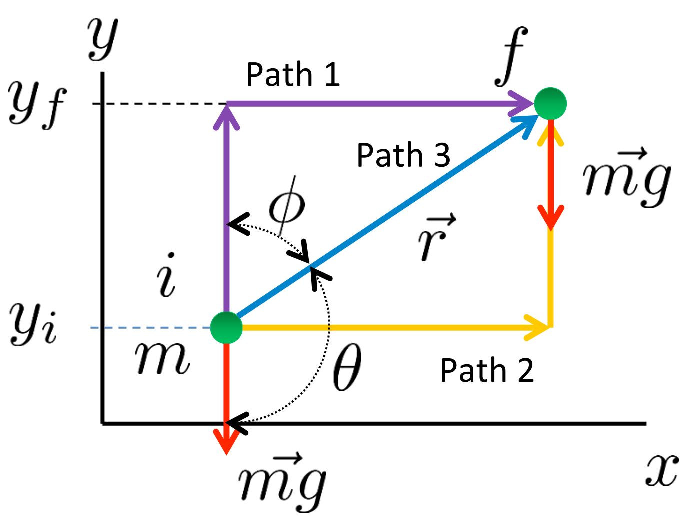
Work & Energy – The Final Look

Wherein, we rediscover our conservation of energy "C. O. M. E." equation in a more useful and powerful form.

Conservative and Nonconservative Forces (we discussed 2 differences last time)

Definition: A **conservative* force** is a force for which the work done by the force in going from some initial point to some final point **does not depend on the path followed**. **[This is another way to state "difference #2" from last time]**

For example, Gravity:



Work done by Gravity going i to f:

$$\text{Path 1: } W_g = -mg(y_f - y_i)$$

$$\text{Path 2: } W_g = -mg(y_f - y_i)$$

$$\begin{aligned}\text{Path 3: } W_g &= \vec{mg} \cdot \vec{r} \\ &= mgr\cos\theta \\ &= -mgr\cos\phi \\ &= -mg(y_f - y_i)\end{aligned}$$

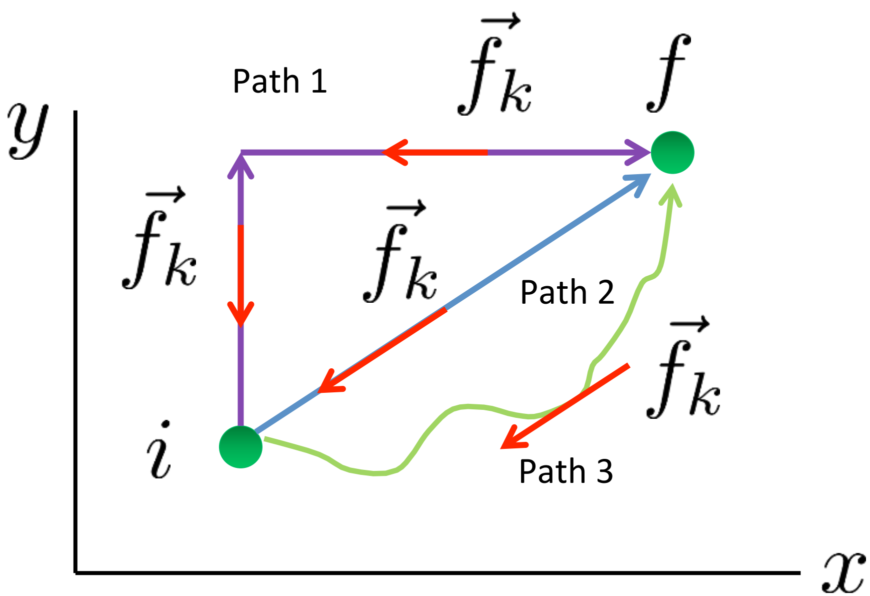
So the work done by gravity does not depend on the path, just the change in y.

*As we'll see, the name comes from the fact that conservative forces conserve the total mechanical energy.

Nonconservative* Forces

Definition: A **nonconservative force** is a force for which the work done by the force in going from some initial point to some final point **does depend on the path followed.** **["difference #2" from last time]**

For example, Friction:



The work done by the friction force is different for each path from i to f.

So, we cannot associate a potential energy with a nonconservative force. **["difference #1" from last time]**

But, we can include nonconservative forces in our conservation of energy technique…
…a more useful, powerful C. O. M. E. !!

*Again, as we'll see, the name comes from the fact that nonconservative forces do not conserve the mechanical energy.

