Chapter 11 Impulse and Momentum

Chapter Goal: To understand and apply the new concepts of impulse and momentum.

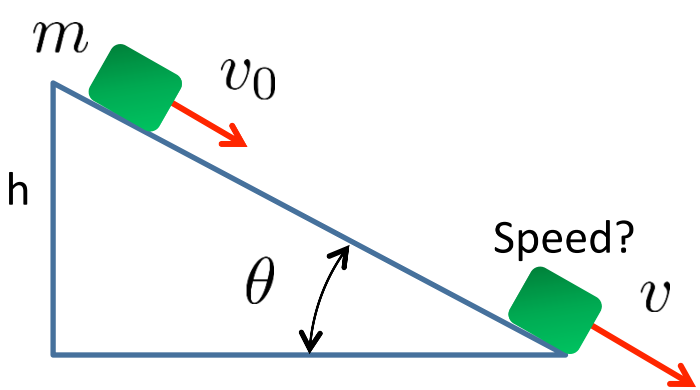
Review: What do we know? Where are we going?

Up to this point in Physics 191, we have concentrated on:

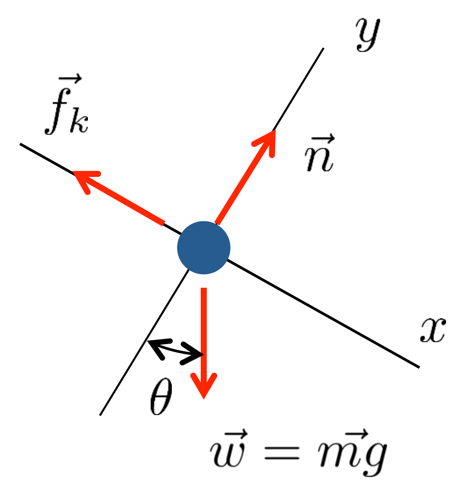
Kinematics: How things move $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ and $\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$

Dynamics: Why things move $\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$

Here's something we can handle:
What's speed of mass *m* at bottom?



FBD:



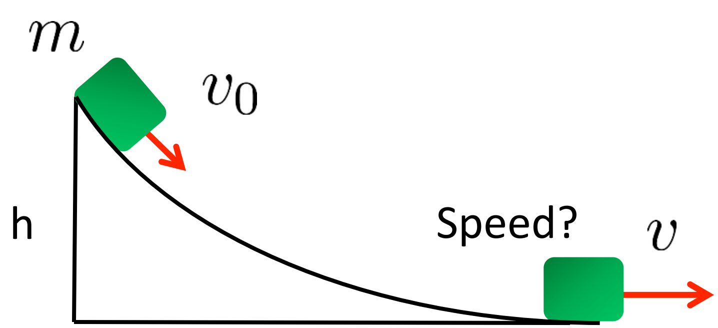
As we've done many times, for no friction: $a_x = g\sin\theta$

Or, if there is friction: $a_x = g(\sin\theta - \mu_k\cos\theta)$

Either way, we can solve for the speed

Review: What do we know? Where are we going?

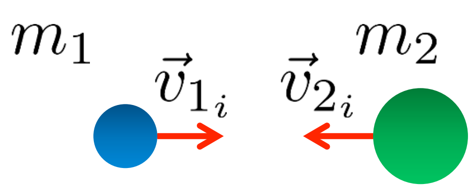
But, what if we had a more complicated trajectory?



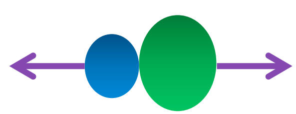
Here, the FBD is not the same at every point on the slide.

This means that the **acceleration is not constant.**

Or, how about this, a collision?
This is much more complicated than blocks tied with a rope, moving in tandem!



Force on m_1 and $m_2 = 0$



$\vec{F}_{2\mathrm{on}1} - \vec{F}_{1\mathrm{on}2}$

(some complicated function of time - watch)

Force on m_1 and $m_2 = 0$

Find: $\vec{v}_{1_f}$ and $\vec{v}_{2_f}$

Both of these examples can be solved using Newton's Laws, but they're very very difficult! We need a better way. **Fortunately, there is one, and it's really easy!**

<u>Conservation Laws help us further understand motion!</u>

Remember, we started Physics 191 with the observation:

"Everything moves"

And we've spent the entire semester so far learning how to describe that motion. We're now ready to add to that observation:

"Everything moves, but in all processes, some quantities stay the same, i.e. are <u>conserved</u>."

A <u>**Conservation Law**</u> tells us that *something* stays the same, and we can use that to solve many types of problems very easily. The trick is find out what that <u>something</u> is.

We'll concentrate on the two conservation laws:

<u>**Conservation of Energy**</u> (Chapter 9 & 10)

<u>**Conservation of Momentum**</u> (Chapter 11)

These will become essential tools that we can use to solve all kinds of problems that would otherwise be very difficult.

<u>Conservation Laws are really cool and important!!</u>

Your author points out that these conservation laws are actually more fundamental than Newton's Laws*; this is true:

In the realm of the very small (i.e. atoms), Newton's 2nd Law fails, but Energy and Momentum conservation are still valid:

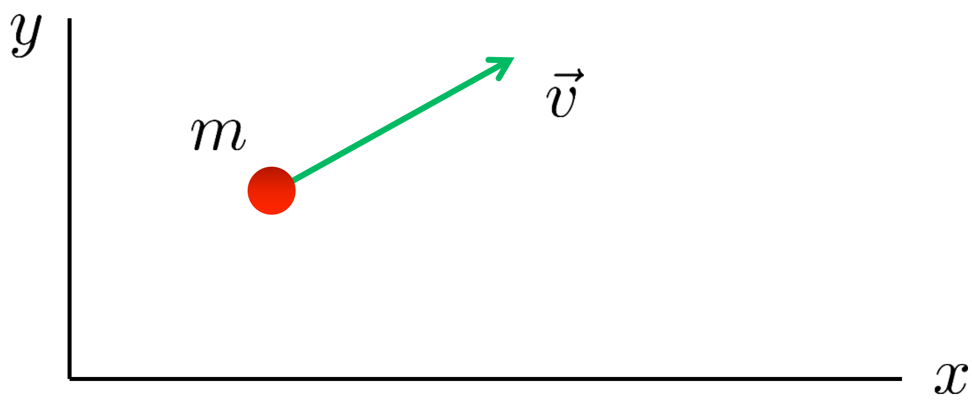
Quantum Mechanics (end of PHY191)

Also, in the realm of the very fast (near the speed of light), both Newton's Laws and the rules of kinematics fail, but with more complete definitions of momentum and energy, the conservation laws are still valid:

Special Relativity (end of PHY192)

*Newton did not use the ideas of momentum and energy.

