Solutions to HW7, Chapter 4

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

4.23. Solve: Since $\omega = (d\theta/dt)$ we have

$$\theta_f = \theta_i + \text{area under the } \omega\text{-versus-}t \text{ graph between } t_i \text{ and } t_f$$

From $t = 0$ s to $t = 2$ s, the area is $(20 \text{ rad/s})(2 \text{ s}) = 40$ rad. From $t = 2$ s to $t = 3$ s, the area is 20 rad. From $t = 3$ s to $t = 4$ s, the area is 10 rad.

Thus, the area under the ω-versus-t graph during the total time interval of 4 s is 66 rad or $(65 \text{ rad}) \times (1 \text{ rev}/2\pi \text{ rad}) = 10$ rev.

4.30. Model: Assume the beach is at sea level so that $r_S = 6400$ km for the surfer and $r_C = 6403$ km for the climber. The angular velocity for each of them is $2\pi \text{ rad}/24 \text{ h} = 7.27 \times 10^{-5}$ rad/s.

Visualize: $v = r\omega$.

Solve:

$$\Delta v = v_C - v_S = r_C\omega - r_S\omega = (r_C - r_S)\omega = (3000 \text{ m})(7.27 \times 10^{-5} \text{rad/s}) = 0.22 \text{ m/s} = 22 \text{ cm/s}$$

Assess: The difference in speed is small because ω is small.

4.35. Solve: The pebble's angular velocity $\omega = (3.0 \text{ rev/s})(2\pi \text{ rad/rev}) = 18.9$ rad/s. The speed of the pebble as it moves around a circle of radius $r = 30 \text{ cm} = 0.30$ m is

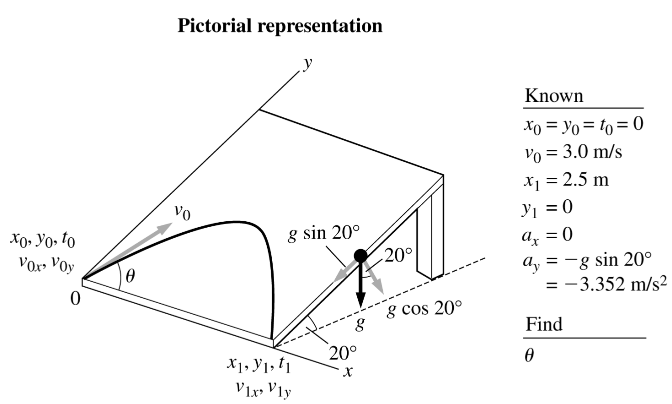
$$v = \omega r = (18.9 \text{ rad/s})(0.30 \text{ m}) = 5.7 \text{ m/s}$$

The radial acceleration is

$$a_r = \frac{v^2}{r} = \frac{(5.7 \text{ m/s})^2}{0.30 \text{ m}} = 108 \text{ m/s}^2$$

4.81. Model: We will use the particle model for the ball's motion under constant-acceleration kinematic equations. Note that the ball's motion on the smooth, flat board is $a_y = -g\sin 20° = -3.352 \text{ m/s}^2$.

Visualize:



Solve: The ball's initial velocity is

$$v_{0x} = v_0\cos\theta = (3.0 \text{ m/s})\cos\theta \qquad v_{0y} = v_0\sin\theta = (3.0 \text{ m/s})\sin\theta$$

Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

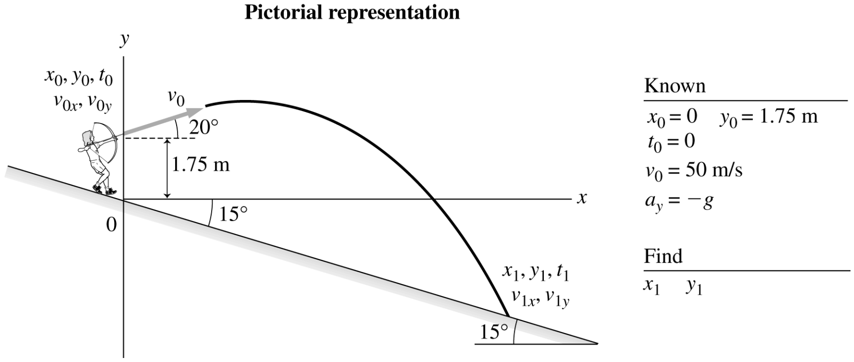
$$2.5 \text{ m} = 0 \text{ m} + (3.0 \text{ m/s})\cos\theta(t_1 - 0 \text{ s}) + 0 \text{ m} \Rightarrow t_1 = \frac{(2.5 \text{ m})}{(3.0 \text{ m/s})\cos\theta} = \frac{0.833 \text{ s}}{\cos\theta}$$

Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$ and the above equation for t_1,

$$0\text{ m} = 0\text{ m} + (3.0\text{ m/s})\sin\theta\left(\frac{0.833\text{ s}}{\cos\theta}\right) - \tfrac{1}{2}(3.352\text{ m/s}^2)\frac{(0.833\text{ s})^2}{\cos^2\theta}$$

$$\Rightarrow (2.5\text{ m})\frac{\sin\theta}{\cos\theta} = \frac{1.164}{\cos^2\theta} \Rightarrow 2.5\sin\theta\cos\theta = 1.164 \Rightarrow 2\theta = 68.6° \Rightarrow \theta = 34.3°$$

4.82. Model: Use the particle model for the arrow and the constant-acceleration kinematic equations. We will assume that the archer shoots from 1.75 m above the slope (about 5′ 9″).
Visualize:



Solve: For the y-motion:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow y_1 = 1.75\text{ m} + (v_0\sin 20°)t_1 - \tfrac{1}{2}gt_1^2$$

$$\Rightarrow y_1 = 1.75\text{ m} + (50\text{ m/s})\sin 20° t_1 - \tfrac{1}{2}gt_1^2$$

For the x-motion:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (v_0\cos 20°)t_1 + 0\text{ m} = (50\text{ m/s})(\cos 20°)t_1$$

Because $y_1/x_1 = -\tan 15° = -0.268$,

$$\frac{1.75\text{ m} + (50\text{ m/s})(\sin 20°)t_1 - \tfrac{1}{2}gt_1^2}{(50\text{ m/s})(\cos 20°)t_1} = -0.268 \Rightarrow t_1 = 6.12\text{ s} \text{ and } -0.058\text{ s (unphysical)}$$

Using $t_1 = 6.12$ s in the x- and y-equations above, we get $y_1 = -77.0$ m and $x_1 = 287$ m. This means the distance down the slope is $\sqrt{x_1^2 + y_1^2} = \sqrt{(287\text{ m})^2 + (-77.0\text{ m})^2} = 297\text{ m}.$

Assess: With an initial speed of 112 mph (50 m/s) for the arrow, which is shot from a 15° slope at an angle of 20° above the horizontal, a horizontal distance of 287 m and a vertical distance of 77.0 m are reasonable numbers.