C. O. M. E. – Better, More Useful....

$\Delta E_{\mathrm{mech}} = \Delta K + \Delta U = 0$, if *only conservative* forces act on system

$\boxed{\Delta E_{\mathrm{mech}} = \Delta K + \Delta U = W_{nc}}$ if N. C. forces, too, act on system

This is our old friend from Chapter 10 with the added feature that **we now know how to include forces like <u>friction and other non-conservative forces</u>** (e.g., tension, normal force, etc). They go right here.

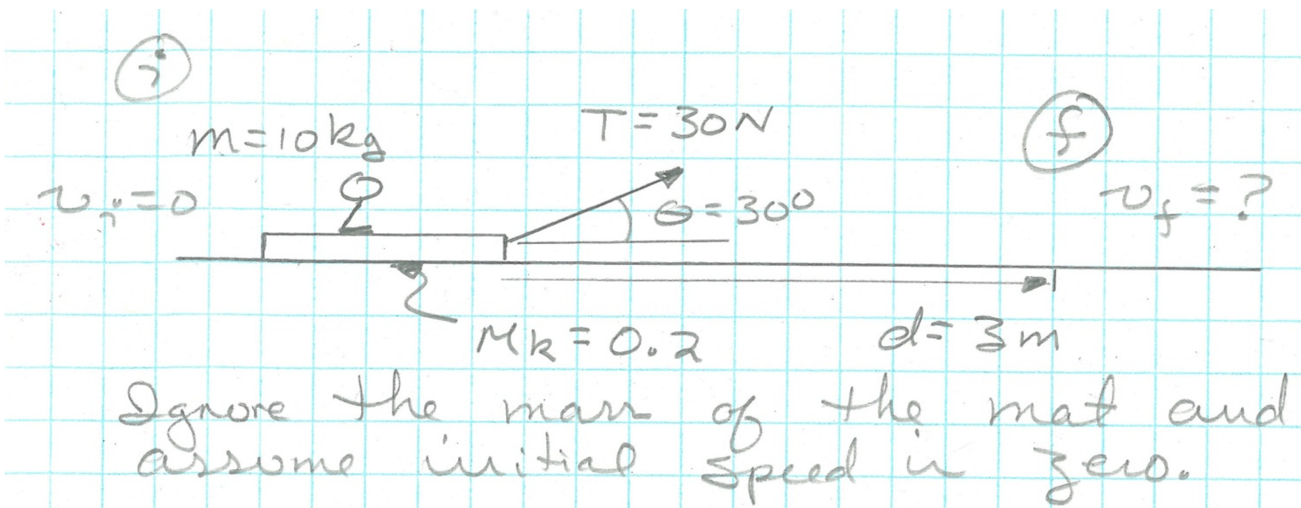
Dissipative forces (like friction) will decrease the total mechanical energy

Other NC forces (like tension, normal force, etc) can increase / decrease the kinetic energy.

Whiteboard Problem 1

46. ||| Susan's 10 kg baby brother Paul sits on a mat. Susan pulls the mat across the floor using a rope that is angled 30° above the floor. The tension is a constant 30 N and the coefficient of friction is 0.20. Use work and energy to find Paul's speed after being pulled 3.0 m.

My Sketch



Pitfall Alert!!!! Proceed with caution while figuring the normal force for calculating the work done by friction!!! How does pulling upward with the rope affect the normal force?

Whiteboard Problem 1

2.4 m/s NC forces: Tension, friction (Note: Normal force does no work here)

46. ||| Susan's 10 kg baby brother Paul sits on a mat. Susan pulls the mat across the floor using a rope that is angled 30° above the floor. The tension is a constant 30 N and the coefficient of friction is 0.20. Use work and energy to find Paul's speed after being pulled 3.0 m.