Momentum and Impulse



Definition: Momentum of the object: $\vec{p} = m\vec{v}$

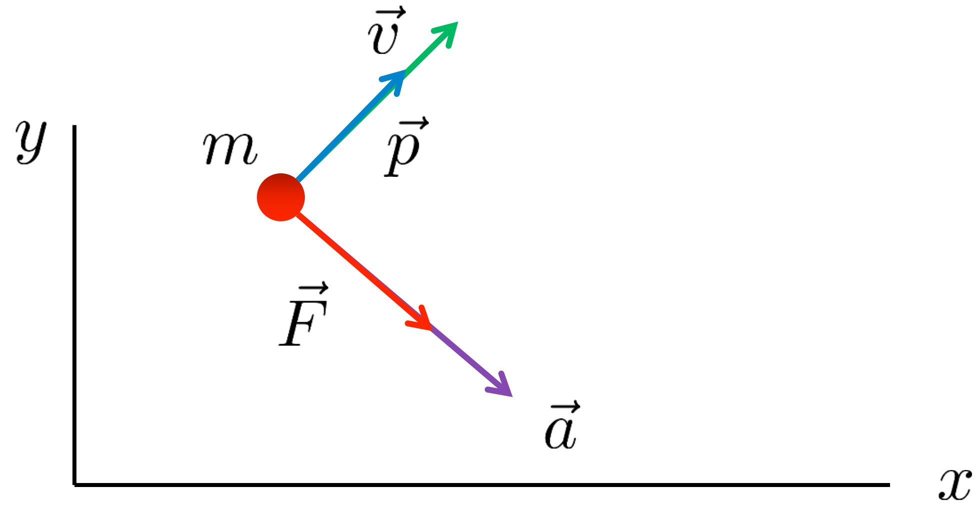
Units: $\left[\frac{kg\ m}{s}\right]$

Note: momentum is a vector quantity, so in most problems, we'll work with it in **component form**:

$$p_x = mv_x$$
$$p_y = mv_y$$

Can be positive, negative, or zero

Newton's 2nd Law Revisited: Better, more precise



$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \quad (\text{for } m = \text{constant})$$

So, another way to write **Newton's 2nd Law** is: $$\vec{F} = \frac{d\vec{p}}{dt}$$

In words: "Force is the time rate of change of the momentum if the object."

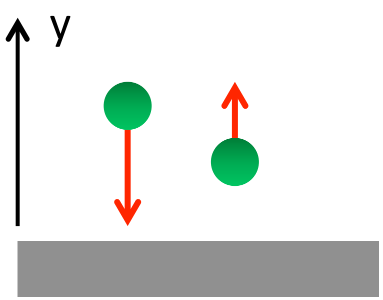
Or even better: "A force causes the momentum of the object to change in time."

This is a much better enunciation of Newton's 2nd Law.

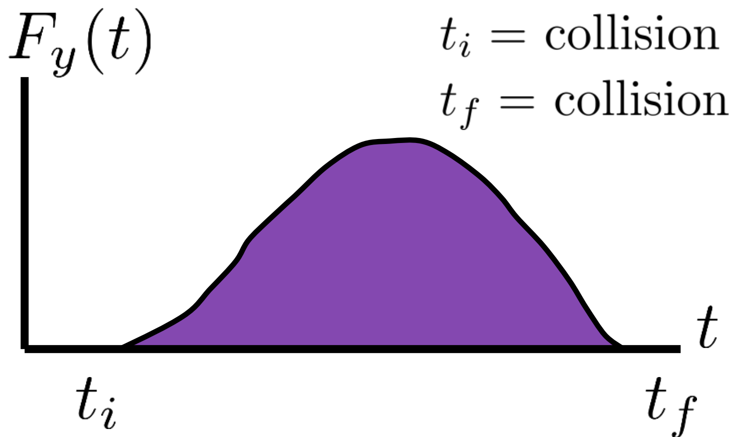
Q: Why? A: B/c *F = dp/dt* predicts rocket propulsion! Harder to see with *F = ma*.

Impulse

Suppose an object is subject to a time-varying force: like a ball bouncing from the floor.



The y-component of the contact force might look like:



t_i = collision with floor begins

t_f = collision with floor ends

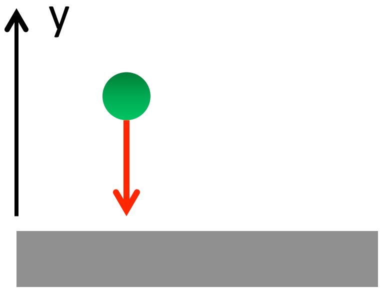
Apply Newton's 2nd in the y-direction: $F_y(t) = \dfrac{dp_y}{dt}$ or $dp_y = F_y(t)dt$

Integrate both sides: $\displaystyle\int_{p_{y_i}}^{p_{y_f}} dp_y = \int_{t_i}^{t_f} F_y(t)dt$

$$p_{y_f} - p_{y_i} = \Delta p_y = \int_{t_i}^{t_f} F_y(t)dt = \text{area under } F_y \text{ vs } t$$

$$\text{Define: Impulse of a Force, } \vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

(Describes what a variable force does over time.)



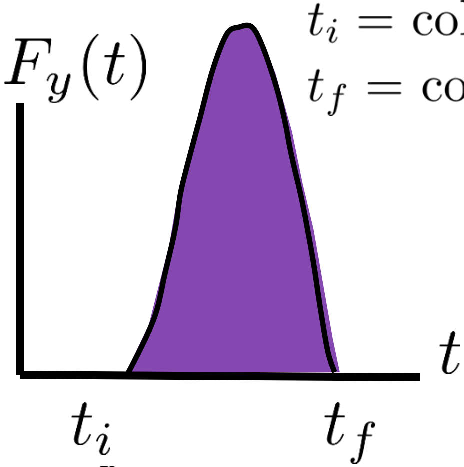
Applications of Impulse

Why do you flex your knees when you jump? Why do boxers wear gloves? Why do airbags save our lives?

