Solutions to HW28, Chapter 16

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

16.1. Model: The wave is a traveling wave on a stretched string.

Solve: The wave speed on a stretched string with linear density μ is $v_{\text{string}} = \sqrt{T_S/\mu}$. The wave speed if the tension is doubled will be

$$v'_{\text{string}} = \sqrt{\frac{T_S}{2\mu}} = \frac{1}{\sqrt{2}} v_{\text{string}} = \frac{1}{\sqrt{2}}(200\text{ m/s}) = 141\text{ m/s}.$$

16.2. Model: The wave is a traveling wave on a stretched string.

Solve: The wave speed on a stretched string with linear density μ is

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow 150\text{ m/s} = \sqrt{\frac{75\text{ N}}{\mu}} \Rightarrow \mu = 3.333\times10^{-3}\text{ kg/m}$$

For a wave speed of 180 m/s, the required tension will be

$$T_S = \mu v_{\text{string}}^2 = (3.333\times10^{-3}\text{ kg/m})(180\text{ m/s})^2 = 110\text{ N}$$

16.3. Solve:

$$L = v\Delta t = \sqrt{\frac{T_S}{\mu}}\,\Delta t = \sqrt{\frac{T_S}{m/L}}\,\Delta t = \sqrt{\frac{T_S L}{m}}\,\Delta t \Rightarrow \sqrt{L} = \sqrt{\frac{T_S}{m}}\,\Delta t \Rightarrow$$

$$L = \frac{T_S}{m}(\Delta t)^2 = \frac{20\text{ N}}{0.025\text{ kg}}(50\text{ ms})^2 = 2.0\text{ m}$$

Assess: 2.0 m seems like a reasonable length for a string.

16.10. Solve: (a) The wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.0\text{ m}} = 3.1\text{ rad/m}$$

(b) The wave speed is

16.11. Solve: (a) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5\text{ rad/m}} = 4.2\text{ m}$$

(b) The frequency is

$$f = \frac{v}{\lambda} = \frac{200\text{ m/s}}{4.19\text{ m}} = 48\text{ Hz}$$

16.12. Model: The wave is a traveling wave.

Solve: (a) A comparison of the wave equation with Equation 16.14 yields: $A = 5.2$ cm, $k = 5.5$ rad/m, $\omega =$ 72 rad/s, and $\phi_0 = 0$ rad. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{72\text{ rad/s}}{2\pi} = 11.5\text{ Hz} \approx 11\text{ Hz}$$

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{5.5\text{ rad/m}} = 1.14\text{ m} \approx 1.1\text{ m}$$

(c) The wave speed $v = \lambda f = 13$ m/s.

16.14. Solve: The amplitude of the wave is the maximum displacement, which is 6.0 cm. The period of the wave is 0.60 s, so the frequency $f = 1/T = 1/0.60\text{ s} = 1.67$ Hz. The wavelength is

$$\lambda = \frac{v}{f} = \frac{2 \text{ m/s}}{1.667 \text{ Hz}} = 1.2 \text{ m}$$

16.45. Solve: (a) We see from the history graph that the period T = 0.20 s and the wave speed v = 4.0 m/s. Thus, the wavelength is

$$\lambda = \frac{v}{f} = vT = (4.0 \text{ m/s})(0.20 \text{ s}) = 0.80 \text{ m}$$

(b) The phase constant ϕ_0 is obtained as follows:

$$D(0 \text{ m, } 0 \text{ s}) = A\sin\phi_0 \Rightarrow -2 \text{ mm} = (2 \text{ mm})\sin\phi_0 \Rightarrow \sin\phi_0 = -1 \Rightarrow \phi_0 = -\tfrac{1}{2}\pi \text{ rad}$$

(c) The displacement equation for the wave is

$$D(x,t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi ft + \phi_0\right) = (2.0 \text{ mm})\sin\left(\frac{2\pi x}{0.80 \text{ m}} - \frac{2\pi t}{0.20 \text{ s}} - \frac{\pi}{2}\right) = (2.0 \text{ mm})\sin(2.5\pi x - 10\pi t - \tfrac{1}{2}\pi)$$

where x and t are in m and s, respectively.

