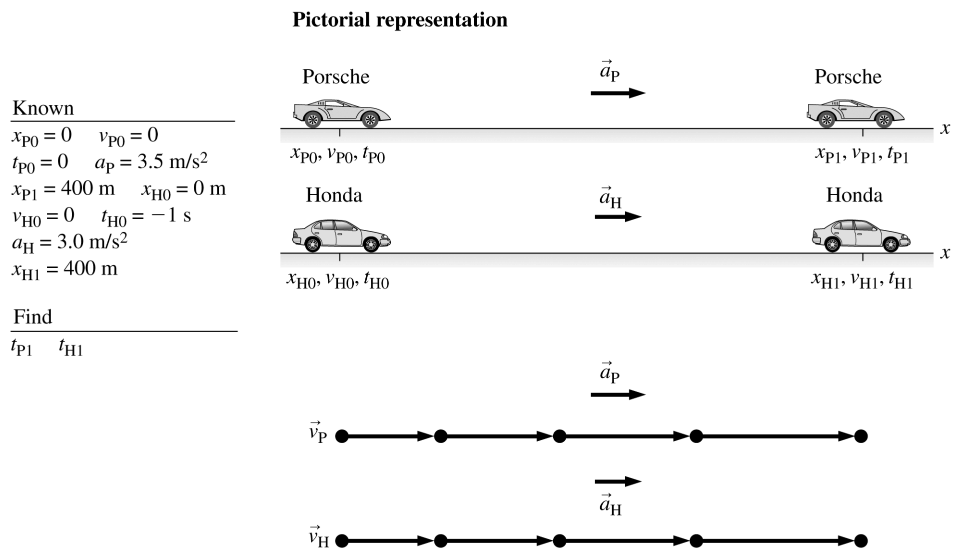
Solutions to HW4, Chapter 2

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

2.18. Model: We are assuming both cars are particles.

Visualize:



Solve: The Porsche's time to finish the race is determined from the position equation

$$x_{P1} = x_{P0} + v_{P0}(t_{P1} - t_{P0}) + \frac{1}{2}a_P(t_{P1} - t_{P0})^2$$

$$\Rightarrow 400 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(3.5 \text{ m/s}^2)(t_{P1} - 0 \text{ s})^2 \Rightarrow t_{P1} = 15.12 \text{ s}$$

The Honda's time to finish the race is obtained from Honda's position equation as

$$x_{H1} = x_{H0} + v_{H0}(t_{H1} - t_{H0}) + \frac{1}{2}a_{H0}(t_{H1} - t_{H0})^2$$

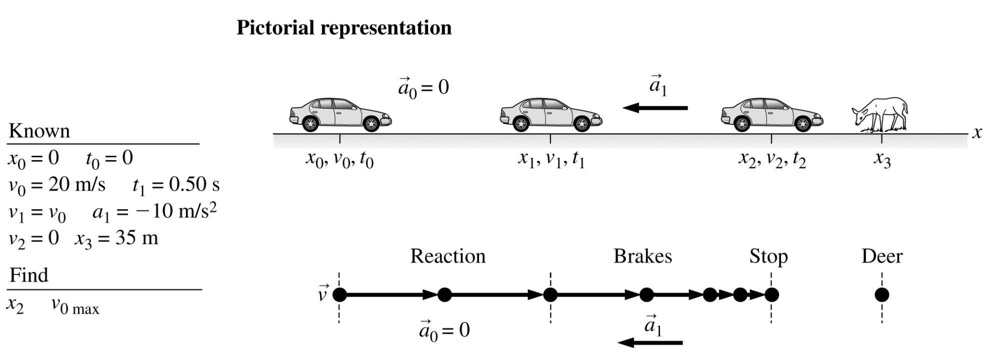
$$400 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(3.0 \text{ m/s}^2)(t_{H1} + 1 \text{ s})^2 \Rightarrow t_{H1} = 15.33 \text{ s}$$

So, the Porsche wins by 0.21 s.

Assess: The numbers are contrived for the Porsche to win, but the time to go 400 m seems reasonable.