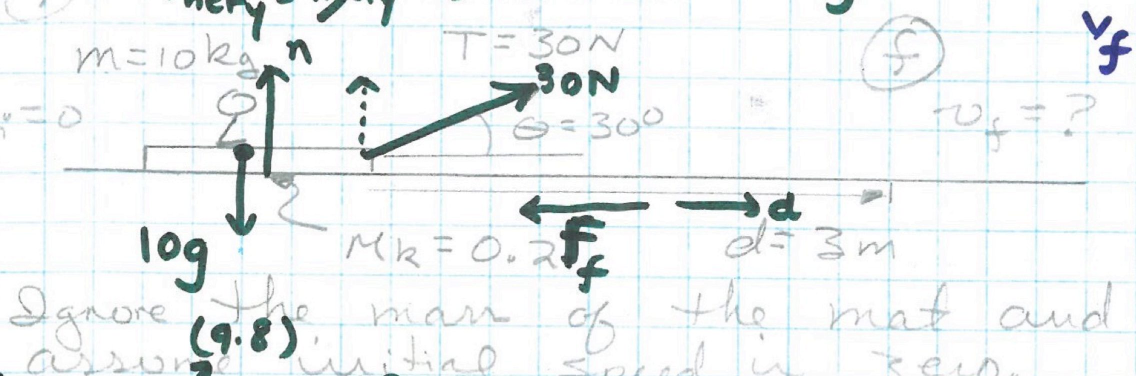
$\Delta K + \cancel{\Delta U}_0 = W_{nc} \Rightarrow \frac{1}{2}mv_f^2 - 0 = W_T + W_f = \underbrace{78 - 49.8}$

$\Rightarrow \frac{1}{2}(10)v_f^2 = 5$

$v_f = 2.4\text{ m/s}$

W_T:

$F_{net\,y} = m\cancel{a_y}^{\,0} \Rightarrow n + 30\sin 30 - 10g = 0$



W_T My Sketch

$= \vec{T}\cdot\Delta\vec{x}$

$= (30)(3)\cos 30°$

$= 78\text{ J}$

$W_f: -\mu_k n d$ $\rightarrow n = 10g - 30\sin 30$ (9.8)

$= 83\text{ N}$

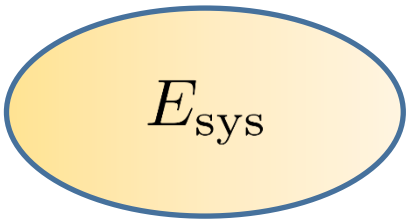
$= -0.2(83)(3)$

$= -49.8\text{ J}$

Pitfall Alert!!!! Proceed with caution while figuring the normal force for calculating the work done by friction!!! How does pulling upward with the rope affect the normal force?

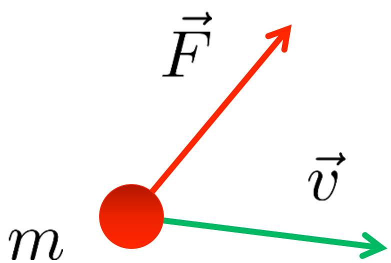
Power: Rate at which work is done

For a system, **Power is the rate that energy is transferred into or out of the system:**



$$\text{Power, P} = \frac{dE_{\text{sys}}}{dt} \qquad \text{Units: } 1\ \frac{J}{s} = 1\ \text{Watt}(W)$$

For a mass for which the energy is changing because a force is doing work on it, your author shows:



$$\boxed{\text{Power, P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}}$$

Power: Rate at which work is done

For a system, **Power is the rate that energy is transferred into or out of the system:**

e.g. 60W light bulb ⇒ Work is being done in transforming electrical energy to light energy (by electric field as it pushes electrons thru the resistive element in light bulb).

E_{sys}

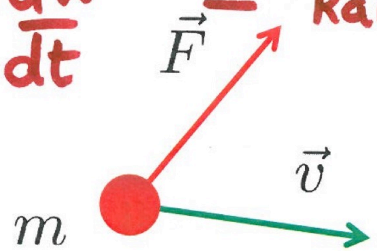
$$\text{Power, P} = \frac{dE_{sys}}{dt} \qquad \text{Units: } 1\ \frac{J}{s} = 1\ \text{Watt}(W)$$

For simplicity, consider case of a constant force.

For a mass for which the energy is changing because a force is doing work on it, your author shows:

$W = \vec{F} \cdot \vec{\Delta x}$

$\frac{dW}{dt}$ = Rate of doing work $= \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \vec{\Delta x}}{\Delta t} = \vec{F} \cdot \vec{v}$



$$\boxed{\text{Power, P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}}$$ ✓

