the direction of $\vec{A} \times \vec{B}$.



Make a loose fist with your *right* hand with your thumb extended outward. Orient your hand so that your thumb is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and your fingers are curling *from* the line of vector $\vec{A}$ *toward* the line of vector $\vec{B}$. Your thumb now points in the direction of $\vec{A} \times \vec{B}$.

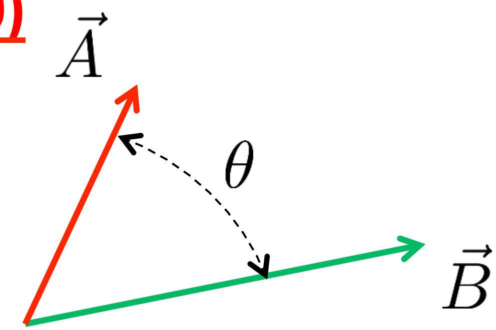


Imagine using a screwdriver to turn the slot in the head of a screw from the direction of $\vec{A}$ to the direction of $\vec{B}$. The screw will move either "in" or "out." The direction in which the screw moves is the direction of $\vec{A} \times \vec{B}$.



There are many ways to do the RHR.
I use this way.

The Right Hand Rule (My Way) (Sec. 12.10)



1. **Use your right hand!**

2. With your four fingers extended and together, point them in the direction of the first vector of the cross product.

3. Rotate your hand about an axis through your forearm until you can close your fingers through the angle θ towards the second vector in the product.

4. Your thumb points in the direction of the cross product.

So, $\vec{C} = \vec{A} \times \vec{B}$ points into the screen.

And, $\vec{D} = \vec{B} \times \vec{A}$ points out of the screen.

Note: $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

And, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Sec. 12.10

$\vec{A} \times \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$?

$\vec{A} \times \vec{B} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$[\hat{i}, \hat{j}, \hat{k}]$

right-handed system

"CYCLIC"

$\hat{i} \times \hat{i}$

$\hat{i} \times \hat{j}$

$\hat{i} \times \hat{k}$

$\hat{j} \times \hat{k}$

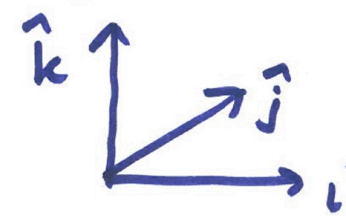
$\vdots$

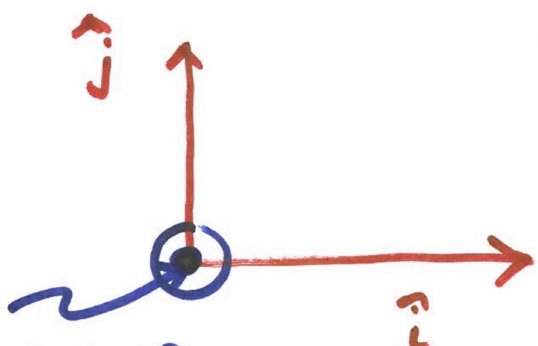
$\hat{j} \times \hat{i} = -\hat{k}$; $\hat{i} \times \hat{j} = \hat{k}$ ✓

$\hat{k} \times \hat{j} = -\hat{i}$; $\hat{j} \times \hat{k} = \hat{i}$ ✓

$\hat{i} \times \hat{k} = -\hat{j}$; $\hat{k} \times \hat{i} = \hat{j}$ ✓

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$





$\hat{k}$, OUT-OF-PAGE!

$\hat{k}$ INTO PAGE NOT RIGHT-HANDED SYSTEM!