2.49. Model: We will use the particle model and the constant-acceleration kinematic equations.

Visualize:



Solve: **(a)** To find x_2, we first need to determine x_1. Using $x_1 = x_0 + v_0(t_1 - t_0)$, we get $x_1 = 0\text{ m} + (20\text{ m/s})(0.50\text{ s} - 0\text{ s}) = 10\text{ m}$. Now,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow 0\text{ m}^2/\text{s}^2 = (20\text{ m/s})^2 + 2(-10\text{ m/s}^2)(x_2 - 10\text{ m}) \Rightarrow x_2 = 30\text{ m}$$

The distance between you and the deer is $(x_3 - x_2)$ or $(35\text{ m} - 30\text{ m}) = 5\text{ m}$.

(b) Let us find $v_{0\text{ max}}$ such that $v_2 = 0\text{ m/s}$ at $x_2 = x_3 = 35\text{ m}$. Using the following equation,

$$v_2^2 - v_{0\text{ max}}^2 = 2a_1(x_2 - x_1) \Rightarrow 0\text{ m}^2/\text{s}^2 - v_{0\text{ max}}^2 = 2(-10\text{ m/s}^2)(35\text{ m} - x_1)$$

Also, $x_1 = x_0 + v_{0\text{ max}}(t_1 - t_0) = v_{0\text{ max}}(0.50\text{ s} - 0\text{ s}) = (0.50\text{ s})v_{0\text{ max}}$. Substituting this expression for x_1 in the above equation yields

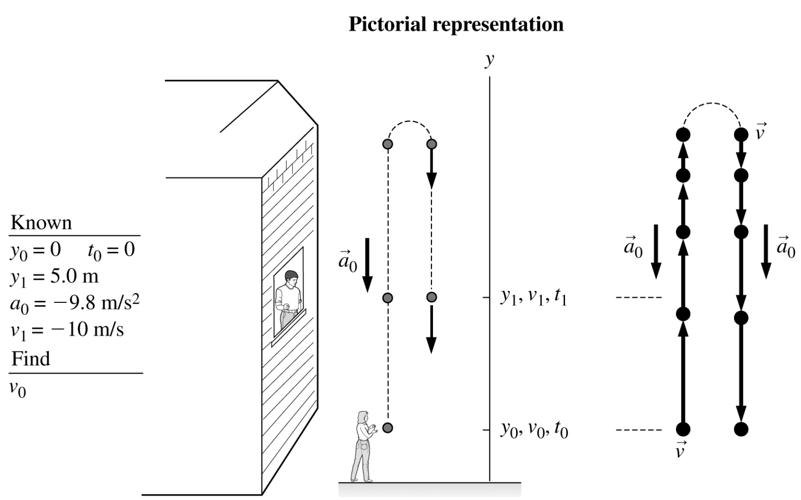
$$-v_{0\text{ max}}^2 = (-20\text{ m/s}^2)[35\text{ m} - (0.50\text{ s})\, v_{0\text{ max}}] \Rightarrow v_{0\text{ max}}^2 + (10\text{ m/s})v_{0\text{ max}} - 700\text{ m}^2/\text{s}^2 = 0$$

The solution of this quadratic equation yields $v_{0\text{ max}} = 22\text{ m/s}$. (The other root is negative and unphysical for the present situation.)

Assess: An increase of speed from 20 m/s to 22 m/s is very reasonable for the car to cover an additional distance of 5 m with a reaction time of 0.50 s and a deceleration of 10 m/s^2.

2.67. Model: The ball is a particle that exhibits freely falling motion according to the constant-acceleration kinematic equations.