Whiteboard Problem 2

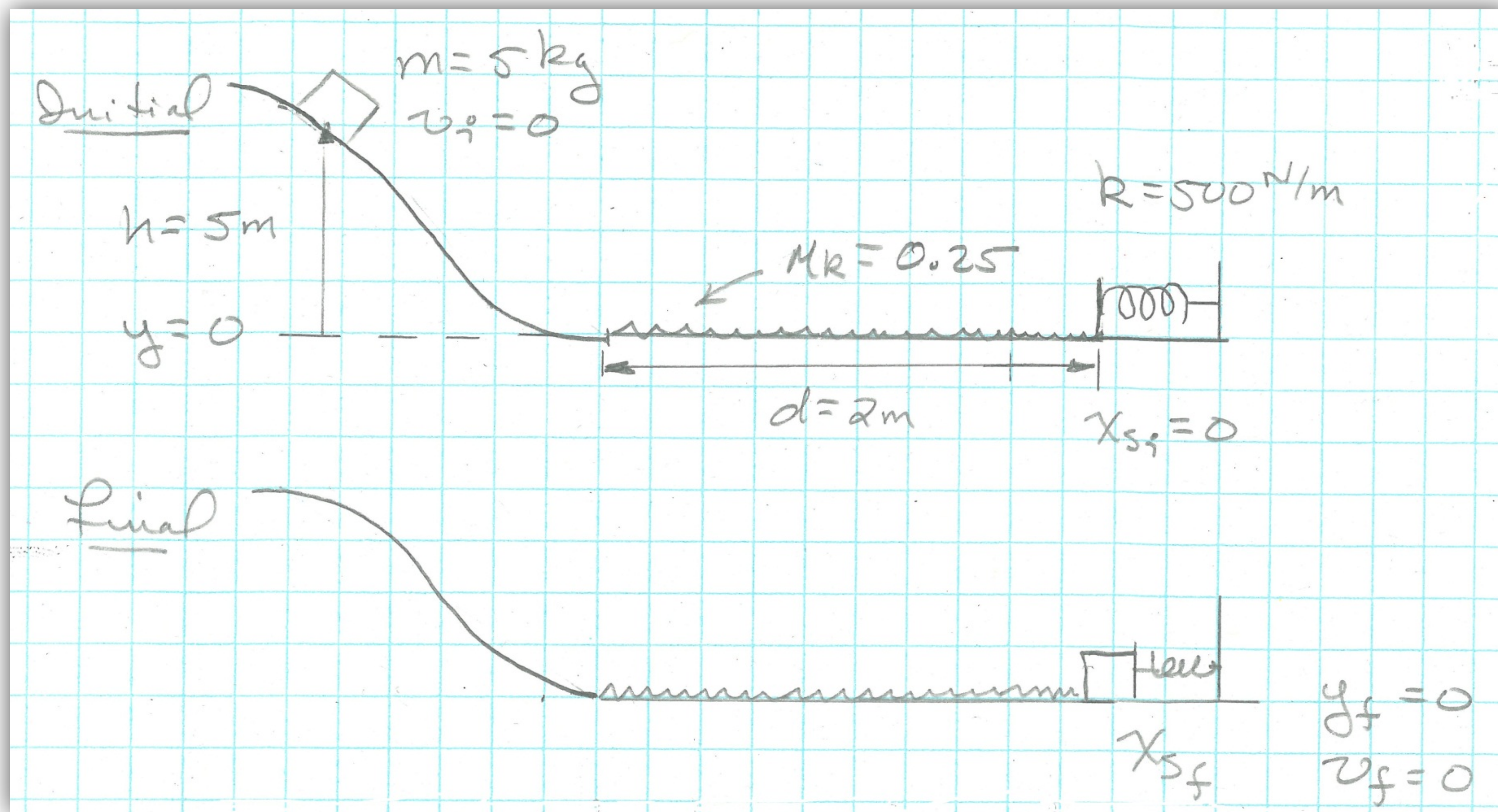
56. || A 5.0 kg box slides down a 5.0-m-high frictionless hill, starting from rest, across a 2.0-m-wide horizontal surface, then hits a horizontal spring with spring constant 500 N/m. The other end of the spring is anchored against a wall. The ground under the spring is frictionless, but the 2.0-m-wide horizontal surface is rough. The coefficient of kinetic friction of the box on this surface is 0.25.

a. ~~What is the speed of the box just before reaching the rough surface?~~
b. ~~What is the speed of the box just before hitting the spring?~~
c. How far is the spring compressed?
d. Including the first crossing, how many *complete* trips will the box make across the rough surface before coming to rest?

You really don't need to do parts a and b to do parts c and d; so just start with part c and then do part d.

