Solutions to HW27, Chapter 15

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

15.43. Model: The astronaut attached to the spring is in simple harmonic motion.

Solve: (a) From the graph, $T = 3.0$ s, so we have

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \left(\frac{T}{2\pi}\right)^2 k = \left(\frac{3.0\ \text{s}}{2\pi}\right)^2 (240\ \text{N/m}) = 55\ \text{kg}$$

(b) Oscillations occur about an equilibrium position of 1.0 m. From the graph, $A = \frac{1}{2}(0.80\ \text{m}) = 0.40\ \text{m}$, $\phi_0 = 0$ rad, and

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.0\ \text{s}} = 2.1\ \text{rad/s}$$

The equation for the position of the astronaut is

$$x(t) = A\cos\omega t + 1.0\ \text{m} = (0.4\ \text{m})\cos[(2.1\ \text{rad/s})t] + 1.0\ \text{m}$$

$$\Rightarrow 1.2\ \text{m} = (0.4\ \text{m})\cos[(2.1\ \text{rad/s})t] + 1.0\ \text{m} \Rightarrow \cos[(2.1\ \text{rad/s})t] = 0.5 \Rightarrow t = 0.50\ \text{s}$$

The equation for the velocity of the astronaut is

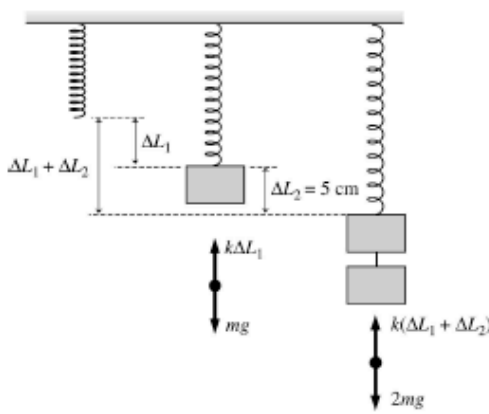
$$v_x(t) = -A\omega\sin(\omega t)$$

$$\Rightarrow v_{0.5\ \text{s}} = -(0.4\ \text{m})(2.1\ \text{rad/s})\sin[(2.1\ \text{rad/s})(0.50\ \text{s})] = -0.73\ \text{m/s}$$

Thus her speed is 0.73 m/s.

15.47. Model: The blocks undergo simple harmonic motion.

Visualize:



The length of the stretched spring due to a block of mass m is ΔL_1. In the case of the two-block system, the spring is further stretched by an amount ΔL_2.

Solve: The equilibrium equations from Newton's second law for the single-block and double-block systems are

$$(\Delta L_1)k = mg \text{ and } (\Delta L_1 + \Delta L_2)k = (2m)g$$

Using $\Delta L_2 = 5.0$ cm, and subtracting these two equations, gives us

$$(\Delta L_1 + \Delta L_2)k - \Delta L_1 k = (2m)g - mg \Rightarrow (0.05\ \text{m})k = mg$$

$$\Rightarrow \frac{k}{m} = \frac{9.8\ \text{m/s}^2}{0.05\ \text{m}} = 196\ \text{s}^{-2}$$

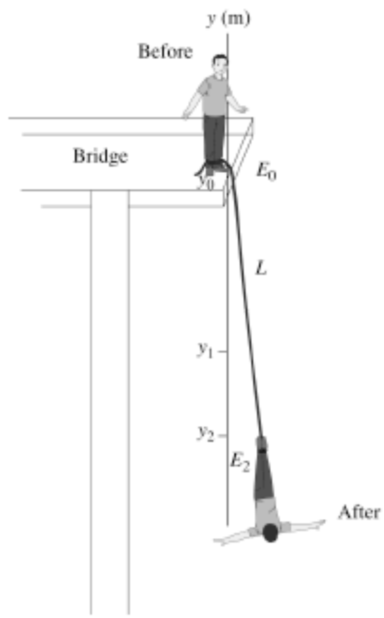
With both blocks attached, giving total mass $2m$, the angular frequency of oscillation is

$$\omega = \sqrt{\frac{k}{2m}} = \sqrt{\frac{1}{2}\frac{k}{m}} = \sqrt{\frac{1}{2}196\ \text{s}^{-2}} = 9.90\ \text{rad/s}$$

Thus the oscillation frequency is $f = \omega/2\pi = 1.6$ Hz.