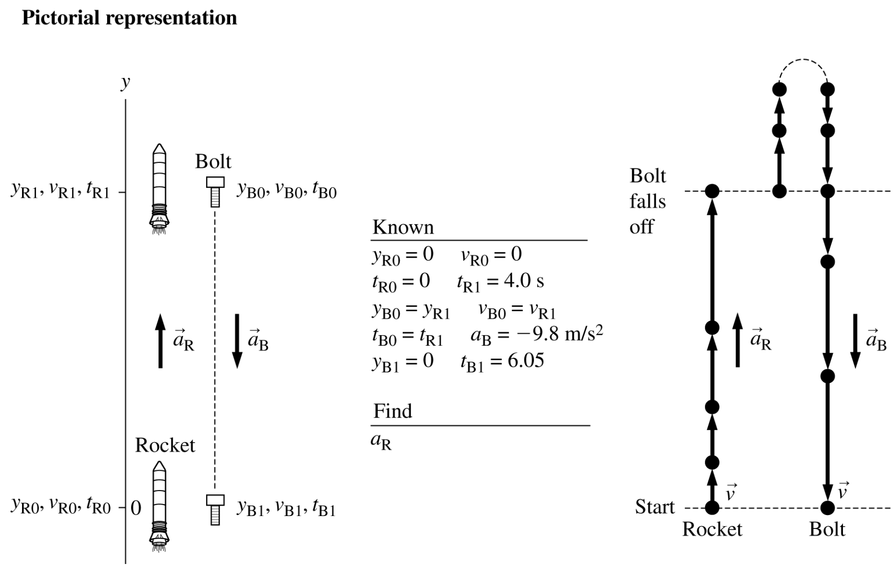
Visualize:



Solve: Using the known values, we have

$$v_1^2 = v_0^2 + 2a_0(y_1 - y_0) \Rightarrow (-10\text{ m/s})^2 = v_0^2 + 2(-9.8\text{ m/s}^2)(5.0\text{ m} - 0\text{ m}) \Rightarrow v_0 = 14\text{ m/s}$$

2.80. Model: The rocket and the bolt will be represented as particles to investigate their motion.
Visualize:



The initial velocity of the bolt as it falls off the side of the rocket is the same as that of the rocket, that is, $v_{B0} = v_{R1}$ and it is positive since the rocket is moving upward. The bolt continues to move upward with a deceleration equal to $g = 9.8 \text{ m/s}^2$ before it comes to rest and begins its downward journey.

Solve: To find a_R we look first at the motion of the rocket:

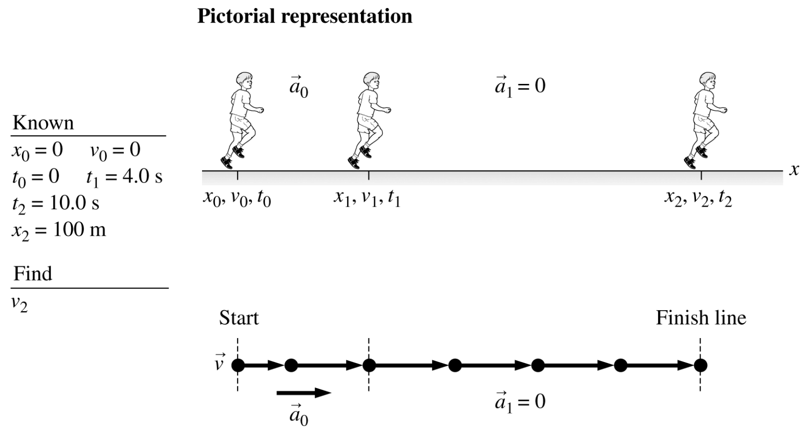
$$y_{R1} = y_{R0} + v_{R0}(t_{R1} - t_{R0}) + \frac{1}{2}a_R(t_{R1} - t_{R0})^2$$
$$= 0 \text{ m} + 0 \text{ m/s} + \frac{1}{2}a_R(4.0 \text{ s} - 0 \text{ s})^2 = 8a_R$$

To find a_R we must determine the magnitude of y_{R1} or y_{B0}. Let us now look at the bolt's motion:

$$y_{B1} = y_{B0} + v_{B0}(t_{B1} - t_{B0}) + \frac{1}{2}a_B(t_{B1} - t_{B0})^2$$
$$0 = y_{R1} + v_{R1}(6.0 \text{ s} - 0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(6.0 \text{ s} - 0 \text{ s})^2$$
$$\Rightarrow y_{R1} = 176.4 \text{ m} - (6.0 \text{ s})\ v_{R1}$$

Since $v_{R1} = v_{R0} + a_R(t_{R1} - t_{R0}) = 0 \text{ m/s} + 4\,a_R = 4\,a_R$ the above equation for y_{R1} yields $y_{R1} = 176.4 - 6.0(4a_R)$. We know from the first part of the solution that $y_{R1} = 8a_R$. Therefore, $8a_R = 176.4 - 24.0a_R$ and hence $a_R = 5.5 \text{ m/s}^2$.