The y-component of the contact force might look like:

$t_i =$ collision with floor begins, initial momentum = p

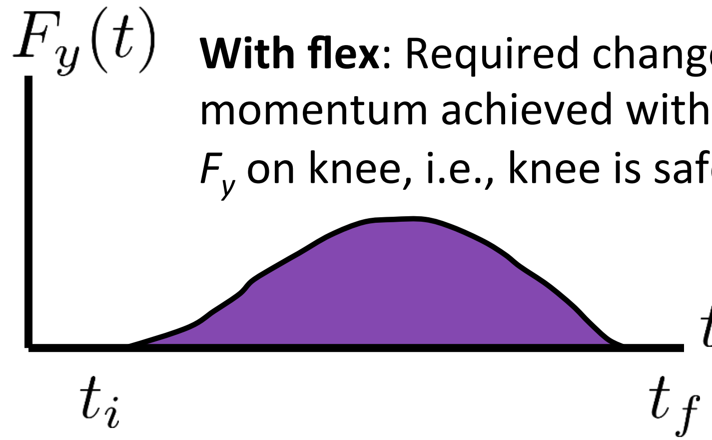
$t_f =$ collision with floor ends, final momentum = 0



No flex: Large $F_{y,\,max}$ may damage knee

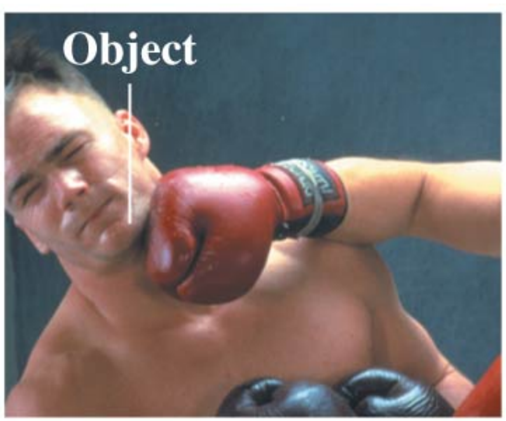
Area under curve is same!

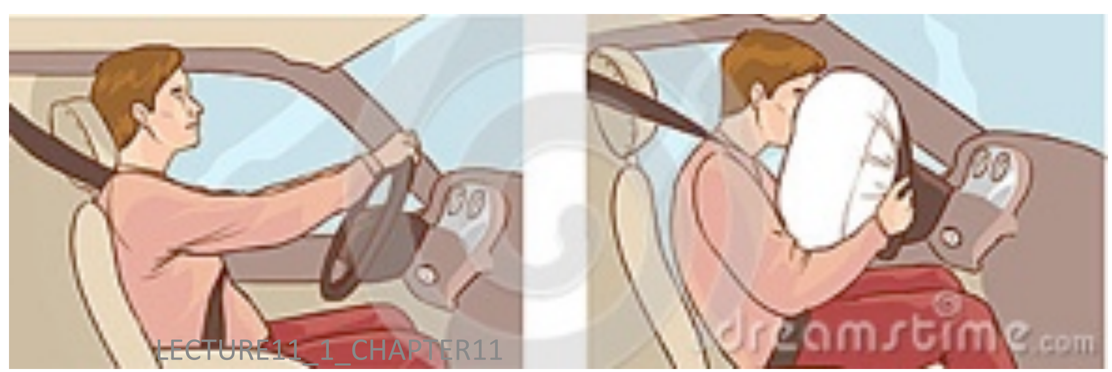
i.e., same momentum change!



With flex: Required change in momentum achieved with low F_y on knee, i.e., knee is safe

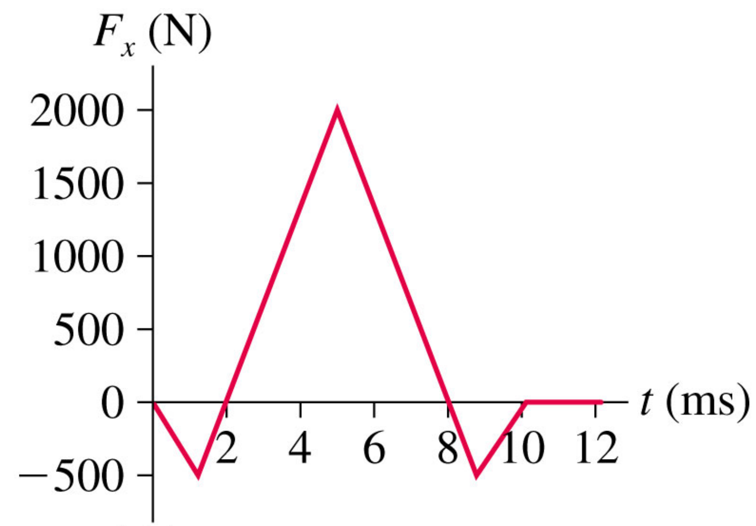
SAME IDEA FOR BOXING GLOVES AND AIRBAGS!





Whiteboard Problems 11-1 and 11-2

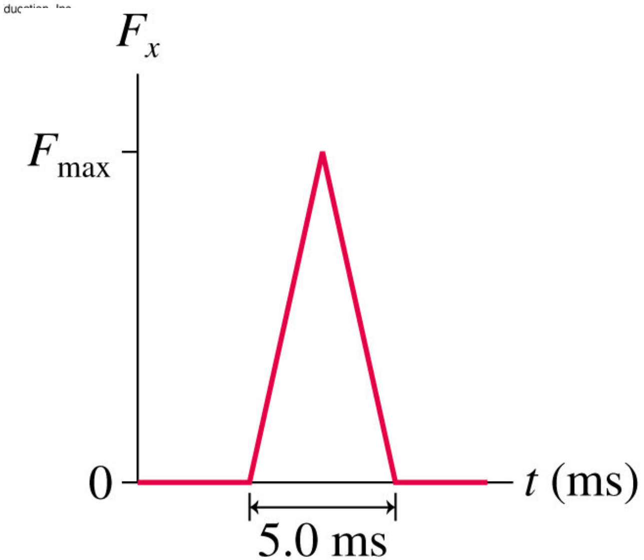
4. ‖ What is the impulse on a 3.0 kg particle that experiences the force shown ?



Chapter 11, Problem 41

||| A 200 g ball is dropped from a height of 2.0 m, bounces on a hard floor, and rebounds to a height of 1.5 m. The figure shows the impulse received from the floor. What maximum force does the floor exert on the ball?

Hint: Remember! Impulse = change in momentum!



Collisions and Explosions…Newton's 3rd law leads to law for Conservation of Momentum!

COLLISIONS

1. KEY POINT: Apply Newton's 3rd Law!

2. Therefore, for the "SYSTEM of m_1 & m_2"

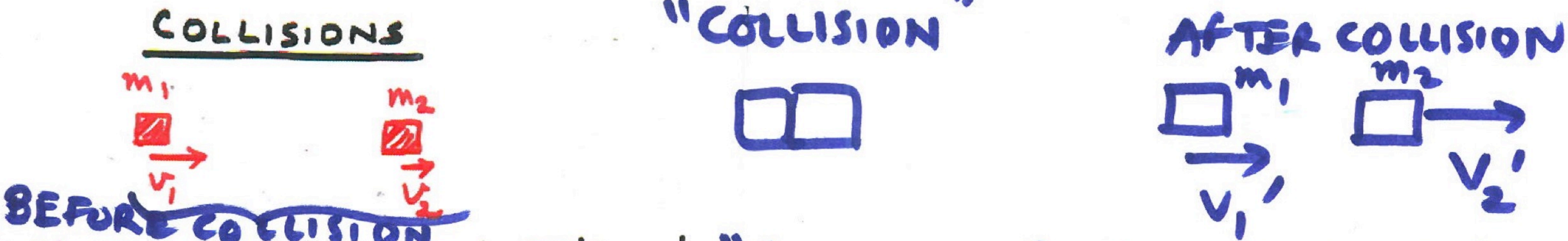
$$\vec{F}_{net} =$$

$\Rightarrow$

LAW OF CONSERVATION OF LINEAR MOMENTUM

IF....

