A Quick Recap of the Last Class

- **Work Kinetic Energy Theorem:**

$$\Delta K = K_f - K_i = W_{net}$$

- **Potential Energy for a Conservative Force:**

$$\Delta U = U_f - U_i \equiv -W_c(i \to f)$$

- **Gravitational Potential Energy*:**

$$U_g = mgy$$

**For g = constant; and we're free to set y = 0 anywhere we like.*

- **Conservation of Mechanical Energy*:**

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

**At this point, includes only conservative forces that can be described by a potential energy.*

A Good Whiteboard Problem: 9-8

A roller coaster car on a frictionless track shown below starts from rest at height h. The track's valley and hill consist of circular shaped segments of radius R.

a) **What is the maximum height h_{max} (in terms of R) from which the car can start so as not to fly off the track going over the hill?**
b) **For the height found in part a, what is the apparent weight (in terms of the actual weight w) felt by the riders at the bottom of the valley?**

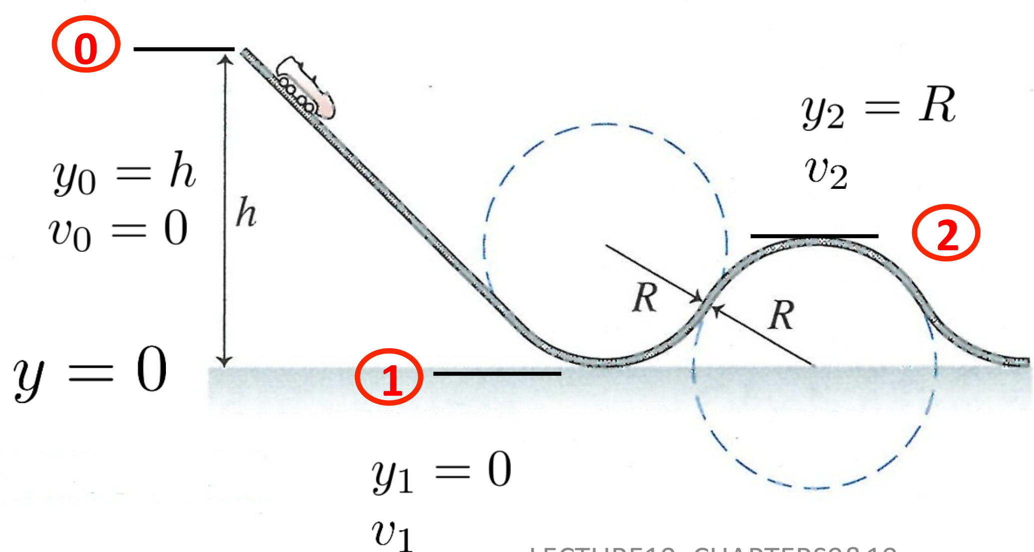

Table Challenge Problem

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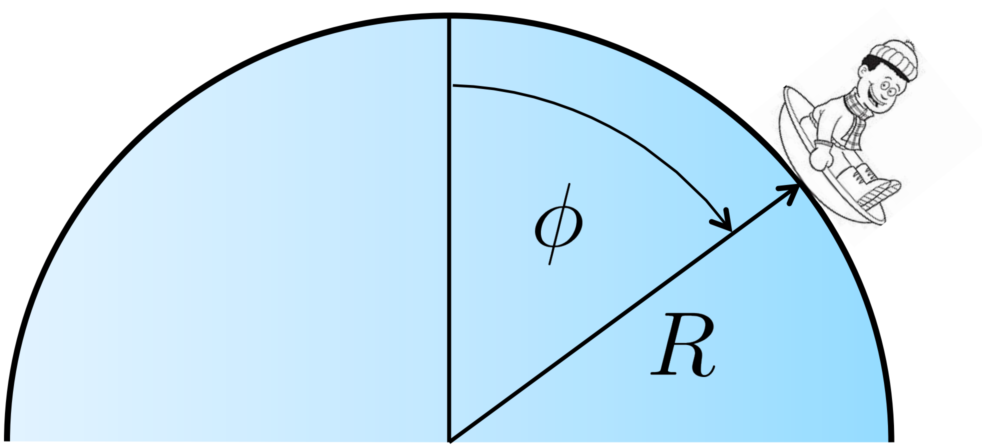
You are sledding on a hemispherical hill that is covered with frictionless snow. If you start from rest at the top of the hill, **at what angle (in degrees) does the sled fly off the hill?**