15.48. Model: Model the bungee cord as massless with a Hooke's law restoring force.

Visualize:



Known
$y_0 = 0$
$y_1 = -L = -12$ m
$m = 75$ kg
$k = 430$ n/m

Find
y_2
Time to reach y_2

Solve: (a) Use energy conservation where $E_0 = 0$ at the top of the bridge. The energy at the lowest point is $E_2 = \frac{1}{2}k(y_2 - y_1)^2 + mgy_2$ where $y_1 = -L$ and $y_2 - y_1$ is the maximum stretch of the bungee and both y_1 and y_2 are negative. We seek y_2.

$$E_0 = E_2 \Rightarrow \tfrac{1}{2}k(y_2 - y_1)^2 + mgy_2 = 0$$

Square the binomial, multiply through by 2/*k*, and collect terms to get this quadratic equation:

$$y_2^2 + \left(2\frac{m}{k}g - 2y_1\right)y_2 + y_1^2 = 0$$

Use the quadratic formula.

$$y_2 = \frac{-\left(2\frac{m}{k}g - 2y_1\right) \pm \sqrt{\left(2\frac{m}{k}g - 2y_1\right)^2 - 4y_1^2}}{2}$$

Cancel a 2 from every term in top and bottom.

$$y_2 = -\left(\frac{m}{k}g - y_1\right) \pm \sqrt{\left(\frac{m}{k}g - y_1\right) - y_1^2}$$

$$= -\left(\frac{(75\text{ kg})}{(430\text{ N/m})}9.8\text{ m/s}^2 - (-12\text{ m})\right)^2 \pm \sqrt{\left(\frac{(75\text{ kg})}{(430\text{ N/m})}9.8\text{ m/s}^2 - (-12\text{ m})\right)^2 - (12\text{ m})^2}$$

$$= -7.1\text{ m or } -20\text{ m}$$

We want the second solution because it is below the length of the unstretched bungee; so the student's lowest point is 20 m below the bridge.

(b) The total time to reach the lowest point will be the time to free fall 12 m plus one-quarter of a period of the SHO. Here's the time to free fall until the bungee begins to stretch.

$$L = \tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2L}{g}} = \sqrt{\frac{2(12\text{ m})}{9.8\text{ m/s}^2}} = 1.56\text{ s}$$

Now find ¼ of a period for the bungee-spring motion (from unstretched to maximum stretch).

$$\frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{75\text{ kg}}{430\text{ N/m}}} = 0.66\text{ s}$$

The total time is thus 1.56 s + 0.66 s = 2.2 s.

Assess: The answers seem reasonable.

15.51. Model: The compact car is in simple harmonic motion.

Solve: **(a)** The mass on each spring is $(1200\text{ kg})/4 = 300\text{ kg}$. The spring constant can be calculated as follows:

$$\omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2 = m(2\pi f)^2 = (300\text{ kg})[2\pi(2.0\text{ Hz})]^2 = 4.74\times10^4\text{ N/m}$$

The spring constant is 4.7×10^4 N/m.

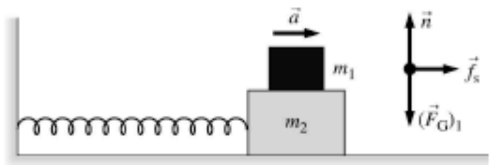
(b) The car carrying four persons means that each spring has, on the average, an additional mass of 70 kg. That is, $m = 300\text{ kg} + 70\text{ kg} = 370\text{ kg}$. Thus,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.74\times10^4\text{ N/m}}{370\text{ kg}}} = 1.8\text{ Hz}$$

Assess: A small frequency change from the additional mass is reasonable because frequency is inversely proportional to the square root of the mass.

15.52. Model: Assume simple harmonic motion for the two-block system without the upper block slipping. We will also use the model of static friction between the two blocks.

Visualize:



Solve: The net force on the upper block m_1 is the force of static friction due to the lower block m_2. The two blocks ride together as long as the static friction doesn't exceed its maximum possible value. The model of static friction gives the maximum force of static friction as

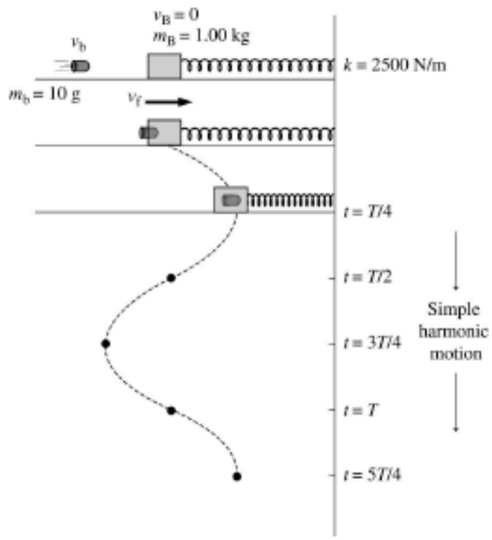
$$(f_s)_{max} = \mu_s n = \mu_s(m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g$$

$$\Rightarrow \mu_s = \frac{a_{max}}{g} = \frac{\omega^2 A_{max}}{g} = \left(\frac{2\pi}{T}\right)^2\left(\frac{A_{max}}{g}\right) = \left(\frac{2\pi}{1.5\text{ s}}\right)^2\left(\frac{0.40\text{ m}}{9.8\text{ m/s}^2}\right) = 0.72$$