Collisions and Explosions...Newton's 3rd law leads to law for Conservation of Momentum!



1. KEY POINT: Apply Newton's 3rd Law! "SYSTEM = PAIR OF COLLIDING OBJECTS"

$$F_{net} = \cancel{m\vec{a}} \; \frac{d\vec{p}}{dt}$$

2. Therefore, for the "SYSTEM of m_1 & m_2"

$$\vec{F}_{net} = F_{\text{by } m_1 \text{ on } m_2} + F_{\text{by } m_2 \text{ on } m_1}$$

CANCEL — "action = reaction" pair

$= 0$

$\Rightarrow d\vec{p}/dt = 0 \Rightarrow$ **$\vec{p}$ is constant, conserved!!**

LAW OF CONSERVATION OF LINEAR MOMENTUM

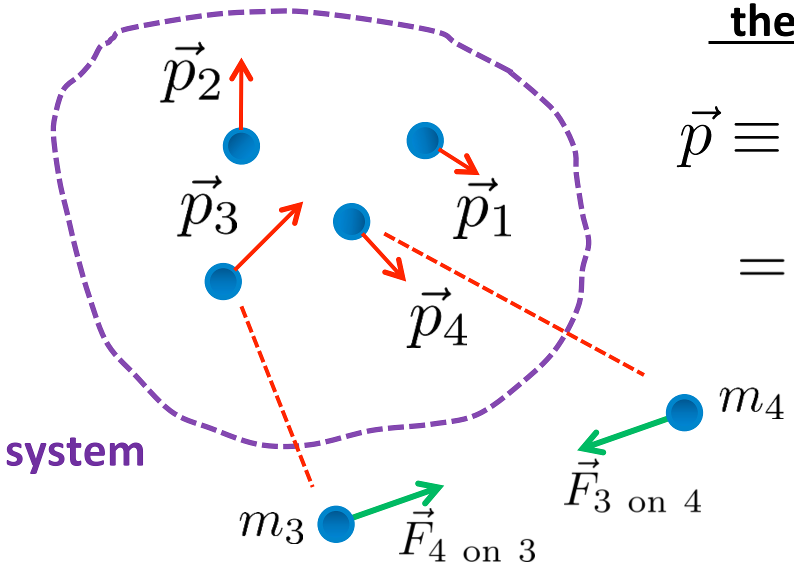
IF....
ALL EXTERNAL FORCES ARE NEGLIGIBLE COMPARED TO FORCE OF IMPACT → WHICH IS INTERNAL ANYWAY!
(i.e. SYSTEM IS ISOLATED)

Conservation of Momentum

Your author shows for a simple two body system and then for a general isolated system of N interacting bodies how momentum is conserved:

For an isolated* system:

***isolated means the net external force on the system is zero**



$$\vec{p} \equiv \text{total momentum of the system}$$

$$= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_N = \sum_{i=1}^{N} \vec{p}_i$$

The forces internal to the system cancel by Newton's 3rd Law.

So for the System:

$$\vec{F}_{\text{net}} = 0 \quad \text{so } \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = 0 \quad \text{or } \vec{p} = \text{Constant}$$

Note: we'll almost always apply this to a system before and after an interaction in the form:

For an Isolated System:

$$\vec{p}_{\text{final}} = \vec{p}_{\text{initial}}$$

PROBLEM-SOLVING STRATEGY 9.1

Conservation of momentum

MP

MODEL Clearly define *the system.*

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapters 10 and 11, conservation of energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

$$p_{x_i} = p_{x_f}$$

$$p_{y_i} = p_{y_f}$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

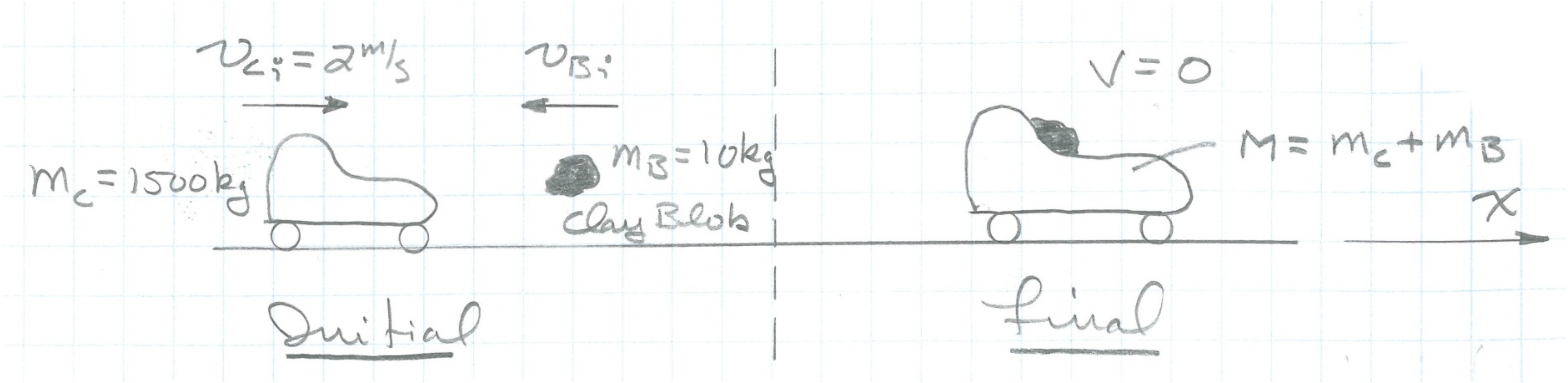
Exercise 16

where $\vec{p}$ is the total momentum of the system

Whiteboard Problem 11-3: Problem 11.20 in book

\# 20 A 1500 kg car is rolling at 2.0 m/s. You would like to stop the car by firing a 10 kg blob of sticky clay at it. How fast should you fire the clay? What a ridiculous way to stop a car, but it would work – might break the windshield!

