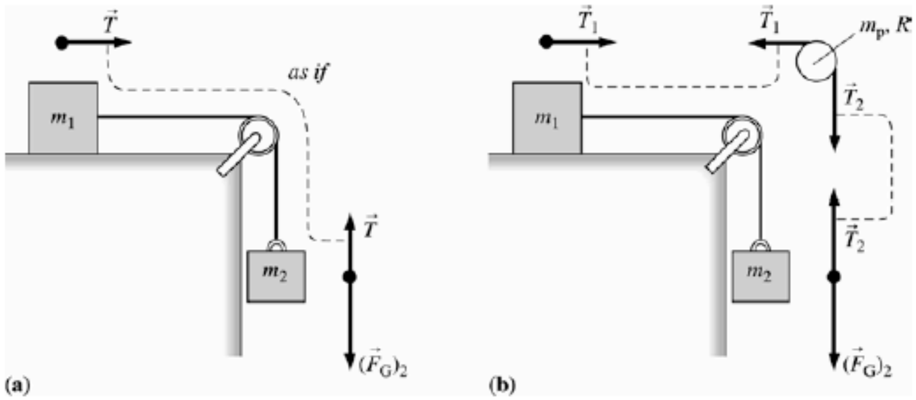
Solutions to HW23, Chapter 12

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

12.65. Model: Assume the string does not slip on the pulley.

Visualize:



The free-body diagrams for the two blocks and the pulley are shown. The tension in the string exerts an upward force on the block m_2, but a downward force on the outer edge of the pulley. Similarly the string exerts a force on block m_1 to the right, but a leftward force on the outer edge of the pulley.

Solve: **(a)** Newton's second law for m_1 and m_2 is $T = m_1 a_1$ and $T - m_2 g = m_2 a_2$. Using the constraint $-a_2 = +a_1 = a$, we have $T = m_1 a$ and $-T + m_2 g = m_2 a$. Adding these equations, we get $m_2 g = (m_1 + m_2)a$, or

$$a = \frac{m_2 g}{m_1 + m_2} \Rightarrow T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) When the pulley has mass m, the tensions $(T_1 \text{ and } T_2)$ in the upper and lower portions of the string are different. Newton's second law for m_1 and the pulley are:

$$T_1 = m_1 a \quad \text{and} \quad T_1 R - T_2 R = -I\alpha$$

We are using the minus sign with α because the pulley accelerates clockwise. Also, $a = R\alpha$. Thus, $T_1 = m_1 a$ and

$$T_2 - T_1 = \frac{I}{R}\frac{a}{R} = \frac{aI}{R^2}$$

Adding these two equations gives

$$T_2 = a\left(m_1 + \frac{I}{R^2}\right)$$

Newton's second law for m_2 is $T_2 - m_2 g = m_2 a_2 = -m_2 a$. Using the above expression for T_2,

$$a\left(m_1 + \frac{I}{R^2}\right) + m_2 a = m_2 g \Rightarrow a = \frac{m_2 g}{m_1 + m_2 + I/R^2}$$

Since $I = \frac{1}{2} m_p R^2$ for a disk about its center,

$$a = \frac{m_2 g}{m_1 + m_2 + \frac{1}{2} m_p}$$

With this value for a we can now find T_1 and T_2:

$$T_1 = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2 + \frac{1}{2} m_p} \qquad T_2 = a(m_1 + I/R^2) = \frac{m_2 g}{(m_1 + m_2 + \frac{1}{2} m_p)}\left(m_1 + \frac{1}{2} m_p\right) = \frac{m_2(m_1 + \frac{1}{2} m_p) g}{m_1 + m_2 + \frac{1}{2} m_p}$$

Assess: For $m = 0\ \text{kg}$, the equations for a, T_1 and T_2 of part (b) simplify to

$$a = \frac{m_2 g}{m_1 + m_2} \quad \text{and} \quad T_1 = \frac{m_1 m_2 g}{m_1 + m_2} \quad \text{and} \quad T_2 = \frac{m_1 m_2 g}{m_1 + m_2}$$

These agree with the results of part (a).

12.66. Model: The disk is a rigid spinning body.
Visualize: Please refer to Figure P12.66. The initial angular velocity is 300 rpm or $(300)(2\pi)/60 = 10\pi$ rad/s. After 3.0 s the disk stops.
Solve: Using the kinematic equation for angular velocity,

$$\omega_1 = \omega_0 + \alpha(t_1 - t_0) \Rightarrow \alpha = \frac{\omega_1 - \omega_0}{t_1 - t_0} = \frac{(0\ \text{rad/s} - 10\pi\ \text{rad/s})}{(3.0\ \text{s} - 0\ \text{s})} = \frac{-10\pi}{3}\ \text{rad/s}^2$$

Thus, the torque due to the force of friction that brings the disk to rest is

$$\tau = I\alpha = -fR \Rightarrow f = -\frac{I\alpha}{R} = -\frac{(\frac{1}{2} mR^2)\alpha}{R} = -\frac{1}{2}(mR)\alpha = -\frac{1}{2}(2.0\ \text{kg})(0.15\ \text{m})\left(-10\frac{\pi}{3}\ \text{rad/s}^2\right) = 1.57\ \text{N} \approx 1.6\ \text{N}$$

The minus sign with $\tau = -fR$ indicates that the torque due to friction acts clockwise.