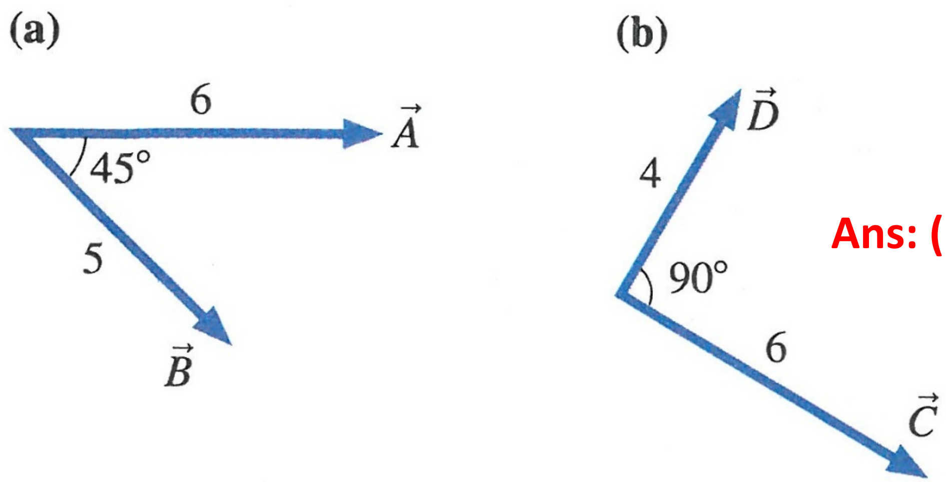
(A Couple of Quick) Whiteboard Problems 3 & 4

Problem 12-38:

37. | Evaluate the cross products $\vec{A} \times \vec{B}$ and $\vec{C} \times \vec{D}$.

Give the direction as in or out.

Ans: (21.21, in)



Ans: (24.0,out)

Problem 12-39:

40. || Vector $\vec{A} = 3\hat{\imath} + \hat{\jmath}$ and vector $\vec{B} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$. What is the cross product $\vec{A} \times \vec{B}$? **Find cross product in Component Form.**

Always use a right-handed co-ordinate system! What does that mean?

(A Couple of Quick) Whiteboard Problems 3 & 4

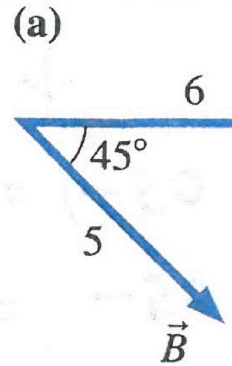
Problem 12-37:

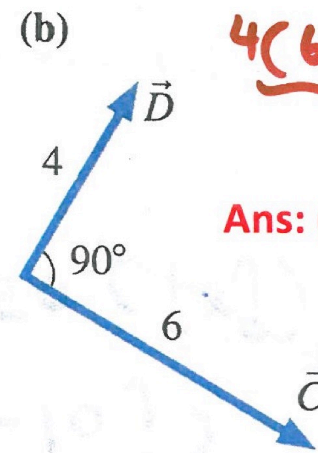
37. | Evaluate the cross products $\vec{A} \times \vec{B}$ and $\vec{C} \times \vec{D}$.

Give the direction as in or out.

6(5)Sin45

Ans: (21.21, in)





4(6)Sin90

Ans: (24.0,out)

Problem 12-39:

40. || Vector $\vec{A} = 3\hat{\imath} + \hat{\jmath}$ and vector $\vec{B} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$. What is the cross product $\vec{A} \times \vec{B}$? Find cross product in Component Form.

Always use a right-handed co-ordinate system! What does that mean?

$$(3\hat{i}+\hat{j}) \times (3\hat{i} - 2\hat{j} + 2\hat{k})$$

$$= \cancel{3\hat{i} \times 3\hat{i}}^{\,0} + 3(-2)(\hat{i} \times \hat{j}) + 3(2)\,\hat{i} \times \hat{k}$$

$$+ 3\,\hat{j} \times \hat{i} + \cancel{(-2)(\hat{j} \times \hat{j})}^{\,0} + 2(\hat{j} \times \hat{k})$$

$$= -6\,\hat{k} + 6(-\hat{j}) + 3(-\hat{k}) + 2\,\hat{i}$$

$$= 2\,\hat{i} - 6\,\hat{j} - 9\,\hat{k}$$

Whiteboard Problem 5: Problem 12-40

42. || Force $\vec{F} = -10\hat{j}$ N is exerted on a particle at $\vec{r} = (5\hat{i} + 5\hat{j})$ m. What is the torque on the particle about the origin?