My Sketch on next page

Sketch for Whiteboard Problem 2



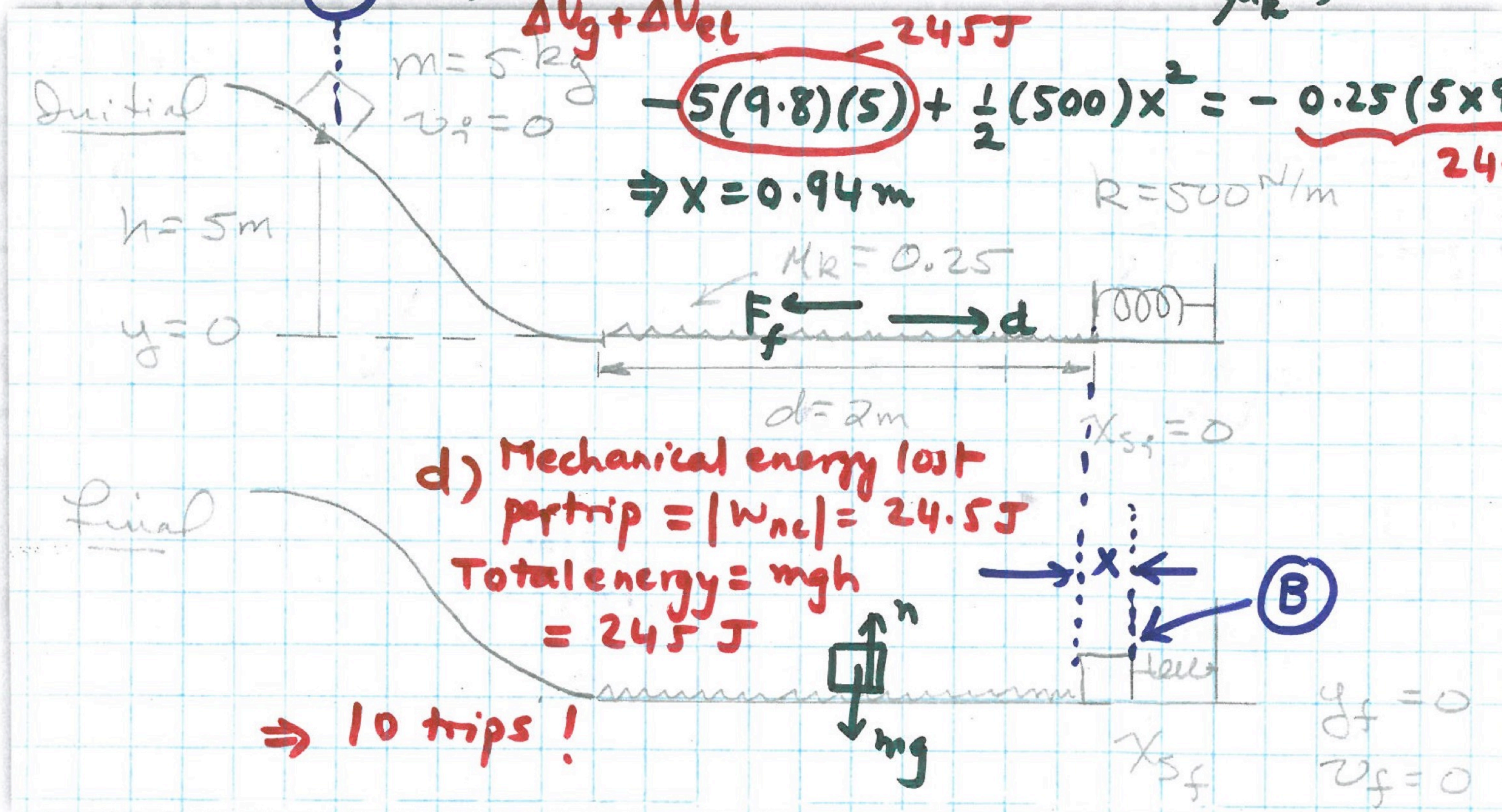
Whiteboard Problem 2

c) $\Delta U + \Delta \cancel{K}_{0} = W_{nc}$ (A) → (B)

$\Delta U = \Delta U_g + \Delta U_{el}$, $W_{nc} = -(\mu_k n)d$

$$-\underbrace{5(9.8)(5)}_{245\,J} + \frac{1}{2}(500)x^2 = -\underbrace{0.25(5\times 9.8)(2)}_{24.5\,J}$$