16.46. Solve: (a) We can see from the graph that the wavelength is λ = 2.0 m. We are given that the wave's frequency is f = 5.0 Hz. Thus, the wave speed is $v = \lambda f$ = 10 m/s.
(b) The snapshot graph was made at t = 0 s. Reading the graph at x = 0 m, we see that the displacement is

$$D(x = 0 \text{ m}, t = 0 \text{ s}) = D(0 \text{ m}, 0 \text{ s}) = 0.5 \text{ mm} = \tfrac{1}{2}A$$

Thus

$$D(0 \text{ m, } 0 \text{ s}) = \tfrac{1}{2}A = A\sin\phi_0 \Rightarrow \phi_0 = \sin^{-1}\left(\tfrac{1}{2}\right) = \frac{\pi}{6}\text{rad or } \frac{5\pi}{6}\text{rad}$$

Note that the value of D(0 m, 0 s) alone gives two possible values of the phase constant. One of the values will cause the displacement to start at 0.5 mm and increase with distance—as the graph shows—while the other will cause the displacement to start at 0.5 mm but *decrease* with distance. Which is which? The wave equation for t = 0 s is

$$D(x, t = 0) = A\sin\left(\frac{2\pi x}{\lambda} + \phi_0\right)$$

16.51. Solve: The difference in the arrival times for the P and S waves is

$$\Delta t = t_S - t_P = \frac{d}{v_S} - \frac{d}{v_P} \Rightarrow 120 \text{ s} = d\left(\frac{1}{4500 \text{ m/s}} - \frac{1}{8000 \text{ m/s}}\right) \Rightarrow d = 1.23\times10^6 \text{ m} = 1230 \text{ km}$$

Assess: d is approximately one-fifth of the radius of the earth and is reasonable.

16.54. Model: This is a sinusoidal wave.
Solve: (a) The equation is of the form $D(y,t) = A\sin(ky + \omega t + \phi_0)$, so the wave is traveling along the y-axis. Because it is $+\omega t$ rather than $-\omega t$ the wave is traveling in the *negative* y-direction.
(b) Sound is a longitudinal wave, meaning that the medium is displaced *parallel* to the direction of travel. So the air molecules are oscillating back and forth along the y-axis.
(c) The wave number is $k = 8.96 \text{ m}^{-1}$, so the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{8.96 \text{ m}^{-1}} = 0.701 \text{ m}$$

The angular frequency is $\omega = 3140 \text{ s}^{-1}$, so the wave's frequency is

$$f = \frac{\omega}{2\pi} = \frac{3140 \text{ s}^{-1}}{2\pi} = 500 \text{ Hz}$$

Thus, the wave speed $v = \lambda f$ = (0.70 m)(500 Hz) = 350 m/s. The period $T = 1/f$ = 0.00200 s = 2.00 ms.
Assess: The wave is a sound wave with speed v = 350 m/s. This is greater than the room-temperature speed of 343 m/s, so the air temperature must be greater than $20°$.

16.55. Model: This is a sinusoidal wave.

Solve: (a) The displacement of a wave traveling in the positive x-direction with wave speed v must be of the form $D(x,t) = D(x - vt)$. Since the variables x and t in the given wave equation appear together as $x + vt$, the wave is traveling toward the left, that is, in the $-x$ direction.

(b) The speed of the wave is

$$v = \frac{\omega}{k} = \frac{2\pi/0.20\text{ s}}{2\pi\text{ rad}/2.4\text{ m}} = 12\text{ m/s}$$

The frequency is

$$f = \frac{\omega}{2\pi} = \frac{2\pi\text{ rad}/0.20\text{ s}}{2\pi} = 5.0\text{ Hz}$$

The wave number is

$$k = \frac{2\pi\text{ rad}}{2.4\text{ m}} = 2.6\text{ rad/m}$$

(c) The displacement is

$$D(0.20\text{ m}, 0.50\text{ s}) = (3.0\text{ cm})\sin\left[2\pi\left(\frac{0.20\text{ m}}{2.4\text{ m}} + \frac{0.50\text{ s}}{0.20\text{ s}} + 1\right)\right] = -1.5\text{ cm}$$