Answer: ____________

Whiteboard Problem: 9/10 - 9

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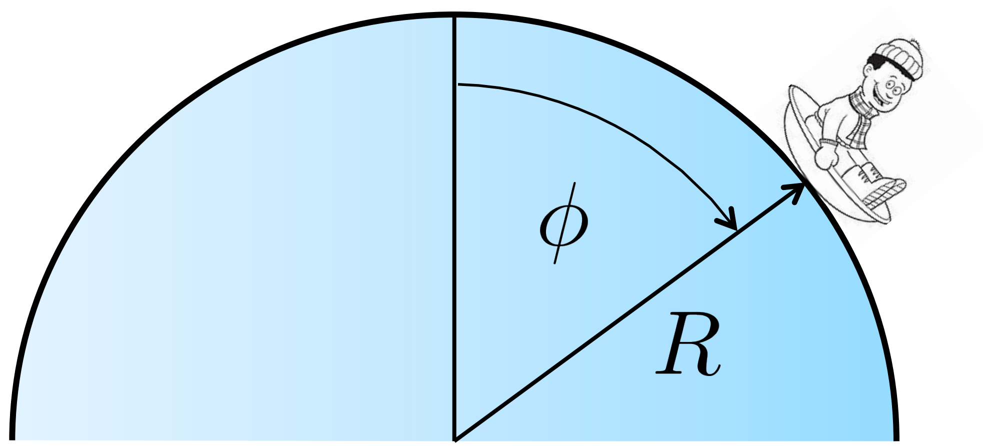
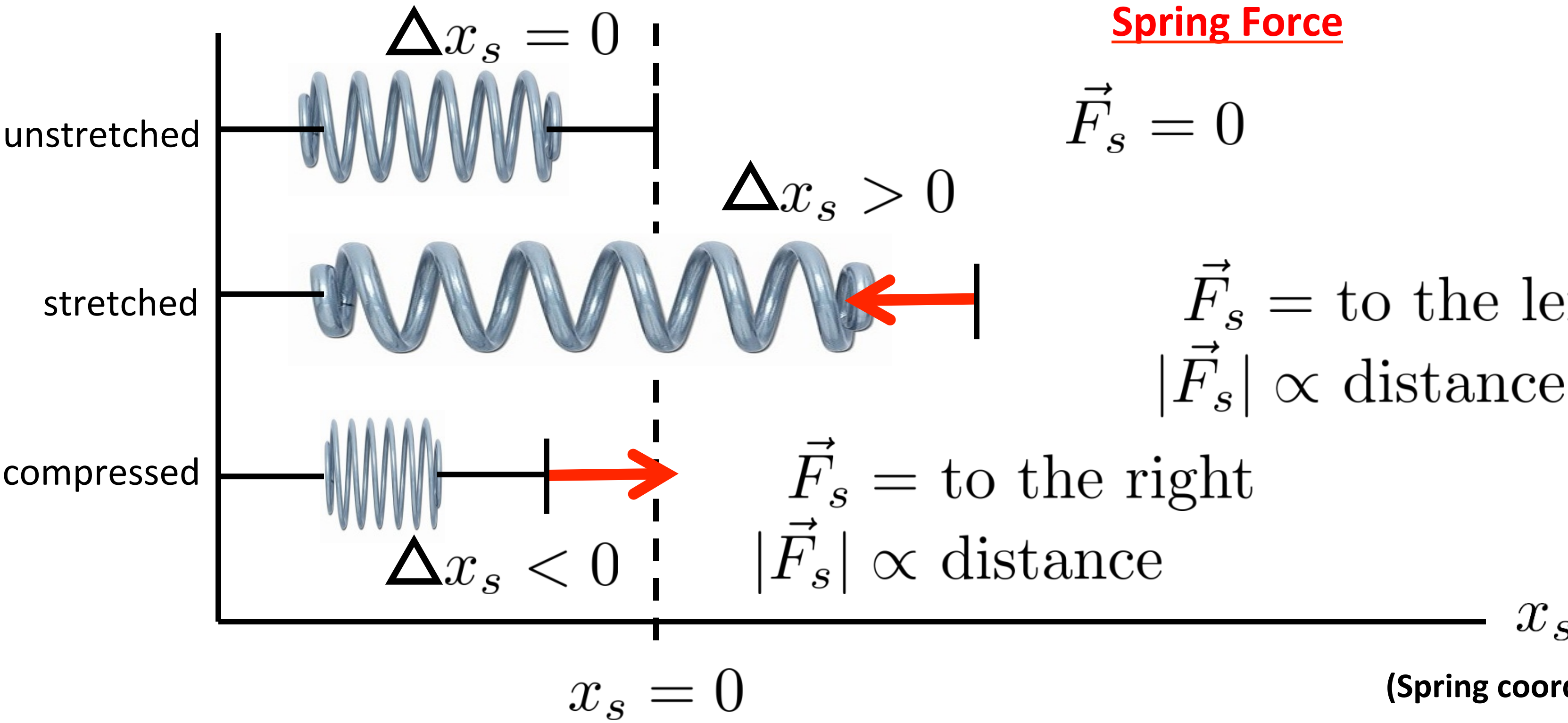


Linear Springs – Hooke's Law



Component of the Spring Force: $\boxed{\cancel{F_s = -kx_s}}$ $F_s = -k\,\Delta x_s$