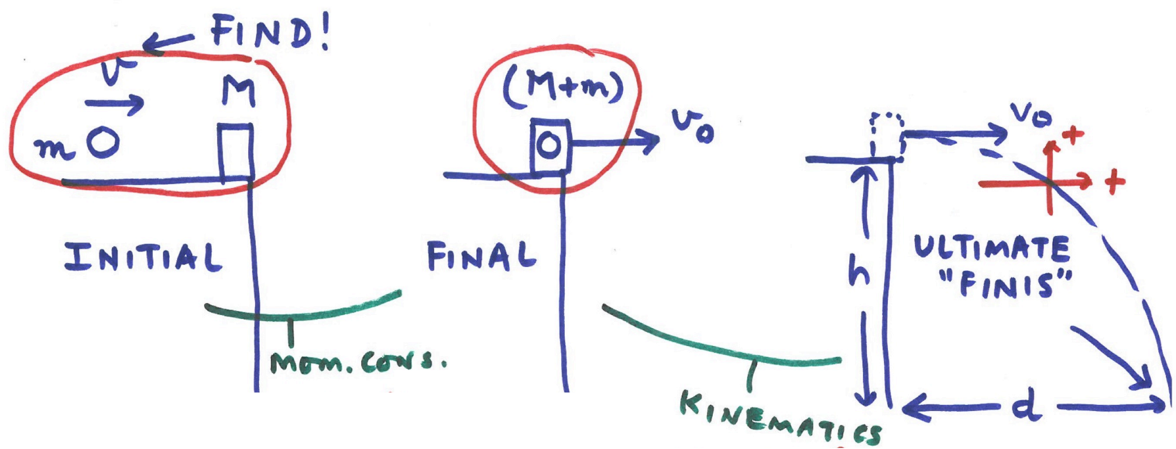
Your sketch should look like this (what is the system?):



Demo: Air Cannon

Consider an air-cannon placed on a horizontal table (just as in the demo). A pingpong ball (mass 2 gms) is launched from an air-cannon with a horizontal velocity *v* straight at an empty soda (mass 9.5 gms) can placed next to it at the edge of the table. The ball stays embedded inside the can, as the can is projected horizontally out a distance 9.27 m along the floor away from the table – we measured this in class. The height of the table is measured to be 0.74 m.

Determine the speed *v* with which the pingpong ball was launched from the air-cannon.

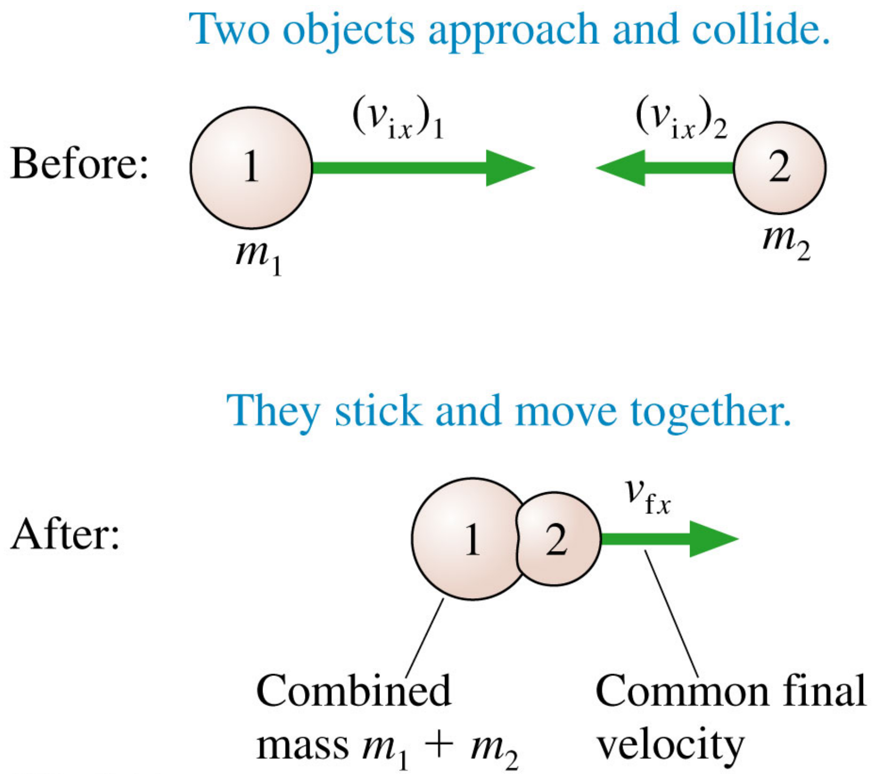


(speed of sound = 340 m/s)

TAKEOFF SPEED OF 747 ≃ 190 mi/hr

Perfectly Inelastic Collisions

The previous problem is an example of a **Perfectly Inelastic Collision***
which means that after the collision the bodies **stick together and have the same final velocity**.



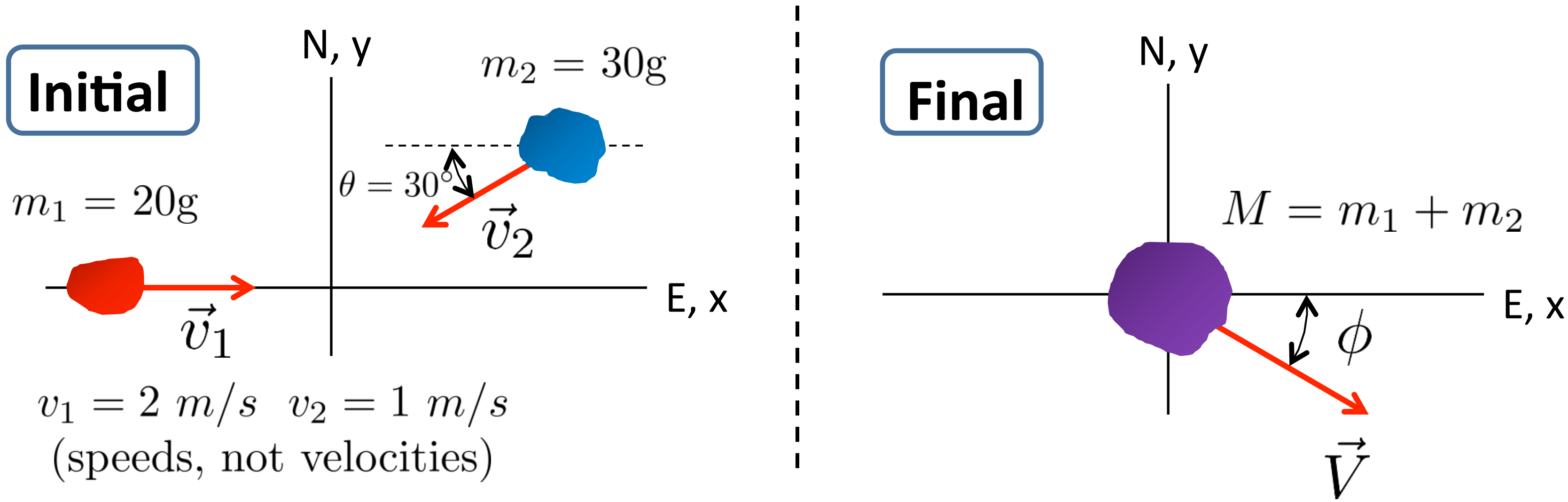
***All collisions conserve momentum**. For problems where the final velocities are different, we need some more information. We'll see the other extreme, perfectly elastic collisions that also conserve *kinetic energy*, next week.

Whiteboard Problem 11-4: Problem 11.70 in book

(Another perfectly inelastic collision, but this time in 2D…so conserve p_x and p_y!)

