Solutions to HW26, Chapter 15

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

15.4. Solve: The object's position as a function of time is $x(t) = A\cos(\omega t + \phi_0)$. Letting $x = 0$ m at $t = 0$ s, gives

$$0 = A\cos\phi_0 \Rightarrow \phi_0 = \pm\tfrac{1}{2}\pi$$

Since the object is traveling to the right, it is in the lower half of the circular motion diagram, giving a phase constant between $-\pi$ and 0 radians. Thus, $\phi_0 = -\frac{1}{2}\pi$ and

$$x(t) = A\cos(\omega t - \tfrac{1}{2}\pi) \Rightarrow x(t) = A\sin\omega t = (0.10\text{ m})\sin(\tfrac{1}{2}\pi t)$$

where we have used $A = 0.10$ m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi\text{ rad}}{4.0\text{ s}} = \frac{\pi}{2}\text{ rad/s}$$

Let us now find t where $x = 0.060$ m:

$$0.060\text{ m} = (0.10\text{ m})\sin\left(\frac{\pi}{2}t\right) \Rightarrow t = \frac{2}{\pi}\sin^{-1}\left(\frac{0.060\text{ m}}{0.10\text{ m}}\right) = 0.41\text{ s}$$

Assess: The answer is reasonable because it is approximately $\frac{1}{8}$ of the period.

15.8. Solve: The position and the velocity of a particle in simple harmonic motion are

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0) = -v_{max}\sin(\omega t + \phi_0)$$

From the graph, $T = 12$ s and the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{12\text{ s}} = \frac{\pi}{6}\text{ rad/s}$$

(a) Because $v_{max} = A\omega = 60$ cm/s, we have

$$A = \frac{60\text{ cm/s}}{\omega} = \frac{60\text{ cm/s}}{\pi/6\text{ rad/s}} = 115\text{ cm}$$

(b) At $t = 0$ s,

$$v_{0x} = -A\omega\sin\phi_0 = 30\text{ cm/s} \Rightarrow -(60\text{ cm/s})\sin\phi_0 = 30\text{ cm/s}$$

$$\Rightarrow \phi_0 = \sin^{-1}(-0.5\text{ rad}) = -\tfrac{1}{6}\pi\text{ rad or } \tfrac{7}{6}\pi\text{ rad}$$

Because the velocity at $t = 0$ s is positive and the particle is slowing down, the particle is in the fourth quadrant of the circular motion diagram. Thus $\phi_0 = -\frac{1}{6}\pi$ rad.

(c) At $t = 0$ s, $x_0 = (115\text{ cm})\cos(-\frac{1}{6}\pi\text{ rad}) = 100$ cm.

15.17. Model: The oscillating mass is in simple harmonic motion.

Solve: **(a)** The amplitude $A = 2.0$ cm.

(b) The period is calculated as follows:

$$\omega = \frac{2\pi}{T} = 10\,\text{rad/s} \Rightarrow T = \frac{2\pi}{10\ \text{rad/s}} = 0.63\ \text{s}$$

(c) The spring constant is calculated as follows:

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.050\ \text{kg})(10\ \text{rad/s})^2 = 5.0\,\text{N/m}$$

(d) The phase constant $\phi_0 = -\frac{1}{4}\pi$ rad.

(e) The initial conditions are obtained from the equations

$$x(t) = (2.0\ \text{cm})\cos(10t - \tfrac{1}{4}\pi) \text{ and } v_x(t) = -(20.0\ \text{cm/s})\sin(10t - \tfrac{1}{4}\pi)$$

At $t = 0$ s, these equations become

$$x_0 = (2.0\ \text{cm})\cos(-\tfrac{1}{4}\pi) = 1.41\ \text{cm and } v_{0x} = -(20\ \text{cm/s})\sin(-\tfrac{1}{4}\pi) = 14.1\,\text{cm/s}$$

In other words, the mass is at $+1.41$ cm and moving to the right with a velocity of 14.1 cm/s.

(f) The maximum speed is $v_{\text{max}} = A\omega = (2.0\ \text{cm})(10\ \text{rad/s}) = 20\,\text{cm/s}$.

(g) The total energy $E = \frac{1}{2}kA^2 = \frac{1}{2}(5.0\ \text{N/m})(0.020\ \text{m})^2 = 1.00 \times 10^{-3}$ J.

(h) At $t = 0.41$ s, the velocity is

$$v_{0x} = -(20\ \text{cm/s})\sin[(10\ \text{rad/s})(0.40\ \text{s}) - \tfrac{1}{4}\pi] = 1.46\,\text{cm/s}$$

15.40. Model: The ball attached to a spring is in simple harmonic motion.

Solve: **(a)** Let $t = 0$ s be the instant when $x_0 = -5.0$ cm and $v_0 = 20$ cm/s. The oscillation frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5\ \text{N/m}}{0.100\ \text{kg}}} = 5.0\ \text{rad/s}$$

Solving Equation 15.26 for the amplitude gives

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-5.0\ \text{cm})^2 + \left(\frac{20\ \text{cm/s}}{5.0\ \text{rad/s}}\right)^2} = 6.4\ \text{cm}$$

(b) The maximum acceleration is $a_{\text{max}} = \omega^2 A = 160\,\text{cm/s}^2$.

(c) For an oscillator, the acceleration is most positive $(a = a_{\text{max}})$ when the displacement is most negative $(x = -x_{\text{max}} = -A)$. So the acceleration is maximum when $x = -6.4$ cm.

(d) We can use the conservation of energy between $x_0 = -5.0$ cm and $x_1 = 3.0$ cm:

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \Rightarrow v_1 = \sqrt{v_0^2 + \frac{k}{m}(x_0^2 - x_1^2)} = 0.283\,\text{m/s}$$

The speed is 28 cm/s. Because k is known in SI units of N/m, the energy calculation *must* be done using SI units of m, m/s, and kg.