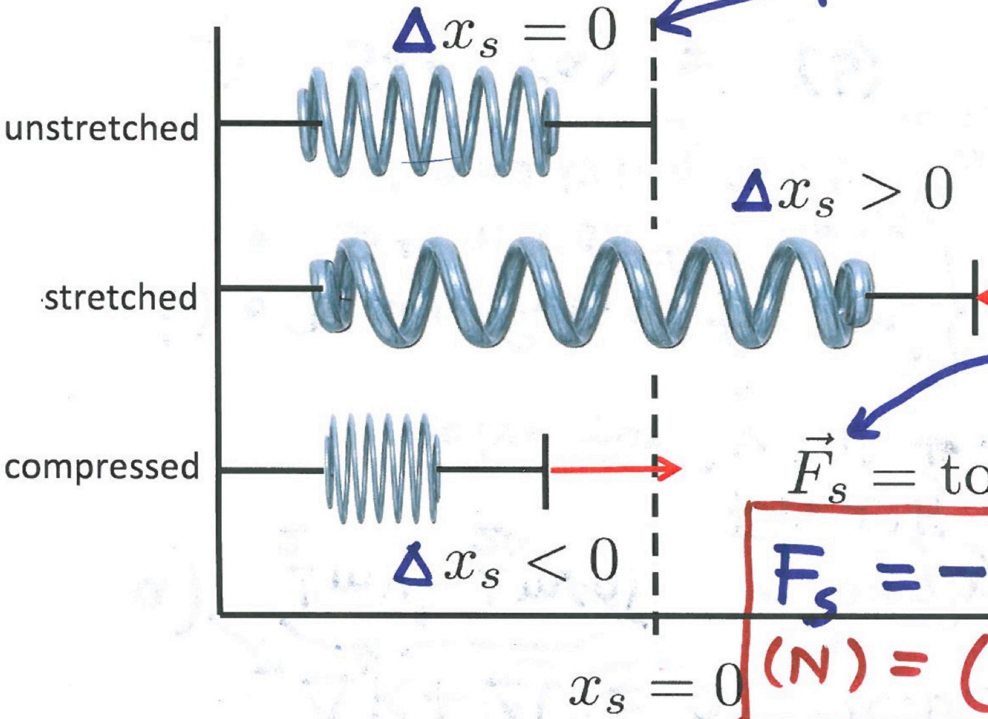
(for $x_s = 0$ at the equilibrium) $k =$ spring constant (≥ 0), [units, $\frac{N}{m}$]

Note: we call a force like this "***a linear restoring force***." (Why?)

Compression or extension of spring = Δx_s

Linear Springs – Hooke's* Law

equilibrium (or unstretched) length of spring



Spring Force

$\vec{F}_s = 0$

$\vec{F}_s =$ to the left

Restoring force (spring force)

$\vec{F}_s =$ to the right

$$\boxed{F_s = -k\,(\underbrace{\Delta x_s}_{(m)})}$$

$$(N) = \left(\frac{N}{m}\right)$$

Force constant of spring [tells us how stiff the spring is]

(Spring coordinate)

Component of the Spring Force: $\cancel{\boxed{F_s = -kx_s}}$

(for $x_s = 0$ at the equilibrium)

$$|k| = \frac{\text{Spring force } F_s}{\text{compression } \Delta x_s \text{ or extension}}$$

$k =$ spring constant, [units, $\frac{N}{m}$]

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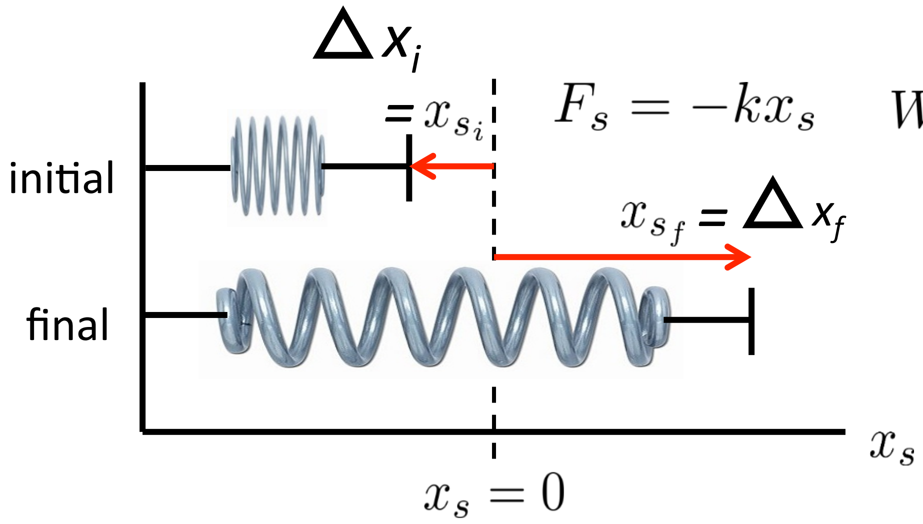
*The same Hooke from the Newton film – his brother was the ship captain in Peter Pan.