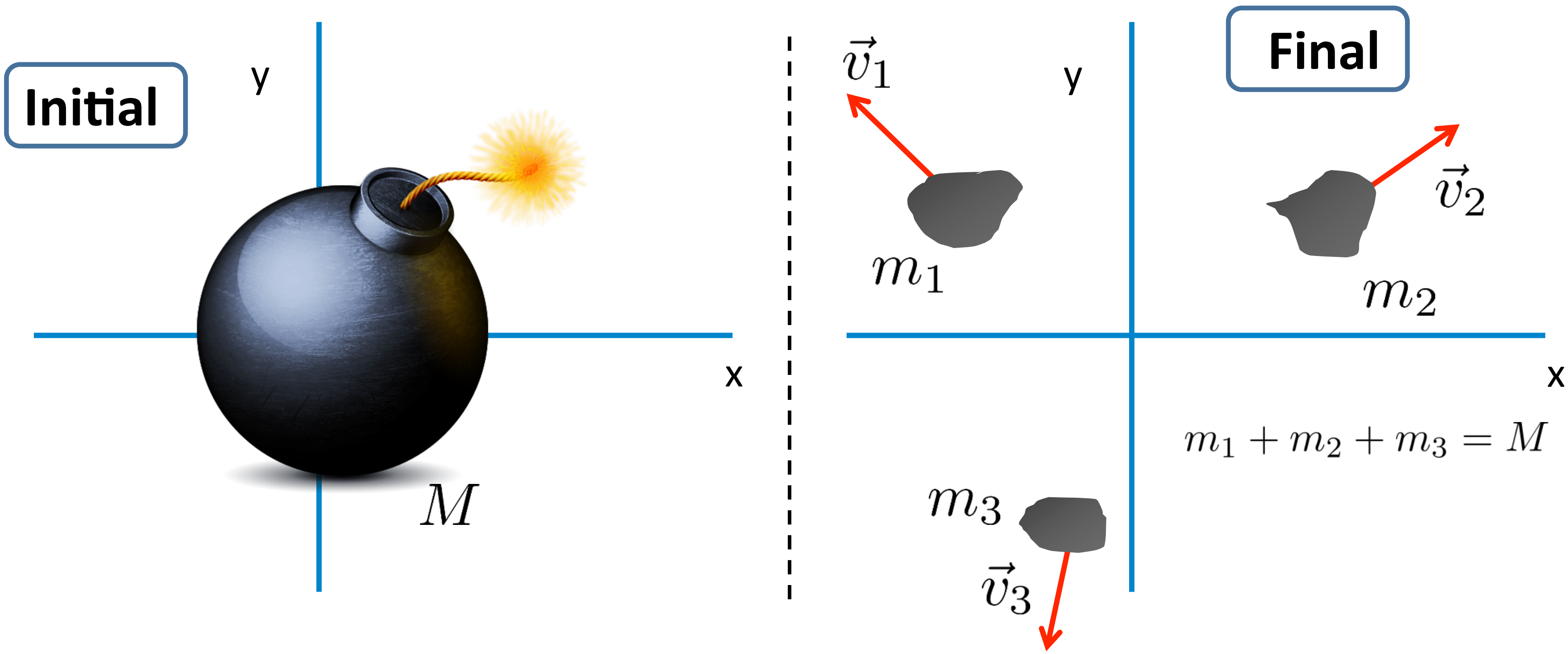
#70 | A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay traveling 30° south of west at 1.0 m/s. What are the speed and direction of the resulting 50 g blob of clay?

Your sketch should look like this:



Explosions

An explosion is just a completely inelastic collision in reverse; like these fireworks. We can use momentum conservation to solve for the velocities of the fragments.



$$\vec{p}_i = 0$$

$$\vec{p}_f = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{p}_i = 0$$

Whiteboard Problem 11-5: Problem 11:47 in book

A firecracker in a coconut blows the coconut into three pieces. Two pieces of equal mass fly off south and west, perpendicular to each other, at speed v_0. The third piece has twice the mass as the other two. What are the speed and direction of the third piece?
Give the direction as an angle east of north.

Draw a sketch of "immediately before" and "immediately after" the collision!

PROBLEM-SOLVING STRATEGY 9.1

Conservation of momentum

Remember this:

MODEL Clearly define *the system.*

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
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$$p_{x_i} = p_{x_f}$$

$$p_{y_i} = p_{y_f}$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

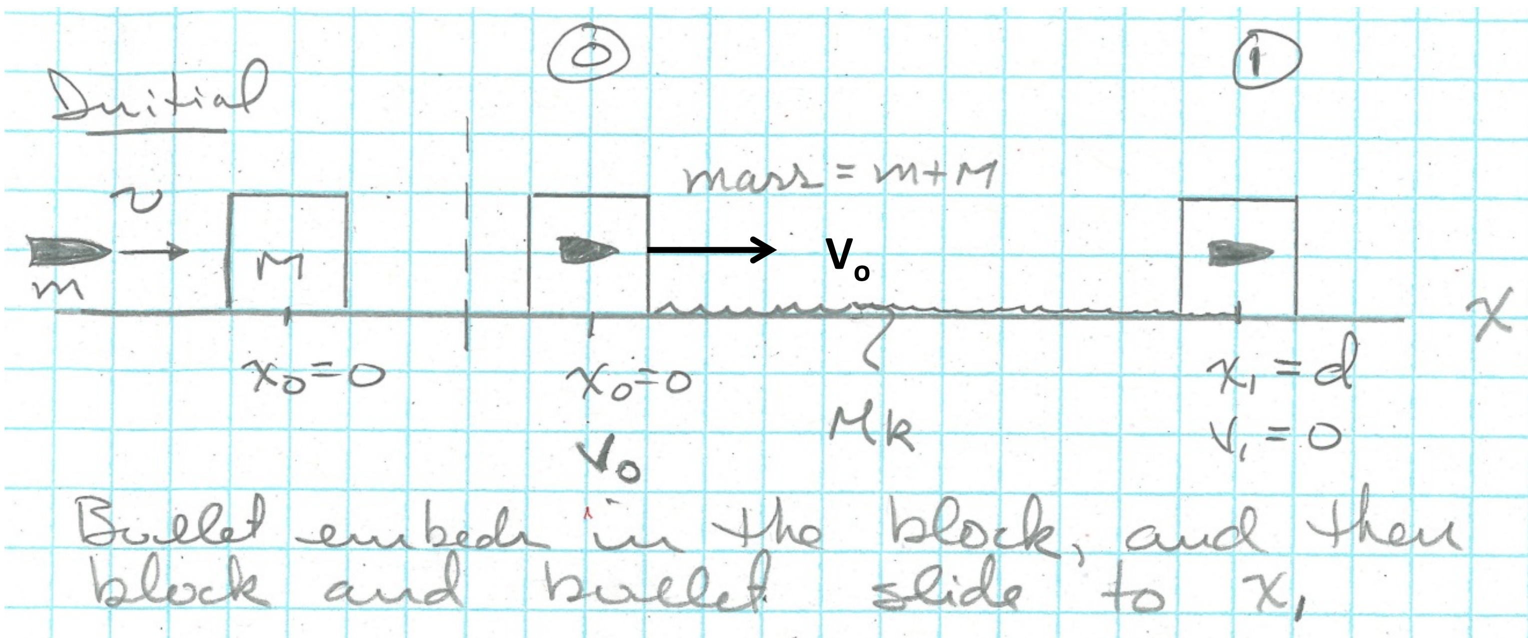
where $\vec{p}$ is the total momentum of the system

