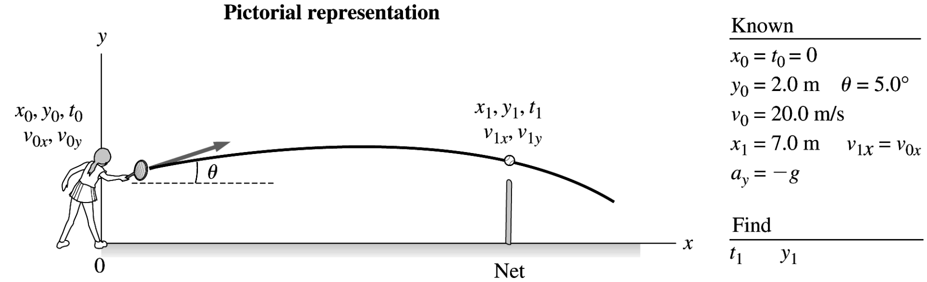
Solutions to HW6, Chapter 4

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

4.51. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.
Visualize:

Pictorial representation



Solve: The initial velocity is

$$v_{0x} = v_0 \cos 5.0° = (20 \text{ m/s})\cos 5.0° = 19.92 \text{ m/s}$$
$$v_{0y} = v_0 \sin 5.0° = (20 \text{ m/s})\sin 5.0° = 1.743 \text{ m/s}$$

The time it takes for the ball to reach the net is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 7.0 \text{ m} = 0 \text{ m} + (19.92 \text{ m/s})(t_1 - 0 \text{ s}) \Rightarrow t = 0.351 \text{ s}$$

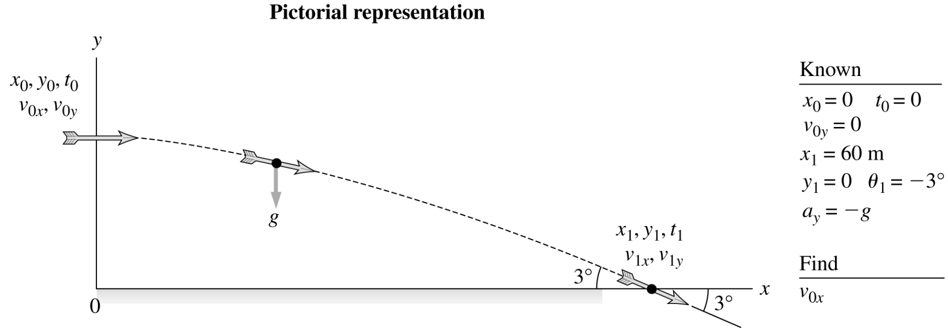
The vertical position at $\vec{v} = \vec{v}' + \vec{V}$ is

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$
$$= (2.0 \text{ m}) + (1.743 \text{ m/s})(0.351 \text{ s} - 0 \text{ s}) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(0.351 \text{ s} - 0 \text{ s})^2 = 2.01 \text{ m}$$

Thus the ball clears the net by $1.01 \text{ m} \approx 1.0 \text{ m}$.
Assess: The vertical free fall of the ball, with zero initial velocity, in 0.351 s is 0.6 m. The ball will clear by approximately 0.4 m it is thrown horizontally. The initial launch angle of 5° provides some initial vertical velocity and the ball clears by a larger distance. The above result is reasonable.

4.54. Model: Use the particle model for the arrow and the constant-acceleration kinematic equations.
Visualize:

Pictorial representation



Solve: Using $v_{1y} = v_{0y} + a_y(t_1 - t_0)$, we get

$$v_{1y} = 0 \text{ m/s} - gt_1 \Rightarrow v_{1y} = -gt_1$$

Also using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$,

$$60 \text{ m} = 0 \text{ m} + v_{0x}t_1 + 0 \text{ m} \Rightarrow v_{0x} = \frac{60 \text{ m}}{t_1} = v_{1x}$$

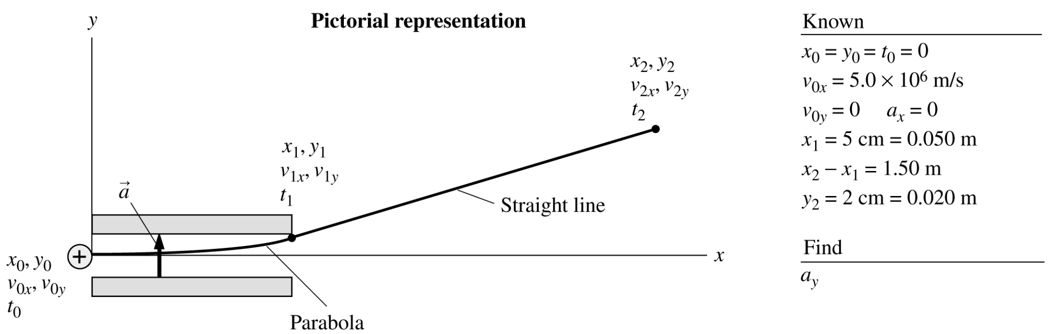
Since $v_{1y}/v_{1x} = -\tan 3.0° = -0.0524,$ using the components of v_0 gives

$$\frac{-gt_1}{(60\text{ m}/t_1)} = -0.0524 \Rightarrow t_1 = \sqrt{\frac{(0.0524)(60\text{ m})}{(9.8\text{ m/s}^2)}} = 0.566\text{ s}$$

Having found $t_1,$ we can go back to the x-equation to obtain $v_{0x} = 60\text{ m}/0.566\text{ s} = 106\text{ m/s} \approx 110\text{ m/s}.$

Assess: In view of the fact that the arrow took only 0.566 s to cover a horizontal distance of 60 m, a speed of 106 m/s or 237 mph for the arrow is understandable.