<u>Work & Potential Energy for a Linear Spring*</u>



$$F_s = -kx_s$$

$$W_{s_{i \to f}} = \int_{x_{s_i}}^{x_{s_f}} F_s dx_s = \int_{x_{s_i}}^{x_{s_f}} -kx_s dx_s$$

$$= -k \int_{x_{s_i}}^{x_{s_f}} x_s dx_s = -k \left(\frac{x_s^2}{2} \right)_{x_{s_i}}^{x_{s_f}}$$

$$W_{s_{i \to f}} = -\frac{1}{2} k \left(x_{s_f}^2 - x_{s_i}^2 \right)$$

$$= -\frac{1}{2} k (\Delta x_s)^2$$

Is the Spring Force Conservative? **Yes**, the work done by F_s does not depend on the path taken between x_{si} and x_{sf}. (Try it)

So, we can define a change in the Spring Potential Energy:

<u>Spring Potential Energy</u>*

$$\cancel{U_s = \frac{1}{2} k x_s^2}$$

For $x_s = 0$ at the spring equilibrium

***<u>Note</u>,** I disagree with your author here. He writes his spring force and potential energy with the spring coordinate at some arbitrary point. <u>You can always choose the spring coordinate to be zero at the equilibrium point</u>; it makes things a lot easier in using this in problems.

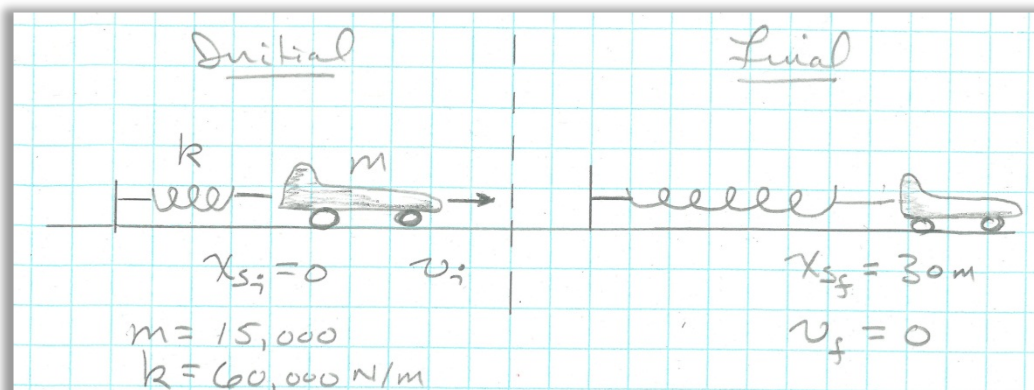
Whiteboard Problem 09/10 - 10

As F-18 fighters land on an aircraft carrier, the jet's tail hook snags a cable to slow it to a stop. The cable is attached to a spring with spring constant 60,000 N/m.

If the spring stretches 30 m to stop the fighter, what was the plane's landing speed?

Like this:

My sketch:



Whiteboard Problem 09/10 - 10

24. ‖ As a 15,000 kg jet plane lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

$$\Delta K + \Delta U = 0$$

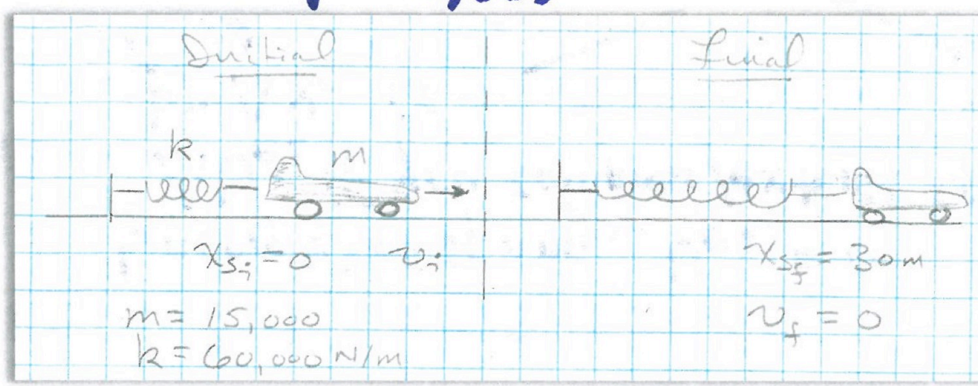
$$\left(\tfrac{1}{2}m\cancel{v_f^2} - \tfrac{1}{2}mv_i^2\right) + \left(\tfrac{1}{2}k(\Delta x_s)^2 - 0\right) = 0$$

$$-\tfrac{1}{2}(15000)(v_i^2) + \tfrac{1}{2}(60{,}000)(30)^2 = 0$$

$$v_i = \sqrt{\frac{60{,}000 \times 30^2}{15{,}000}} = 60\ \text{m/s}$$

Like this:

My sketch:



Whiteboard Problem 09/10-11

A vertical spring with k = 490 N/m is standing on the ground. You are holding a 5 kg block just above the spring, not quite touching it.

a) How far does the spring compress if you let go of the block suddenly?
b) How far does the spring compress if you slowly lower the block to the point where you can remove your hand without disturbing it?
c) Why are your two answers different?
d) In part (a), *at what compression of the spring does the box have its maximum speed?*

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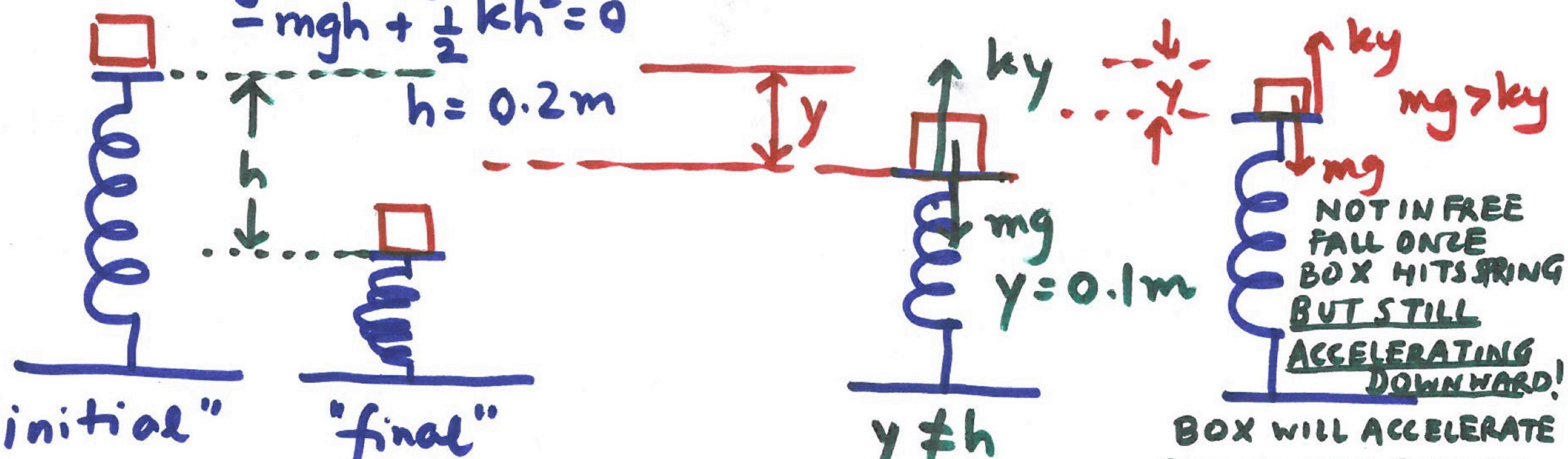
a)

$$\cancel{\Delta K}^{0} + \Delta U = 0$$
$$\Delta U_g + \Delta U_{ee} = 0$$
$$-mgh + \tfrac{1}{2}kh^2 = 0$$

b) NOT isolated! CANNOT USE C.O.M.E.



BOX WILL ACCELERATE DOWNWARD, BUT AT AN EVER-DECREASING RATE, UNTIL $mg = ky$ i.e. the equilibrium point in (b).

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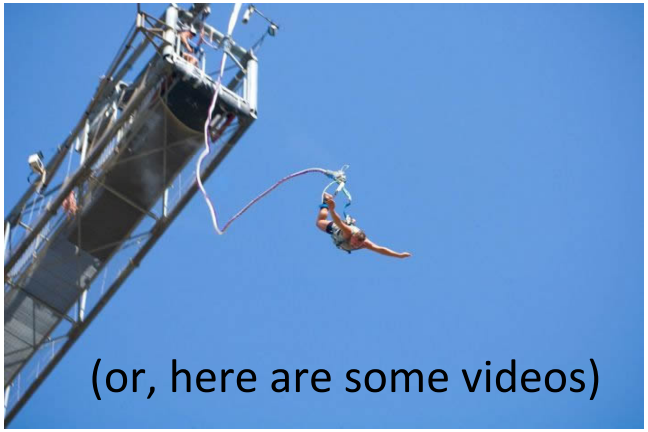
Table:

Names:

In a moment of insanity, you decide to go bungee jumping. You stand on a bridge 100 m above a raging river and attach a 30 m long bungee cord to your harness. A bungee cord, for practical purposes, is just a long spring, and this has a spring constant of 40 N/m. Assume that your mass is 80 kg. After a long hesitation, you dive off the bridge. How far are you above the water when the cord reaches its maximum elongation?



Answer: ____________

Force from Potential Energy

We know how to find the Work done by a Force:

$$W = \int \vec{F} \cdot d\vec{s}$$

By the way:
This should be your answer if you are ever asked "What is Work" – not F x d!