Whiteboard Problem 5: Problem 12-42

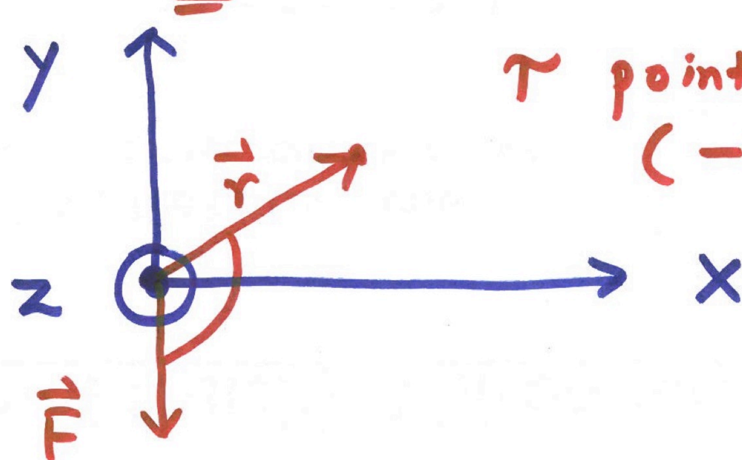
42. || Force $\vec{F} = -10\hat{j}$ N is exerted on a particle at $\vec{r} = (5\hat{i} + 5\hat{j})$ m. What is the torque on the particle about the origin?

$\vec{\tau}$ = Torque = $\vec{r} \times \vec{F}$

$= (5\hat{i} + 5\hat{j}) \times (-10\hat{j})$

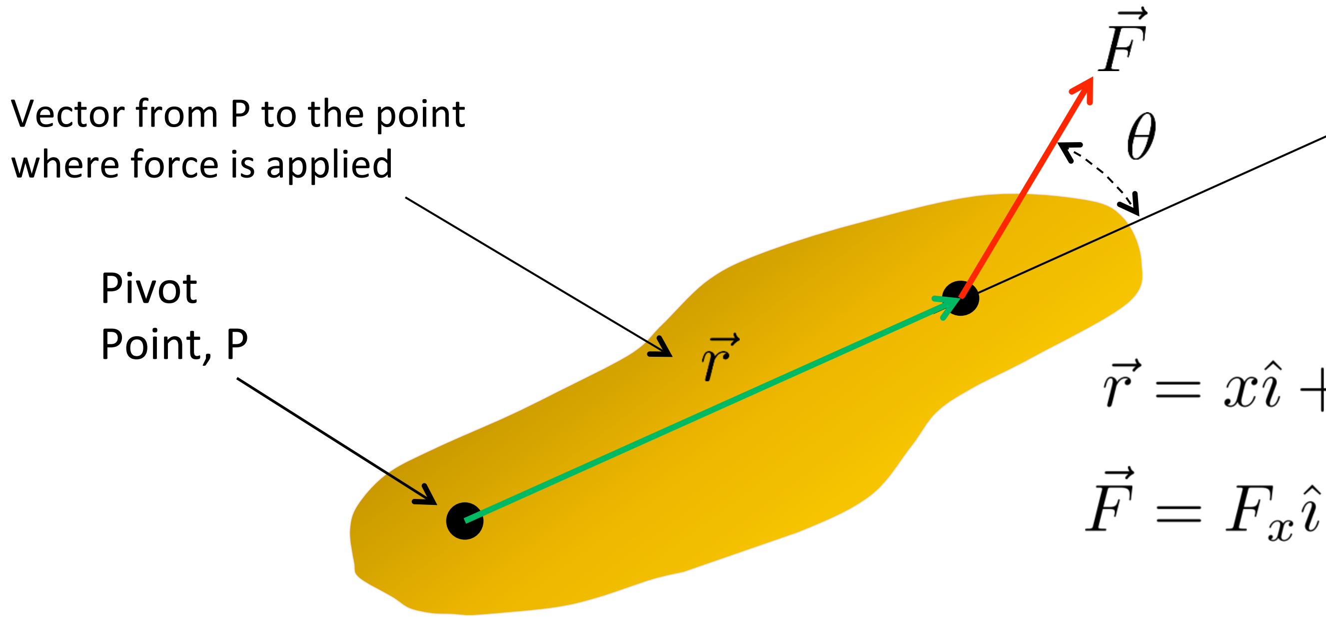
$= -50(\hat{i} \times \hat{j}) - 50(\hat{j} \times \hat{j}) \rightarrow 0$

$= (-50\hat{k})$ Nm



τ points INTO Page (−z-direction)

Back to Torque (Sections 12.5 and 12.10)



$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

$$\text{Torque:} \quad \vec{\tau} = \vec{r} \times \vec{F} = (rF\sin\theta, \text{direction by RHR})$$

Recall that once we knew kinematics (the 3 constant acceleration kinematic equations), and we knew ***F*** = m***a***, we moved on to Dynamics.