16.56. Model: This is a sinusoidal wave traveling on a stretched string in the $+x$ direction.

Solve: (a) From the displacement equation of the wave, $A = 2.0$ cm, $k = 12.57$ rad/m, and $\omega = 638$ rad/s. Using the equation for the wave speed in a stretched string,

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v_{\text{string}}^2 = \mu\left(\frac{\omega}{k}\right)^2 = (5.00\times10^{-3}\text{ kg/m}^3)\left(\frac{638\text{ rad/s}}{12.57\text{ rad/m}}\right)^2 = 12.6\text{ N}$$

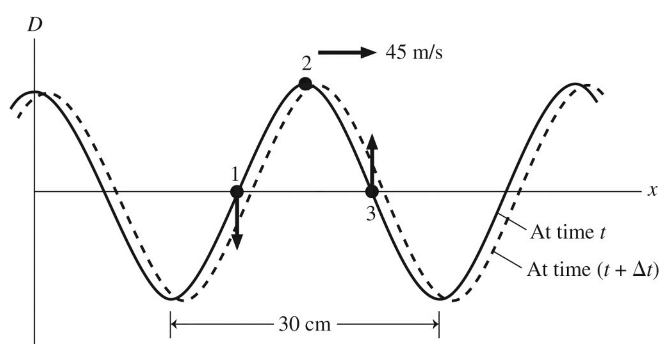
(b) The maximum displacement is the amplitude $D_{\max}(x,t) = 2.00$ cm.

(c) From Equation 16.17,

$$v_{y\max} = \omega A = (638\text{ rad/s})(2.0\times10^{-2}\text{ m}) = 12.8\text{ m/s}$$

16.57. Model: We have a wave traveling to the right on a string.

Visualize:



Solve: The snapshot of the wave as it travels to the right for an infinitesimally small time Δt shows that the velocity at point 1 is downward, at point 3 is upward, and at point 2 is zero. Furthermore, the speed at points 1 and 3 is the maximum speed given by Equation 16.17: $v_1 = v_3 = \omega A$. The frequency of the wave is

$$\omega = 2\pi f = 2\pi\frac{v}{\lambda} = \frac{2\pi(45\text{ m/s})}{0.30\text{ m}} = 300\pi\text{ rad/s} \Rightarrow \omega A = (300\pi\text{ rad/s})(2.0\times10^{-2}\text{ m}) = 19\text{ m/s}$$

Thus, $v_1 = -19$ m/s, $v_2 = 0$ m/s, and $v_3 = +19$ m/s.

16.61. Model: The wave is traveling on a stretched string.

Solve: The wave speed on the string is

$$v = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{50\text{ N}}{0.005\text{ kg/m}}} = 100\text{ m/s}$$

The speed of the particle on the string, however, is given by Equation 16.17. The maximum speed is calculated as follows:

$$v_y = -\omega A\cos(kx - \omega t + \phi_0) \Rightarrow v_{y\ \max} = \omega A = 2\pi f A = 2\pi \frac{v}{\lambda} A = 2\pi\left(\frac{100\ \text{m/s}}{2.0\ \text{m}}\right)(0.030\ \text{m}) = 9.4\ \text{m/s}$$

16.63. Model: A sinusoidal wave is traveling along a stretched string.

Solve: Equation 16.17 gives $v_{\max} = \omega A.$ The derivative of Equation 16.17 gives $a = \frac{dv}{dt} = -\omega^2 A\sin(kx - \omega t + \phi_0) \Rightarrow a_{\max} = \omega^2 A.$ These two equations can be combined to give

$$\omega = \frac{a_{\max}}{v_{\max}} = \frac{200\ \text{m/s}^2}{2.0\ \text{m/s}} = 100\ \text{rad/s} \Rightarrow f = \frac{\omega}{2\pi} = 15.9\ \text{Hz} \approx 16\ \text{Hz} \qquad A = \frac{v_{\max}}{\omega} = \frac{2.0\ \text{m/s}}{100\ \text{rad/s}} = 2.0\ \text{cm}$$

Assess: This frequency and amplitude are typical for a wave on a string.