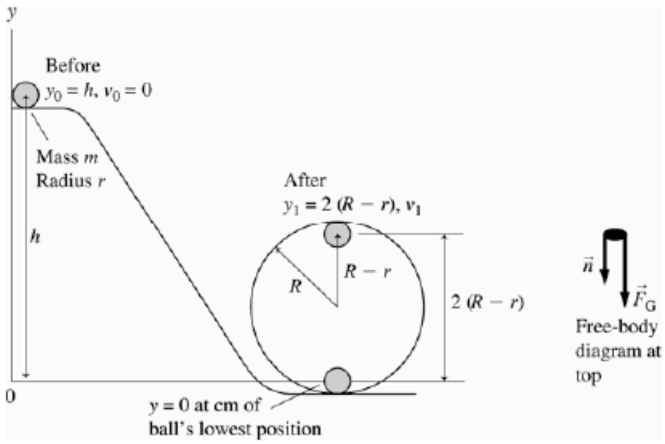
Solutions to HW24, Chapter 12

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

12.75. Model: Assume that the marble does not slip as it rolls down the track and around a loop-the-loop. The mechanical energy of the marble is conserved.

Visualize:



Solve: The marble's center of mass moves in a circle of radius $R - r$. The free-body diagram on the marble at its highest position shows that Newton's second law for the marble is

$$mg + n = \frac{mv_1^2}{R - r}$$

The minimum height (h) that the track must have for the marble to make it around the loop-the-loop occurs when the normal force of the track on the marble tends to zero. Then the weight will provide the centripetal acceleration needed for the circular motion. For $n \to 0$ N,

$$mg = \frac{mv^2}{(R - r)} \Rightarrow v_1^2 = g(R - r)$$

Since rolling motion requires $v_1^2 = r^2\omega_1^2$, we have

$$\omega_1^2 r^2 = g(R - r) \Rightarrow \omega_1^2 = \frac{g(R - r)}{r^2}$$

The conservation of energy equation is

$$(K_f + U_{gf})_{\text{top of loop}} = (K_i + U_{gi})_{\text{initial}} \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgy_1 = mgy_0 = mgh$$

Using the above expressions and $I = \frac{2}{5}mr^2$ the energy equation simplifies to

$$\frac{1}{2}mg(R - r) + \frac{1}{2}\left(\frac{2}{5}\right)mr^2\left(\frac{g(R - r)}{r^2}\right) + mg2(R - r) = mgh \Rightarrow h = 2.7(R - r)$$

12.81. Model: Model the merry-go-round as a rigid disk rotating on frictionless bearings about an axle in the center and John as a particle. For the (merry-go-round + John) system, no external torques act as John jumps on the merry-go-round. Angular momentum is thus conserved.

Visualize: The initial angular momentum is the sum of the angular momentum of the merry-go-round and the angular momentum of John. The final angular momentum as John jumps on the merry-go-round is equal to $I_{\text{final}}\omega_{\text{final}}$.

Solve: John's initial angular momentum is that of a particle: $L_J = m_J v_J R \sin\beta = m_J v_J R$. The angle $\beta = 90°$ since John runs tangent to the disk. The conservation of angular momentum equation $L_f = L_i$ is

$$I_{\text{final}}\omega_{\text{final}} = L_{\text{disk}} + L_J = \left(\frac{1}{2}MR^2\right)\omega_i + m_J v_J R$$

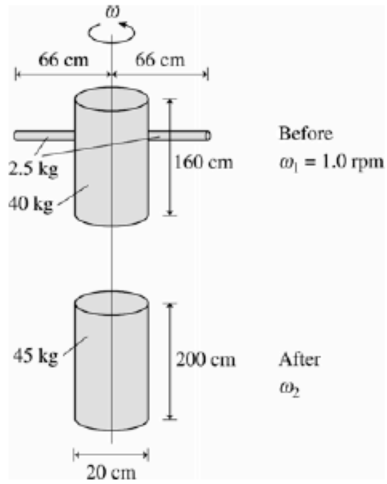
$$= \left(\frac{1}{2}\right)(250\text{ kg})(1.5\text{ m})^2(20\text{ rpm})\frac{2\pi}{60}\left(\frac{\text{rad}}{\text{rpm}}\right) + (30\text{ kg})(5.0\text{ m/s})(1.5\text{ m}) = 814\text{ kg m}^2/\text{s}$$

$$\Rightarrow \omega_{\text{final}} = \frac{814\text{ kg m}^2/\text{s}}{I_{\text{final}}}$$

$$I_{\text{final}} = I_{\text{disk}} + I_J = \frac{1}{2}MR^2 + m_J R^2 = \frac{1}{2}(250\text{ kg})(1.5\text{ m})^2 + (30\text{ kg})(1.5\text{ m})^2 = 349\text{ kgm}^2$$

$$\omega_{\text{final}} = \frac{814\text{ kg m}^2/\text{s}}{349\text{ kg m}^2} = 2.33\text{ rad/s} = 22\text{ rpm}$$