Assess: Because the period is given, we did not need to use the block masses or the spring constant in our calculation.

15.53. Model: A completely inelastic collision between the bullet and the block resulting in simple harmonic motion.

Visualize:



Solve: (a) The equation for conservation of energy after the collision is

$$\frac{1}{2}kA^2 = \frac{1}{2}(m_b + m_B)v_f^2 \Rightarrow v_f = \sqrt{\frac{k}{m_b + m_B}}A = \sqrt{\frac{2500\text{ N/m}}{1.010\text{ kg}}}(0.10\text{ m}) = 5.0\text{ m/s}$$

The momentum conservation equation for the perfectly inelastic collision $p_{after} = p_{before}$ is

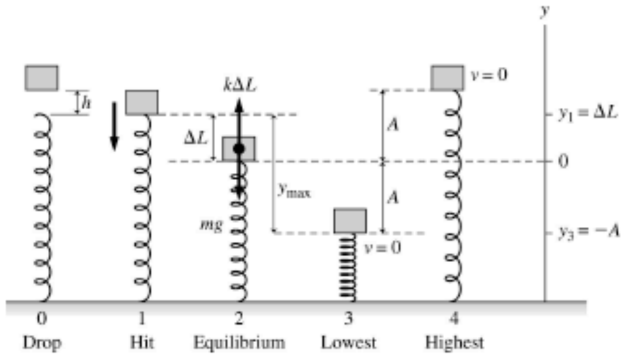
$$(m_b + m_B)v_f = m_b v_b + m_B v_B$$

$$(1.010\text{ kg})(5.0\text{ m/s}) = (0.010\text{ kg})v_b + (1.00\text{ kg})(0\text{ m/s}) \Rightarrow v_b = 5.0\times10^2\text{ m/s}$$

(b) No. The oscillation frequency $\sqrt{k/(m_b + m_B)}$ depends on the masses but not on the speeds.

15.79. Model: The block undergoes simple harmonic motion after sticking to the spring. Energy is conserved throughout the motion.

Visualize:



It's essential to carefully visualize the motion. At the highest point of the oscillation the spring is *stretched* upward.

Solve: We've placed the origin of the coordinate system at the equilibrium position, where the block would sit on the spring at rest. The spring is compressed by ΔL at this point. Balancing the forces requires $k\Delta L = mg$. The angular frequency is $w^2 = k/m = g/\Delta L$, so we can find the oscillation frequency by finding ΔL. The block hits the spring (1) with kinetic energy. At the lowest point (3), kinetic energy and gravitational potential energy have been transformed into the spring's elastic energy. Equate the energies at these points:

$$K_1 + U_{1g} = U_{3s} + U_{3g} \Rightarrow \tfrac{1}{2}mv_1^2 + mg\Delta L = \tfrac{1}{2}k(\Delta L + A)^2 + mg(-A)$$

We've used $y_1 = \Delta L$ as the block hits and $y_3 = -A$ at the bottom. The spring has been compressed by $\Delta y = \Delta L + A$. Speed v_1 is the speed after falling distance h, which from free-fall kinematics is $v_1^2 = 2gh$. Substitute this expression for v_1^2 and $mg/\Delta L$ for k, giving

$$mgh + mg\Delta L = \frac{mg}{2(\Delta L)}(\Delta L + A)^2 + mg(-A)$$

The mg term cancels, and the equation can be rearranged into the quadratic equation

$$(\Delta L)^2 + 2h(\Delta L) - A^2 = 0$$

The positive solution is

$$\Delta L = \sqrt{h^2 + A^2} - h = \sqrt{(0.030\text{ m})^2 + (0.100\text{ m})^2} - 0.030\text{ m} = 0.0744\text{ m}$$

Now that ΔL is known, we can find

$$\omega = \sqrt{\frac{g}{\Delta L}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.0744\text{ m}}} = 11.48\text{ rad/s} \Rightarrow f = \frac{\omega}{2\pi} = 1.8\text{ Hz}$$