Exercise 16

Whiteboard Problem 11-6: Problem 11.49 in book

A bullet of mass m is fired at speed v into a block of wood of mass M that is at rest. The block, with the bullet embedded, slides a distance d across a horizontal surface. The coefficient of kinetic friction between the block and the surface is μ_k. Find an expression for the bullet's initial speed v.
(The expression should be in terms of m, M, μ_k, d, and, of course, g)

My sketch



Whiteboard Problem 11-7 (Problem 11:56 in book):

|| You have been asked to design a "ballistic spring system" to measure the speed of bullets. A bullet of mass m is fired into a block of mass M. The block, with the embedded bullet, then slides across a frictionless table and collides with a horizontal spring whose spring constant is k. The opposite end of the spring is anchored to a wall. The spring's maximum compression d is measured.

a. Find an expression for the bullet's speed v_B in terms of m, M, k, and d.

b. What was the speed of a 5.0 g bullet if the block's mass is 2.0 kg and if the spring, with $k = 50$ N/m, was compressed by 10 cm?

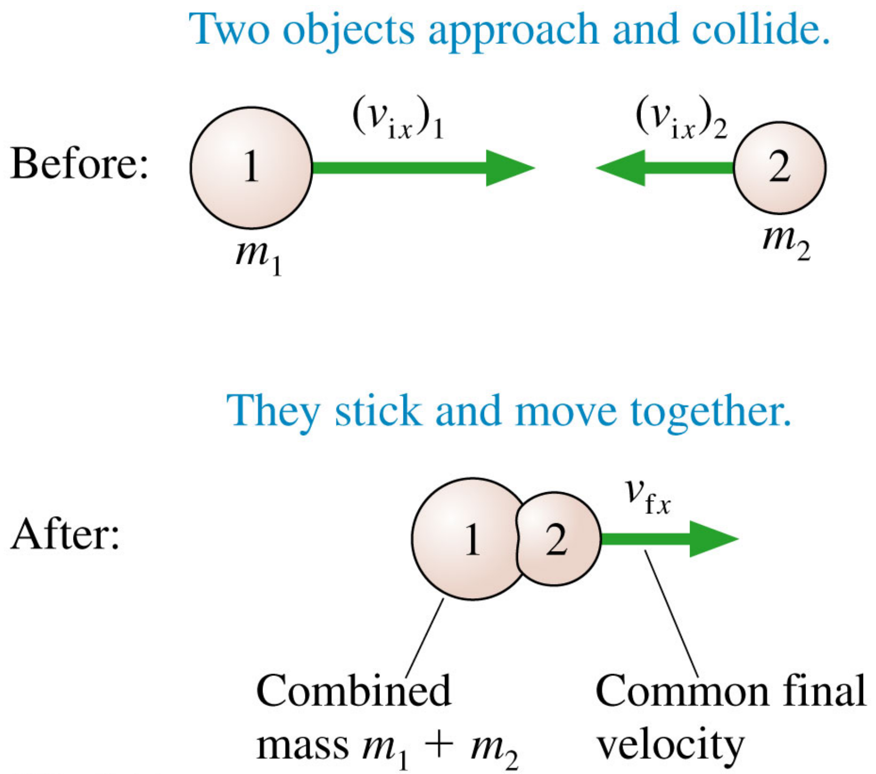
c. What fraction of the bullet's energy is "lost"? Where did it go?

My sketch:

How do you proceed?

Perfectly Inelastic Collisions

The previous problem (9-19) is an example of a **Perfectly Inelastic Collision***
which means that after the collision the bodies **stick together and have the same final velocity**.

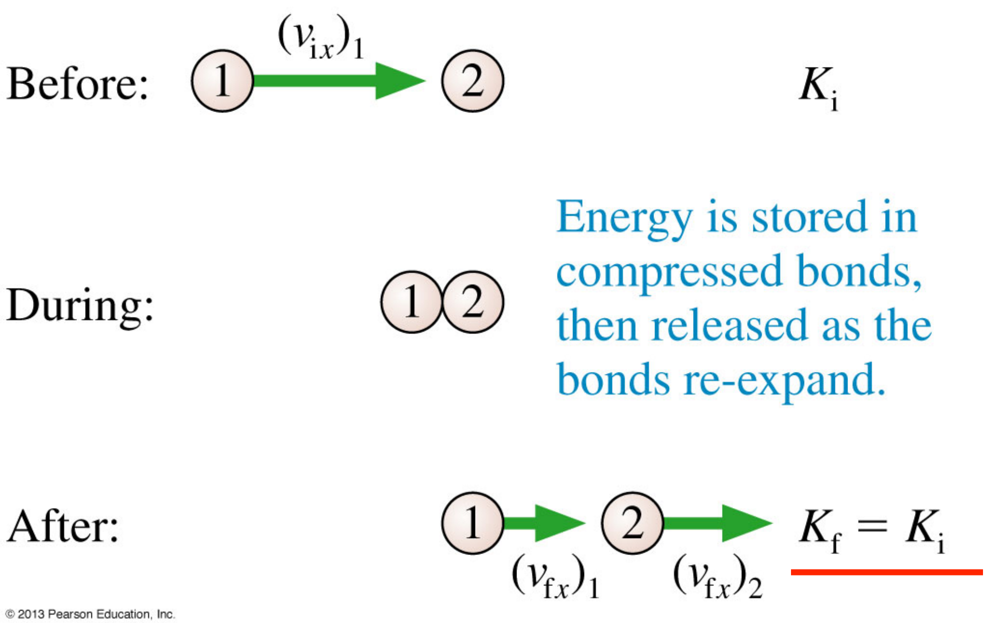


***All collisions conserve momentum**. For problems where the final velocities are different, we need some more information. We'll see the other extreme, perfectly elastic collisions that also conserve *kinetic energy*, next week.

Perfectly Elastic* Collisions

Recall the **Perfectly Inelastic Collision** where two objects stick together and have the same final velocity. These can be solved with momentum conservation alone.

A **Perfectly Elastic Collision** conserves both momentum and mechanical energy.



During the collision, some KE is transferred to PE; then this is transferred back to KE with 100% efficiency.