12.82. Model: Model the skater as a cylindrical torso with two rod-like arms that are perpendicular to the axis of the torso in the initial position and collapse into the torso in the final position.
Visualize:



Solve: For the initial position, the moment of inertia is $I_1 = I_{\text{Torso}} + 2I_{\text{Arm}}$. The moment of inertia of each arm is that of a 66-cm-long rod rotating about a point 10 cm from its end, and can be found using the parallel-axis theorem. In the final position, the moment of inertia is $I_2 = \frac{1}{2}MR^2$. The equation for the conservation of angular momentum $L_f = L_i$ can be written $I_2\omega_2 = I_1\omega_1 \Rightarrow \omega_2 = (I_1/I_2)\omega_1$. Calculating I_1 and I_2,

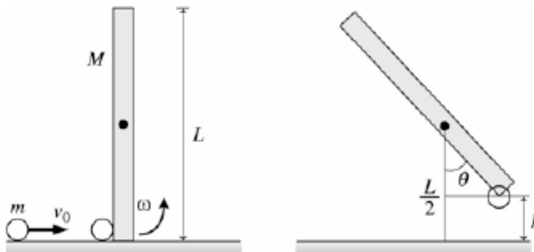
$$I_1 = \frac{1}{2}M_T R^2 + 2\left[\frac{1}{12}M_A L_A^2 + M_A d^2\right]$$
$$= \frac{1}{2}(40 \text{ kg})(0.10 \text{ m})^2 + 2\left[\frac{1}{12}(2.5 \text{ kg})(0.66 \text{ m})^2 + (2.5 \text{ kg})(0.33 \text{ m} + 0.10 \text{ m})^2\right] = 1.306 \text{ kg m}^2$$
$$I_2 = \frac{1}{2}MR^2 = \frac{1}{2}(45 \text{ kg})(0.10 \text{ m})^2 = 0.225 \text{ kg m}^2 \Rightarrow \omega_2 = \frac{(1.306 \text{ kg m}^2)}{(0.225 \text{ kg m}^2)}(1.0 \text{ rev/s}) = 5.8 \text{ rev/s}$$

reasonable.

For Problem 12.90 below, also see my detailed solutions posted in Lecture entitled "Chapter12_Lecture12.2" posted on Nov 7, Slides 38 and 39.

12.90. Model: The clay ball is a particle. The rod is a uniform thin rod rotating about its center. Angular momentum is conserved in the collision.

Visualize:



Known
$L = 30$ cm
$M = 75$ g
$m = 10$ g
$V_0 = 2.5$ m/s
$h = \frac{L}{2}(1-\cos\theta)$
Find
θ

Solve: This is a two-part problem. Angular momentum is conserved in the collision, and energy is conserved as the ball rises like a pendulum. The angular momentum conservation equation about the rod's pivot point is

$$L_i = L_f \Rightarrow mv_0 r = (I_{\text{ball+rod}})\omega$$

Note $r = \frac{L}{2} = 15$ cm. The rod and ball are a composite object. From Table 12.2, $I_{\text{rod}} = \frac{1}{12}ML^2$, so

$$I_{\text{ball+rod}} = I_{\text{ball}} + I_{\text{rod}} = mr^2 + \frac{1}{12}ML^2 = m\frac{L^2}{4} + \frac{1}{12}ML^2 = \frac{L^2}{4}\left(m + \frac{M}{3}\right)$$

If v_f is the final velocity of the clay ball, $\omega = \frac{v_f}{r} = \frac{2v_f}{L}$ since the ball sticks to the rod. Thus

$$\frac{mv_0 L}{2} = \frac{L^2}{4}\left(m + \frac{M}{3}\right)\left(\frac{2v_f}{L}\right)$$

$$\Rightarrow v_f = \frac{mv_0}{m + \frac{M}{3}} = \frac{(0.010\text{ kg})(2.5\text{ m/s})}{(0.010\text{ kg}) + \frac{(0.075\text{ kg})}{3}} = 0.714\text{ m/s}$$

Energy is conserved as the clay ball rises. Compare the energy of the ball-rod system just after the collision to when the ball reaches the maximum height. Note that the center of mass of the rod does not change position.

$$E_i = E_f \Rightarrow \frac{1}{2}(I_{\text{rod + ball}})\omega^2 = mgh$$

Thus

$$\frac{1}{2}\frac{L^2}{4}\left(m + \frac{M}{3}\right)\left(\frac{2v_f}{L}\right)^2 = mgh \Rightarrow v_f^2\left(m + \frac{M}{3}\right) = mgL(1-\cos\theta)$$

$$\Rightarrow 1 - \frac{v_f^2}{mgL}\left(m + \frac{M}{3}\right) = \cos\theta$$

Using the various values, $\cos\theta = 0.393 \Rightarrow \theta = 67°$.

Assess: The clay ball rises $h = \frac{L}{2}(1-\cos\theta) = 9.1$ cm. This is about 2/3 of the height of the pivot point, and is reasonable.