Similarly, here, now that we know about rotational kinematics and how to calculate torque, we can **begin to look at rotational dynamics problems**…
…What's the rotational analog of ***F*** = m***a***??

Q: Well, first…what's the rotational analog of mass?

A: Moment of Inertia (Sec. 12.4)

$$I = \text{Moment of Inertia, Units} = [\text{kg m}^2]$$

The moment of inertia of an object is determined by its mass and how the mass is distributed about the axis of rotation.

Key concept: For rotational motion, the spatial distribution of mass about pivot axis matters!

Example: Carrying a heavy long ladder or rod!

Rotational Dynamics (here's the analog for *F* = m*a*) (Sec. 12.6):

A net torque on an object causes an angular acceleration. Or:

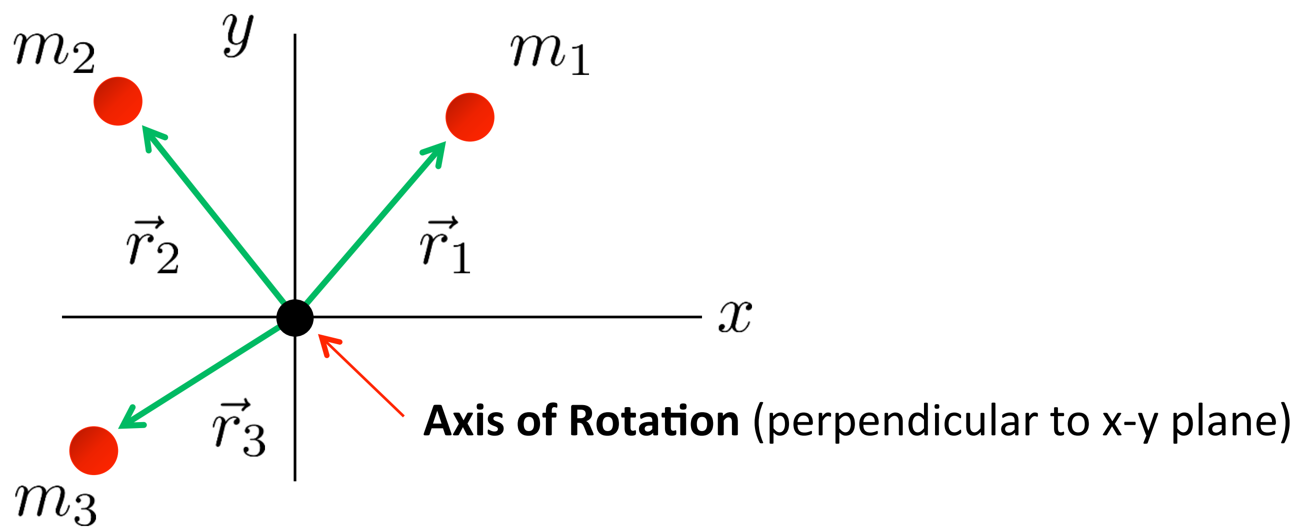
$$\alpha = \frac{\tau_{\text{net}}}{I} \quad \text{or} \quad \tau_{\text{net}} = I\alpha$$

This is Newton's 2nd Law for Rotation.

But…what's the definition of Moment of Inertia? How do you calculate it?

Moment of Inertia (Sec. 12.4)

Collection of Discrete Point Masses:



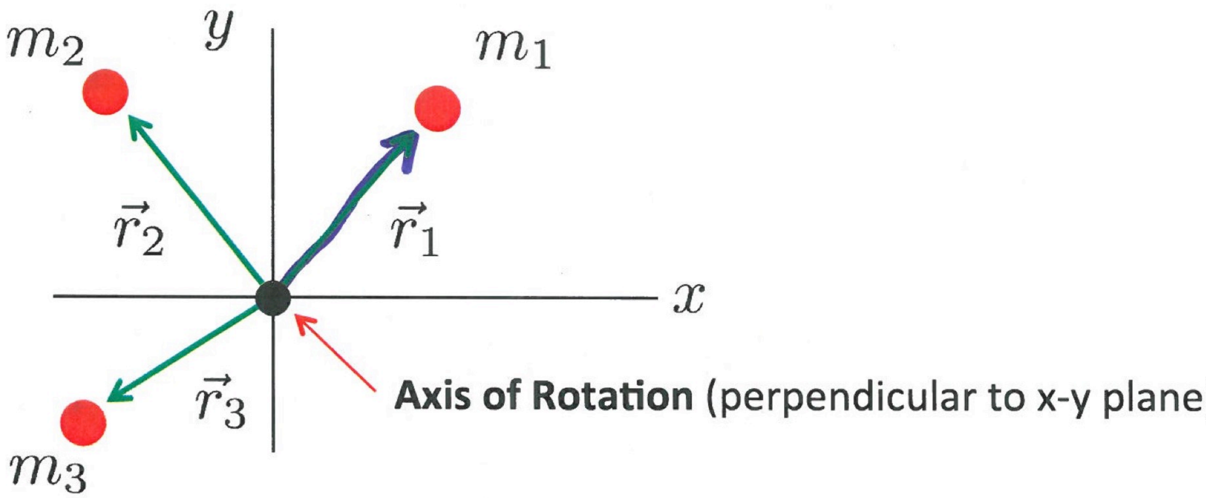
$$\text{Moment of Inertia, } I = \sum_{i=1}^{N} m_i r_i^2$$

$$\text{where: } r_i = \text{perpendicular distance from the axis to } m_i$$

$$= \sqrt{x_i^2 + y_i^2}$$

Moment of Inertia

Collection of Discrete Point Masses:



$$\text{Moment of Inertia, } I = \sum_{i=1}^{N} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

where: r_i = perpendicular distance from the axis to m_i

$$= \sqrt{x_i^2 + y_i^2}$$

Whiteboard Problem 6: Problem 12-13b

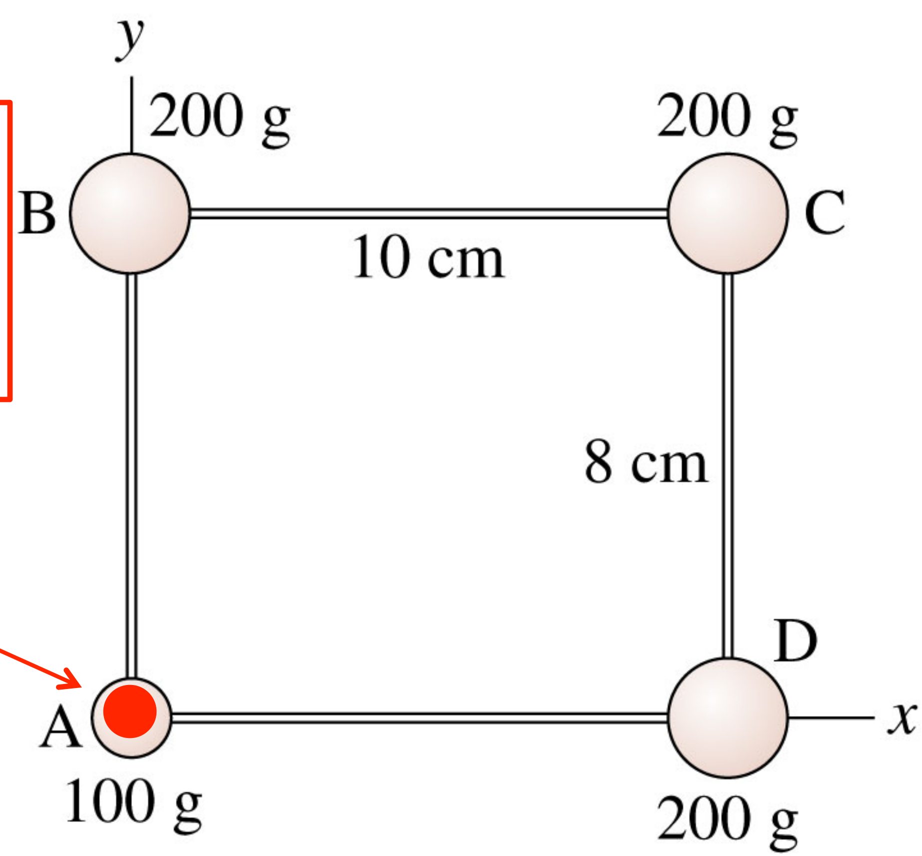
13. || The four masses shown in FIGURE EX12.13 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.

b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.