Or, in one dimension:

$$W = \int F_x dx$$

And for a conservative force:

$$\Delta U = -W(\text{done by } \mathrm{F_x}) = -\int F_x dx$$

$i.e.$ if we know F_x, we can find ΔU.

Can we go the other way? i.e. if we know the potential energy, can we find the force?
As your author shows (for one dimension):

$$\boxed{F_x = -\frac{dU}{dx} = -(\text{slope of the } U_x \text{ vs. } x \text{ curve})}$$

Everyone forgets this negative sign, but it's important; you just have to remember it.
It will even come back to haunt us in PHY192 when we do electric potential!

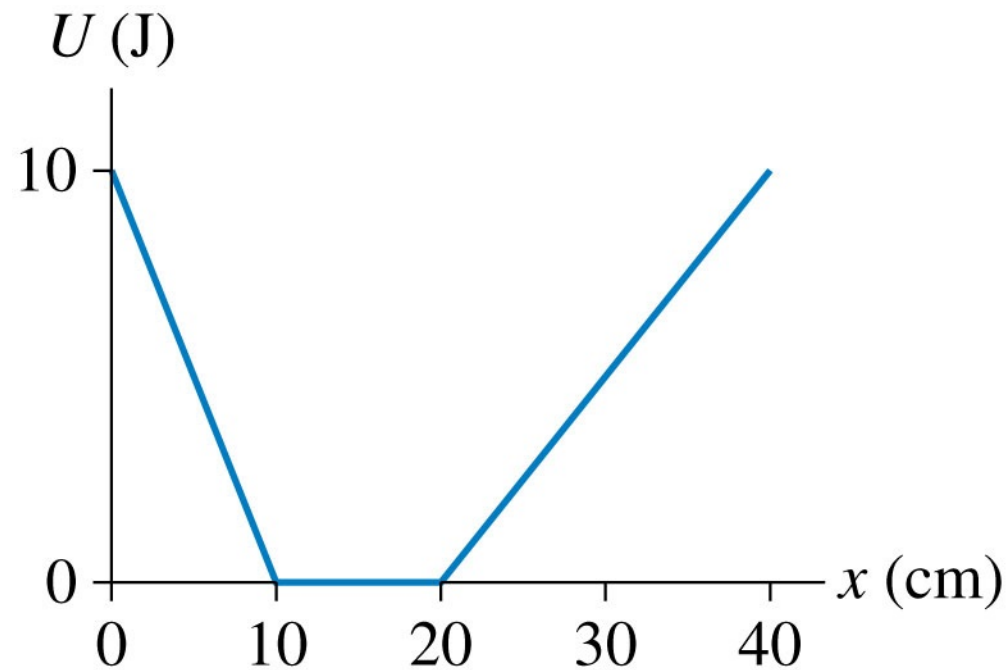
Whiteboard Problem 9/10 - 12

A particle has the potential energy show below.

a) What is the x-component of the force on the particle at x = 5 cm?

b) What is the x-component of the force on the particle at x = 15 cm?

c) What is the x-component of the force on the particle at x = 30 cm?



Whiteboard Problem 9/10 - 13

A particle moving along the x-axis has the potential energy:

$$U(x) = \frac{10}{x}\ J$$

where x is in m.

What is the x-component of the force on the particle at x = 5 m?

Where We Stand with Conservation of Energy

For a system going from state initial state *i* to a final state *f*:

i K_i & U_i → K_f & U_f f

If there are no dissipative forces (e.g. friction) or applied forces, the Mechanical Energy is Conserved.

$$E_{mech} = K + U = \text{constant}$$

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$$

Where: $\Delta \overset{\text{(always)}}{=} \text{(final) - (initial)}$

Where:

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

For Uniform Gravity:

$$\Delta U_g = mgy_f - mgy_i = mg\left(y_f - y_i\right)$$

For a Linear Spring:

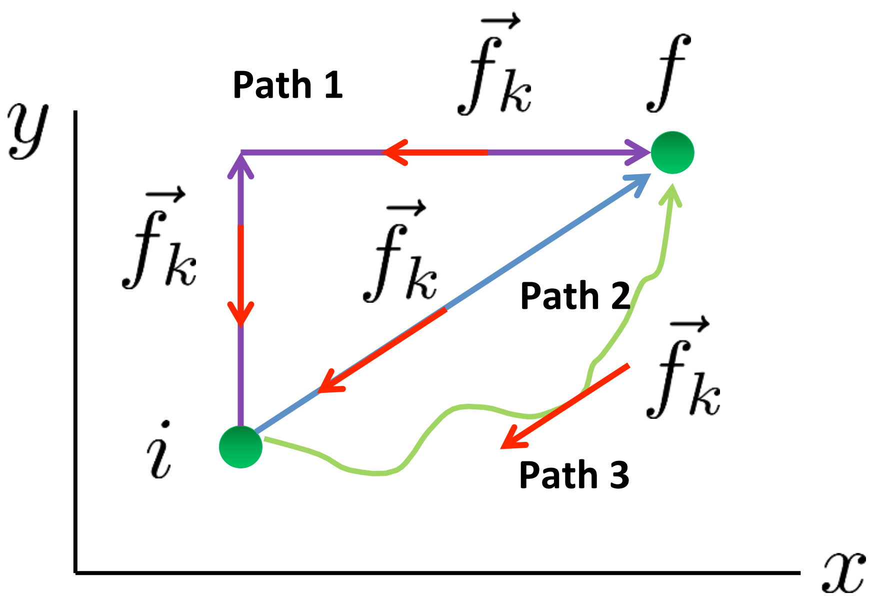
$$\Delta U_s = \frac{1}{2}kx_{s_f}^2 - \frac{1}{2}kx_{s_i}^2 = \frac{1}{2}k\left(x_{s_f}^2 - x_{s_i}^2\right)$$