2.83. Model: We will use the particle-model to represent the sprinter and the equations of kinematics.
Visualize:



Solve: Substituting into the constant-acceleration kinematic equations,

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_0(4 \text{ s} - 0 \text{ s})^2 = \frac{1}{2}a_0 t_1^2 = \frac{1}{2}a_0(4.0 \text{ s})^2$$

$$\Rightarrow x_1 = (8 \text{ s}^2)a_0$$

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + a_0(4.0 \text{ s} - 0 \text{ s}) \Rightarrow v_1 = (4.0 \text{ s})\ a_0$$

From these two results, we find that $x_1 = (2 \text{ s})v_1$. Now,

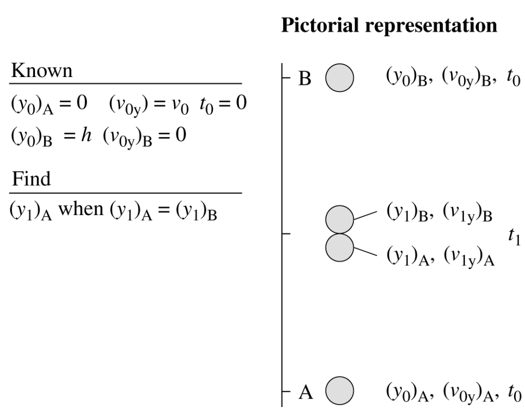
$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

$$\Rightarrow 100 \text{ m} = (2 \text{ s})v_1 + v_1(10 \text{ s} - 4 \text{ s}) + 0 \text{ m} \Rightarrow v_1 = 12.5 \text{ m/s}$$

Assess: Using the conversion 2.24 mph = 1 m/s, $v_1 = 12.5 \text{ m/s} = 28 \text{ mph}$. This speed as the sprinter reaches the finish line is physically reasonable.

2.84. Look at discussion notes from class (on the challenge problems) for tips and hints, and diagrams that clarify what's going on!

Model: The balls are particles undergoing constant acceleration.
Visualize:



Solve: **(a)** The positions of each of the balls at t_1 is found from kinematics.

$$(y_1)_A = (y_0)_A + (v_{0y})_A t_1 - \frac{1}{2}g{t_1}^2 = v_0 t_1 - \frac{1}{2}g{t_1}^2$$

$$(y_1)_B = (y_0)_B + (v_{0y})_B t_1 - \frac{1}{2}g{t_1}^2 = h - \frac{1}{2}g{t_1}^2$$

In the particle model the balls have no physical extent, so they meet when $(y_1)_A = (y_1)_B$. This means

$$v_0 t_1 - \frac{1}{2}g{t_1}^2 = h - \frac{1}{2}g{t_1}^2 \Rightarrow t_1 = \frac{h}{v_0}$$

Thus the collision height is $y_{coll} = h - \frac{1}{2}gt_1^2 = h - \frac{gh^2}{2{v_0}^2}$.

(b) We need the collision to occur while $y_{coll} \ge 0$. Thus

$$h - \frac{gh^2}{2{v_0}^2} \ge 0 \Rightarrow 1 \ge \frac{gh}{2{v_0}^2} \Rightarrow h \le \frac{2{v_0}^2}{g}$$

So $h_{max} = \frac{2{v_0}^2}{g}$.

(c) Ball A is at its highest point when its velocity $(v_{1y})_A = 0$.

$$(v_{1y})_A = (v_{0y})_A - gt_1 \Rightarrow 0 = v_0 - gt_1 \Rightarrow t_1 = \frac{v_0}{g}$$

In **(a)** we found that the collision occurs at $t_1 = \frac{h}{v_0}$. Equating these, $\frac{h}{v_0} = \frac{v_0}{g} \Rightarrow h = \frac{{v_0}^2}{g}$.

Assess: Interestingly, the height at which a collision occurs while Ball A is at its highest point is exactly half of h_{max}.