4.60. Model: The ions are particles that move in a plane. They have vertical acceleration while between the acceleration plates, and they move with constant velocity from the plates to the tumor. The flight time will be so small, because of the large speeds, that we'll ignore any deflection due to gravity.
Visualize:



Solve: There's never a horizontal acceleration, so the horizontal motion is constant velocity motion at $v_x = 5.0\times10^6\text{ m/s}.$ The times to pass between the 5.0-cm-long acceleration plates and from the plates to the tumor are

$$t_1 - t_0 = t_1 = \frac{0.050\text{ m}}{5.0\times10^6\text{ m/s}} = 1.00\times10^{-8}\text{ s}$$

$$t_2 - t_1 = \frac{1.50\text{ m}}{5.0\times10^6\text{ m/s}} = 3.00\times10^{-7}\text{ s}$$

Upon leaving the acceleration plates, the ion has been deflected sideways to position y_1 and has velocity $v_{1y}.$ These are

$$y_1 = y_0 + v_{0y}t_1 + \tfrac{1}{2}a_y t_1^2 = \tfrac{1}{2}a_y t_1^2$$

$$v_{1y} = v_{0y} + a_y t_1 = a_y t_1$$

In traveling from the plates to the tumor, with no vertical acceleration, the ion reaches position

$$y_2 = y_1 + v_{1y}(t_2 - t_1) = \tfrac{1}{2}a_y t_1^2 + (a_y t_1)(t_2 - t_1) = (\tfrac{1}{2}t_1^2 + t_1(t_2 - t_1))\, a_y$$

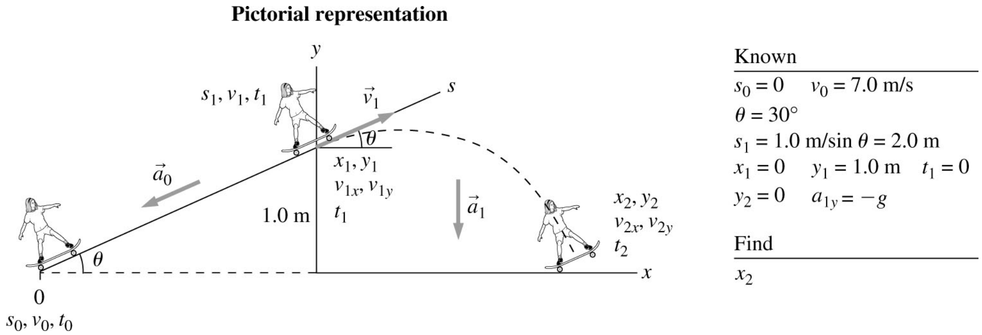
We know $y_2 = 2.0\text{ cm} = 0.020\text{ m},$ so we can solve for the acceleration a_y that the ion had while between the plates:

$$a_y = \frac{y_2}{\frac{1}{2}t_1^2 + t_1(t_2 - t_1)} = \frac{0.020\text{ m}}{\frac{1}{2}(1.00\times10^{-8}\text{ s})^2 + (1.00\times10^{-8}\text{ s})(3.00\times10^{-7}\text{ s})} = 6.6\times10^{12}\text{ m/s}^2$$

Assess: This acceleration is roughly 10^{12} times larger than the acceleration due to gravity. This justifies our assumption that the acceleration due to gravity can be neglected.

4.83. Model: Treat the skateboarder as a particle.

Visualize: This is a two-part problem. Use an s-axis parallel to the slope for the first part, regular xy-coordinates for the second. The skateboarder's final velocity at the top of the ramp is her initial velocity as she becomes airborne.



Solve: Without friction, the skateboarder's acceleration on the ramp is $a_0 = -g\sin 30° = -4.90 \text{ m/s}^2$. The length of the ramp is $s_1 = (1.0 \text{ m})/\sin 30° = 2.0$ m. We can use kinematics to find her speed at the top of the ramp:

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = v_0^2 + 2a_0 s_1$$

$$\Rightarrow v_1 = \sqrt{(7.0 \text{ m/s})^2 + 2(-4.90 \text{ m/s}^2)(2.0 \text{ m})} = 5.4 \text{ m/s}$$

This is the skateboarder's initial speed into the air, giving her velocity components $v_{1x} = v_1 \cos 30° = 4.7$ m/s and $v_{1y} = v_1 \cos 30° = 2.7$ m/s. We can use the y-equation of projectile motion to find her time in the air:

$$y_2 = 0 \text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 1.0 \text{ m} + (2.7 \text{ m/s})t_2 - (4.90 \text{ m/s}^2)t_2^2$$

This quadratic equation has roots $t_2 = -0.253$ s (unphysical) and $t_2 = 0.805$ s. The x-equation of motion is thus

$$x_2 = x_1 + v_{1x}t_2 = 0 \text{ m} + (4.7 \text{ m/s})t_2 = 3.8 \text{ m}$$

She touches down 3.8 m from the end of the ramp.