What about other forces that can't be described with a Potential Energy?

Nonconservative* Forces

Definition: A **nonconservative force** is a force for which the work done by the force in going from some initial point *i* to some final point *f* **does depend on the path followed.**

For example, Friction:



The work done by the friction force is different for each path from *i* to *f*.

So, we cannot associate a potential energy with a nonconservative force.

But, we can include nonconservative forces in our conservation of energy technique.

**Again, as we'll see, the name comes from the fact that nonconservative forces do not conserve the mechanical energy.*

<u>The Complete Conservation of Mechanical Energy</u>

We started with the **<u>Work – Kinetic Energy Theorem</u>** for a system going from an initial state, *i*, to a final state, *f:*

$$\Delta K = W_{\text{net}}(i \to f) = W_c(i \to f) + W_{nc}(i \to f)$$

conservative forces

nonconservative forces

$$\text{For Conservative Forces}^*\text{:}\ \Delta U = -W_c(i \to f)$$

$$\text{Define: Total Mechanical Energy: } E_{\text{mech}} = K + U$$

$$\text{so, } \Delta E_{\text{mech}} = \Delta K + \Delta U$$

So, the <u>Work – Kinetic Energy Theorem</u> can be written as:

$$\boxed{\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}}$$

This is the equation we've been using with the added feature that **we now know how to include forces like <u>friction or applied forces</u>.** *They go right here.*

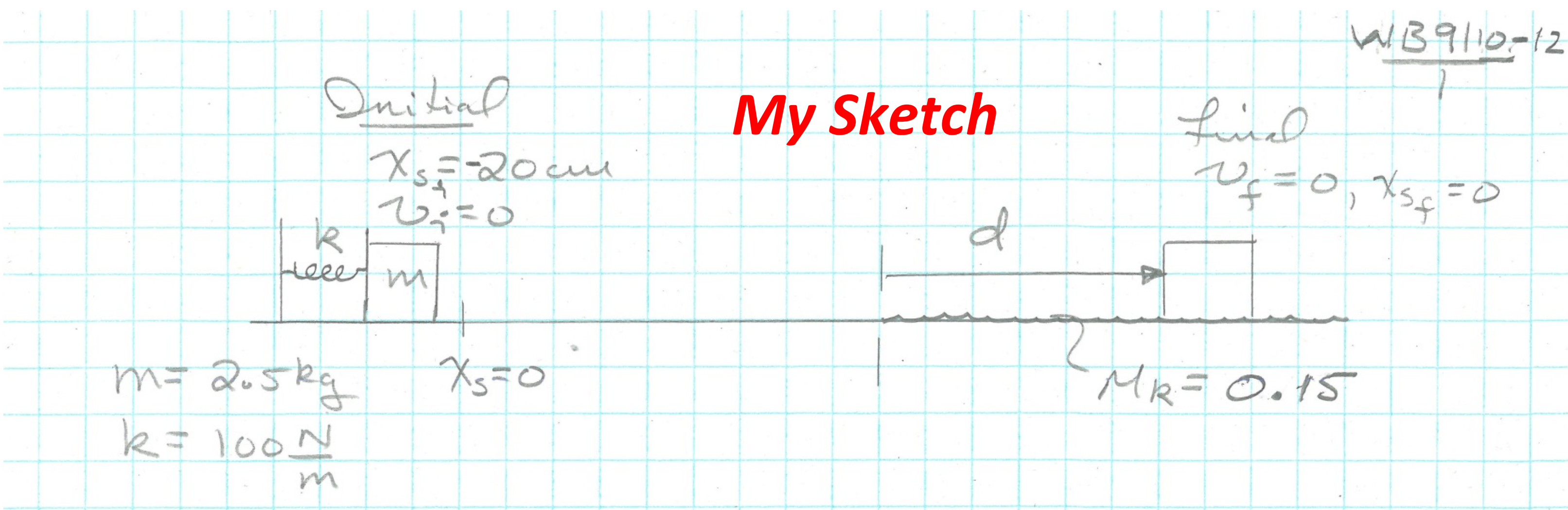
We see that potential energy is energy that can be stored and returned to kinetic. Applied forces can change the kinetic or the potential energies, and hence the mechanical energy. On the other hand, dissipative forces, like friction, will always decrease the mechanical energy and increase the thermal energy.