We already did part a. Don't need it to do part b.

Rotation Axis:



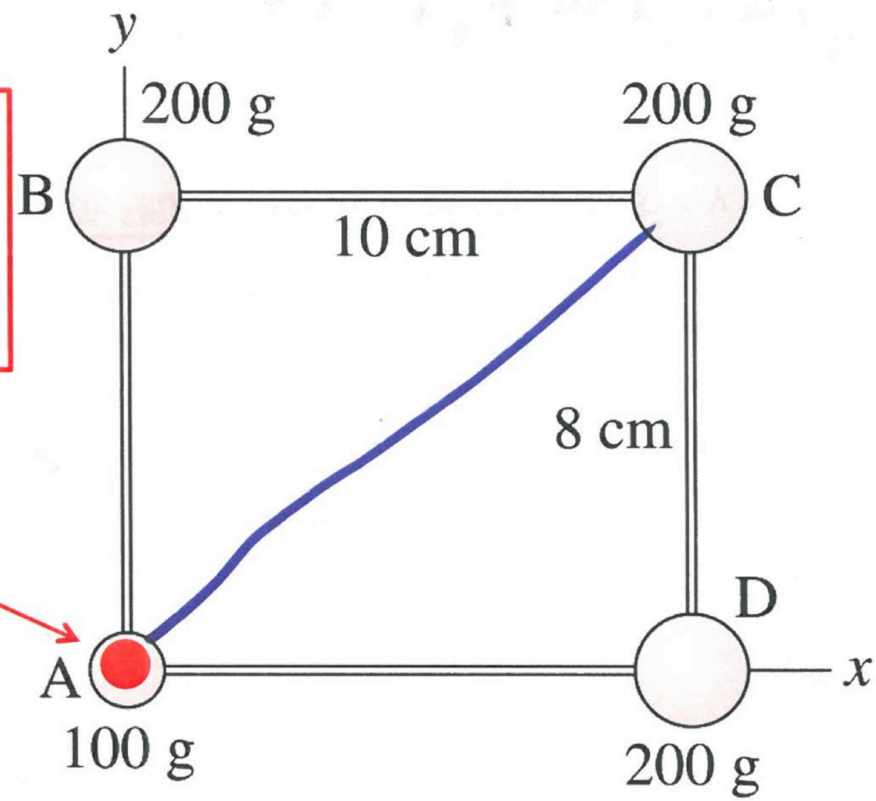
Whiteboard Problem 6: Problem 12-13b

13. || The four masses shown in FIGURE EX12.13 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.

b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.

We already did part a. Don't need it to do part b.



Rotation Axis: (A)

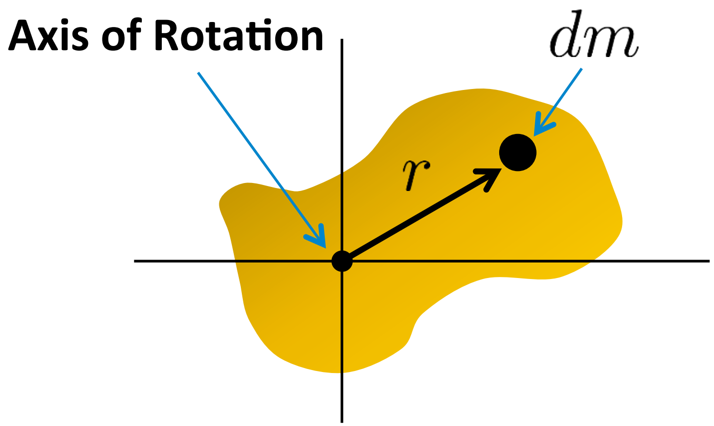
$$I = \underbrace{(0.100)\,0^2}_{A} + \underbrace{0.2(0.08)^2}_{B}$$

$$+ \underbrace{0.2(0.08^2 + 0.1^2)}_{C} + \underbrace{0.2(0.1^2)}_{D}$$

$$= 0.00656\ \text{kgm}^2$$

Moment of Inertia (Sec. 12.4)

Continuous Bodies:

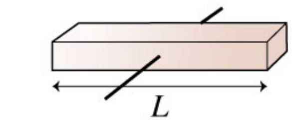
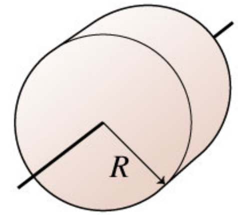
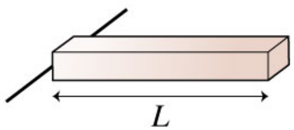
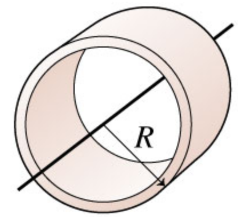
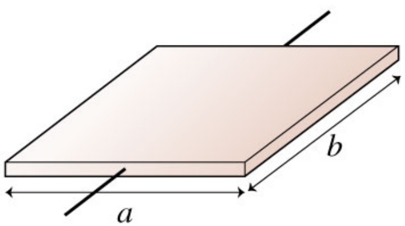
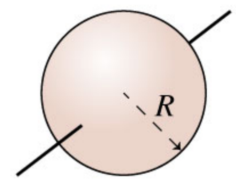
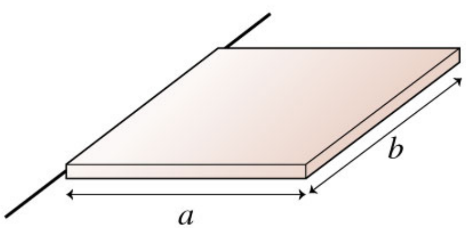
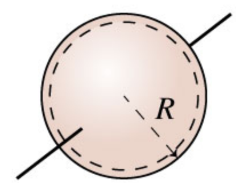


$$\text{Moment of Inertia, } I = \int_{\text{Volume}} r^2 dm$$

These can be difficult integrals. **We won't do any***, but we will use ones already calculated:

*If you are an engineering major, you'll do these in an engineering mechanics class.

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

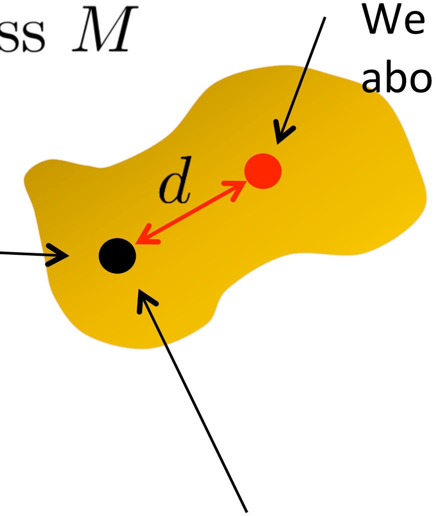
Parallel Axis Theorem (Sec 12.4)

If you know the moment of inertia about an axis through the object's center of mass, you can find the moment of inertia *about any parallel axis*.

body of mass M

Axis through the center of mass,
We know the moment of inertia (I_{cm}) about this axis

Axis parallel to axis through CM

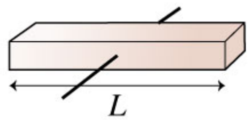
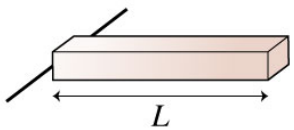


The moment of inertia about this parallel axis is:

$$I = I_{cm} + Md^2$$

where d is the distance between the axes

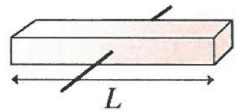
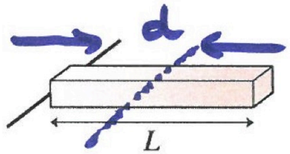
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$
Thin rod, about end		$\frac{1}{3}ML^2$

Apply Parallel Axis Theorem to calculate Moment of Inertia for thin rod about end:

Corollary:
Same object but different pivot axis, leads to a different value for the moment of inertia.

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$
Thin rod, about end		$\frac{1}{3}ML^2$

Apply Parallel Axis Theorem to calculate Moment of Inertia for thin rod about end:

Find I about axis passing thru end, given

$$I_{cm} = \frac{1}{12}ML^2$$

$$I = I_{cm} + M\left(\frac{L}{2}\right)^2$$ by application of Parallel-Axis Theorem

$$= \frac{ML^2}{12} + \frac{ML^2}{4} = ML^2\left(\frac{1+3}{12}\right) = ML^2\frac{4}{12} = \frac{ML^2}{3}$$

Corollary:

Same object but different pivot axis, leads to a different value for the moment of inertia.