Along with:

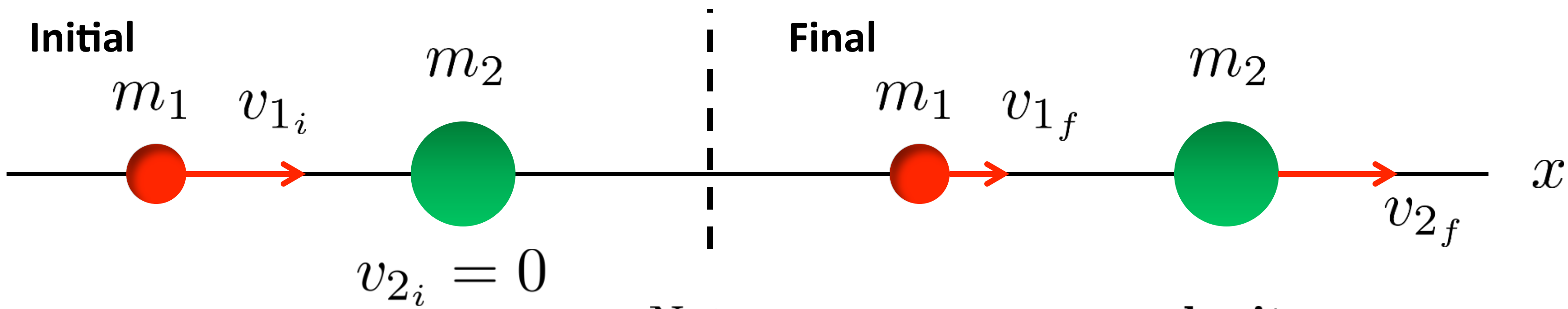
$$\vec{p}_f = \vec{p}_i$$

Note, in real collisions, there is always some energy lost to thermal energy.

*Watch the word **elastic** here. It has nothing to do with the elasticity of a spring. For collisions, it means that the mechanical energy is conserved along with the momentum.

Perfectly Elastic Collisions – A Special Case

Your author considers the Special Case of a **1D perfectly elastic collision between two bodies where one is initially at rest**.



Note: v_{1_i}, v_{1_f}, v_{2_f} are **velocity components**

Using momentum and kinetic energy conservation, the final velocities are:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest) (11.29)

(with slight differences in subscripts)

(OK, OK, I'll put these on the equation sheet!)

For anyone who is interested, the derivation looks like this (it's actually quite fun!):

KNOWNS: m_1, m_2, v_{1i}, v_{1f}

UNKNOWNS: v_{1f}, v_{2f}

Momentum conservation: $p_i = p_f$

$$\Rightarrow m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{—— (i)}$$

Perfectly Elastic collision! ∴ KE is conserved, i.e. $KE_i = KE_f$

$$\Rightarrow \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{—— (ii)}$$

All the physics is done! All that remains is algebra!

We want to solve for v_{1f} & v_{2f}. So solve for v_{2f}, say, from eqn (i):

$$v_{2f} = \frac{m_1}{m_2}(v_{1i} - v_{1f}) \quad \text{—— (iii)}$$

and plonk into eqn. (ii):

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 \cdot \frac{m_1^2}{m_2^2}(v_{1i} - v_{1f})^2$$

$$\Rightarrow v_{1f}^2 - \left(\frac{2m_1}{m_1+m_2} v_{1i}\right) v_{1f} - \left(\frac{m_2 - m_1}{m_1+m_2} \cdot v_{1i}^2\right) = 0$$

This is a quadratic equation in v_{2f} with solutions:

$$v_{1f} = \frac{\frac{2m_1 v_{1i}}{m_1+m_2} \pm \sqrt{\left(\frac{2m_1}{m_1+m_2}\right)^2 v_{1i}^2 + 4\left(\frac{m_2-m_1}{m_1+m_2}\right) v_{1i}^2}}{2} = \frac{m_1}{m_1+m_2} v_{1i} \pm v_{1i}\sqrt{\frac{m_1^2}{(m_1+m_2)^2} + \frac{m_2-m_1}{m_1+m_2}}$$

$$\therefore v_{1f} = v_{1i}\left(\frac{m_1 \pm m_2}{m_1+m_2}\right) \Rightarrow$$ the 2 solutions for v_{1f} are

$v_{1f} = v_{1i}$ — But this means there was NO collision!! So we discard this solution!

or $$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1+m_2}\right) v_{1i}} \quad \text{—— (iv)}$$ — KEEP THIS!

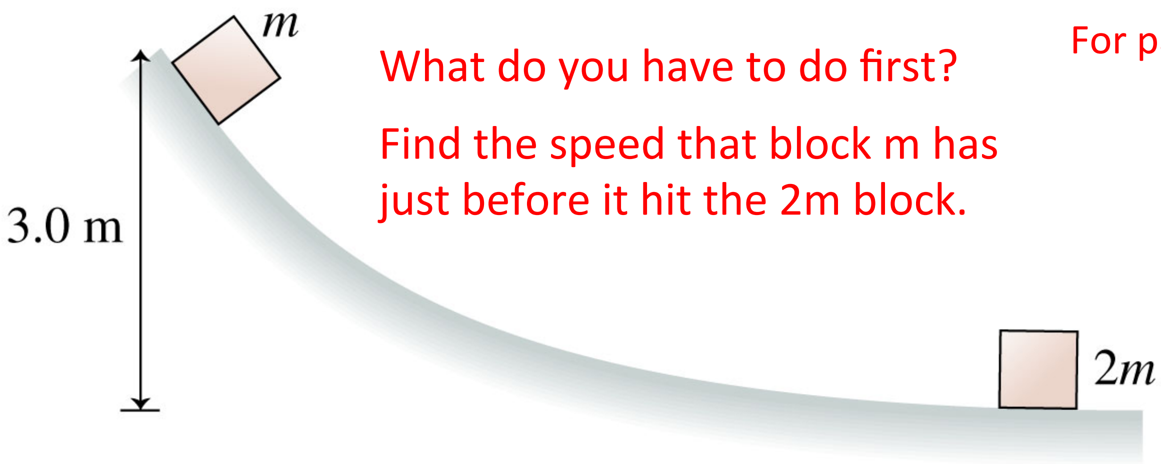
Substituting Eqn.(iv) into Eqn. (iii) we obtain $$\boxed{v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i}} \quad \text{—— (v)}$$

Whiteboard Problem 11-8 (Problem 11: 25 in book)

|| A package of mass m is released from rest at a warehouse loading dock and slides down the 3.0-m-high, frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass $2m$, from the bottom of the chute.

a. Suppose the packages stick together. What is their common speed after the collision?

b. Suppose the collision between the packages is perfectly elastic. To what height does the package of mass m rebound?



What do you have to do first?

Find the speed that block m has just before it hit the 2m block.

For part b, you might want eqns 11-29:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$