*If you have more than one conservative force, you'll need a potential energy for each.

Whiteboard Problem: 9/10-14

A horizontal spring with spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction between the box and the surface is 0.15.

How far does the box slide across the rough surface before it comes to rest?



Whiteboard Problem: 9/10-15

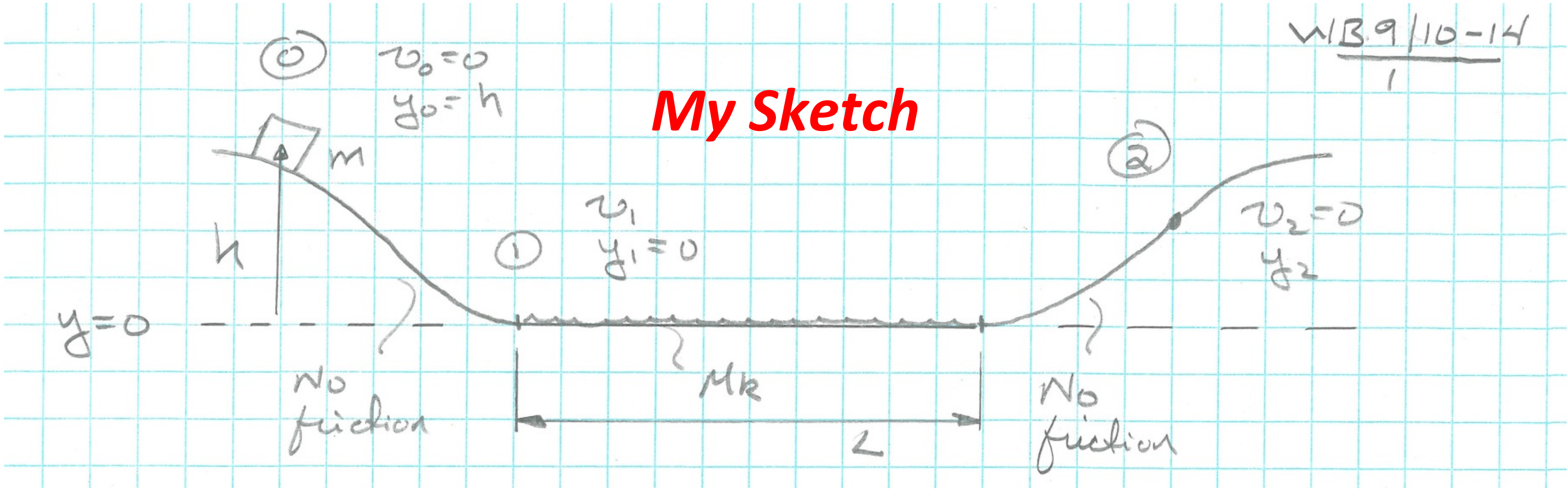
Kathy's baby brother sits on a sled. The combined mass of the sled and the baby is m. Kathy pulls on the rope that is at an angle θ above the horizontal. The tension T is constant and the coefficient of kinetic friction between the sled and the snow is μ_k.

Find an expression for the speed of the sled after Kathy has pulled it a distance d starting from rest. **(LC)**

There are two traps in this problem - - don't fall into them!

Whiteboard Problem: 9/10-16

A block of mass m starts from rest at height h. It slides down a frictionless slope, across a rough horizontal surface of length L, then up another frictionless slope. The coefficient of kinetic friction on the rough surface is μ_k.
Find an expression for the height that the block goes up the second slope.



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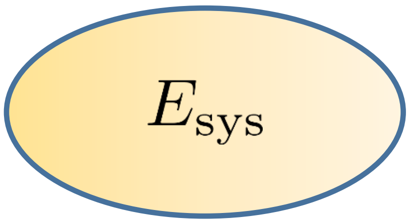
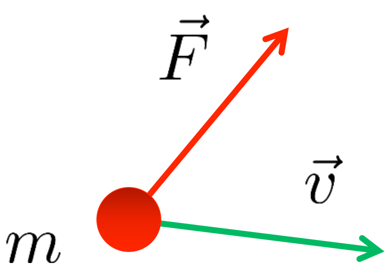
Power

For a system, **Power is the rate that energy is transferred into or out of the system:**



$$\text{Power, P} = \frac{dE_{\text{sys}}}{dt} \qquad \text{Units: } 1\ \tfrac{J}{s} = 1\ \text{Watt}(W)$$

For a mass for which the energy is changing because a force is doing work on it, your author shows:



$$\boxed{\text{Power, P} = \frac{dW}{dt} = \vec{F}\cdot\vec{v}}$$

Whiteboard Problem 9/10 - 17

You push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s.

How much work did you do, and what was your power output while pushing?

For steel on steel: $\mu_k = 0.6$