$\Rightarrow x = 0.94\,m$



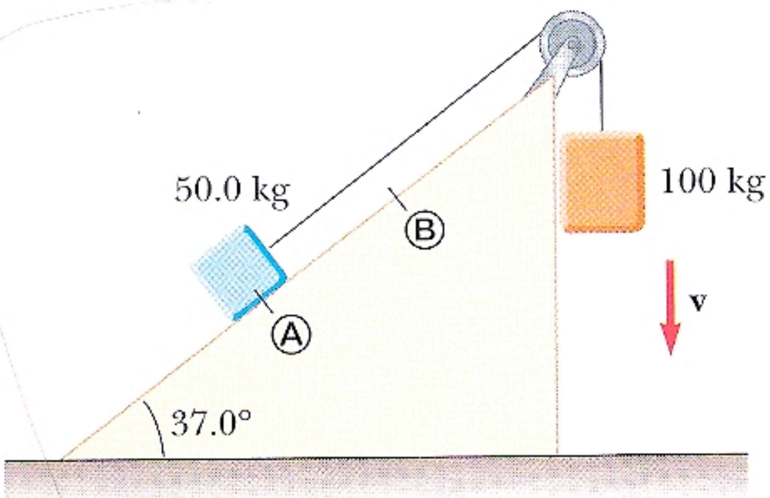
d) Mechanical energy lost per trip $= |W_{nc}| = 24.5\,J$

Total energy $= mgh = 245\,J$

$\Rightarrow$ 10 trips!

Whiteboard problem 3

44. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P7.44. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50-kg block and incline is 0.250. Determine the change in the kinetic energy of the 50-kg block as it moves from Ⓐ to Ⓑ, a distance of 20.0 m.



Whiteboard problem 3

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WHOLE SYSTEM:

$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}(50+100)v^2 - 0$$

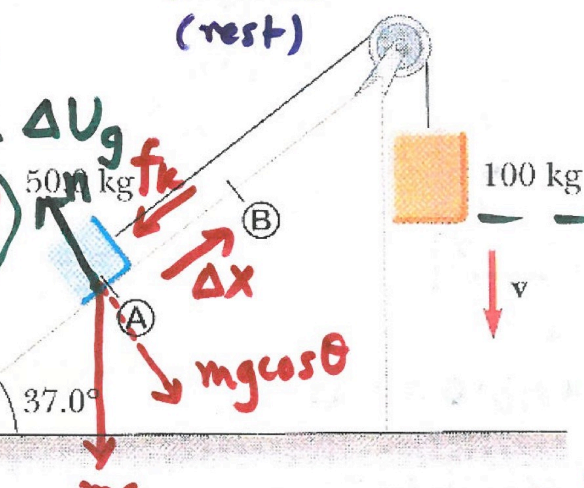
$$\underbrace{+\,50(9.8)(20\sin 37) - 100(9.8)(20)}_{\Delta U_g}$$

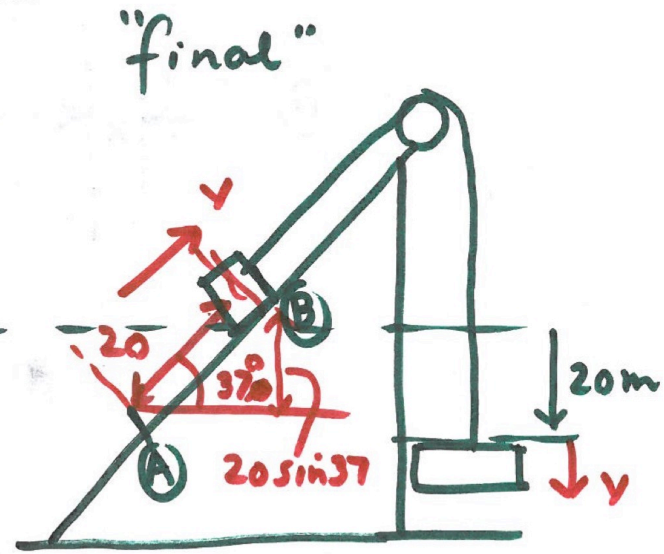
$$= -\mu_k \, n \, \Delta x \quad (\Delta x = 20)$$

$\mu_k = 0.25$

$n = mg\cos\theta$

$n = 50(9.8)\cos 37$





$$\Rightarrow \frac{1}{2}(150)v^2 + 50(9.8)(20\sin 37) - 100(9.8)(20) = -0.25(50\times 9.8\cos 37)(20)$$

$$\Rightarrow v = 12.5 \text{ m/s}$$

$$\Delta KE_{50} = \frac{1}{2}(50)(12.5)^2 = 3.91 \text{ kJ}$$