12.70. Model: Assume there is no air resistance or friction.
Visualize: Use conservation of energy in each case. Initially each object has translational and rotational kinetic energy.

Known
$\theta = 15°$
$m_{di} = 750$ g
$m_{ri} = 760$ g
$r_{di} = r_{ri} = 7.5$ cm
$v_i = 1.5$ m/s

Find
L

Solve:

$$K_i = U_f$$

$$\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 = mg\Delta y$$

$$\Rightarrow \Delta y = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2}{mg} = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\left(\frac{v_i}{r}\right)^2}{mg}$$

For the disk insert $I_{di} = \tfrac{1}{2}m_{di}r_{di}^2$:

$$\Delta y = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_{di}r_{di}^2\right)\left(\frac{v_i}{r_{di}}\right)^2}{mg} = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{4}mv_i^2}{mg} = \frac{\tfrac{3}{4}v_i^2}{g} = \frac{\tfrac{3}{4}(1.5\text{ m/s})^2}{9.8\text{ m/s}^2} = 17.2\text{ cm}$$

Now find how far up the slope the disk goes to achieve a height of 17.2 cm.

$$L = \frac{\Delta y}{\sin\theta} = \frac{17.2\text{ cm}}{\sin 15°} = 67\text{ cm}$$

For the ring insert $I_{ri} = m_{ri}r_{ri}^2$:

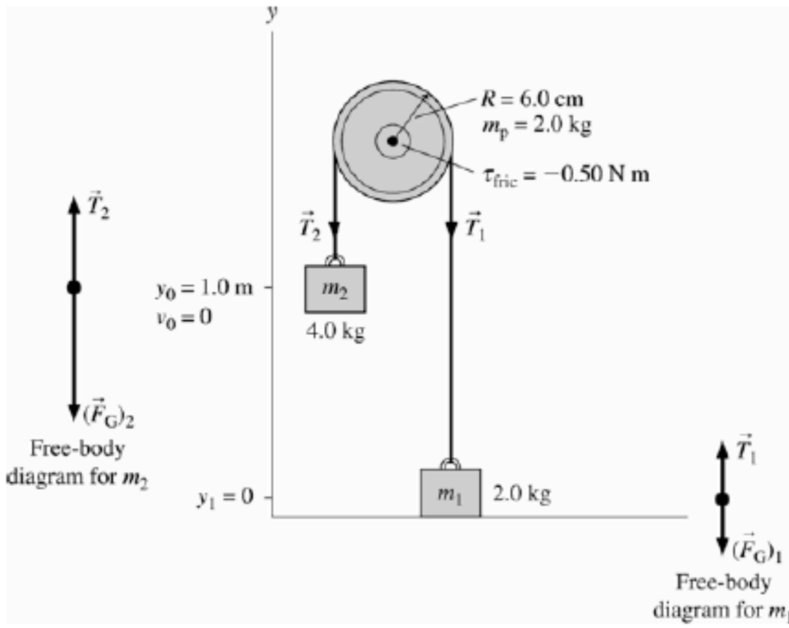
$$\Delta y = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}(m_{ri}r_{ri}^2)\left(\frac{v_i}{r_{ri}}\right)^2}{mg} = \frac{\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}mv_i^2}{mg} = \frac{v_i^2}{g} = \frac{(1.5\text{ m/s})^2}{9.8\text{ m/s}^2} = 23.0\text{ cm}$$

Now find how far up the slope the ring goes to achieve a height of 23 cm.

$$L = \frac{\Delta y}{\sin\theta} = \frac{23.0\text{ cm}}{\sin 15°} = 89\text{ cm}$$

Assess: Because the ring has a greater moment of inertia, it has greater total kinetic energy than the disk moving at the same translational speed. Notice both answers are independent of the mass and the radius.

12.86. Model: The pulley is a rigid rotating body. We also assume that the pulley has the mass distribution of a disk and that the string does not slip.
Visualize:



Because the pulley is not massless and frictionless, tension in the rope on both sides of the pulley is *not* the same.
Solve: Applying Newton's second law to $m_1, m_2,$ and the pulley yields the three equations:

$$T_1 - (F_G)_1 = m_1 a_1 \qquad -(F_G)_2 + T_2 = m_2 a_2 \qquad T_2 R - T_1 R - 0.50 \text{ Nm} = I\alpha$$

Noting that $-a_2 = a_1 = a,$ $I = \frac{1}{2} m_p R^2,$ and $\alpha = a/R,$ the above equations simplify to

$$T_1 - m_1 g = m_1 a \qquad m_2 g - T_2 = m_2 a \qquad T_2 - T_1 = \left(\frac{1}{2} m_p R^2\right)\left(\frac{a}{R}\right)\frac{1}{R} + \frac{0.50 \text{ Nm}}{R} = \frac{1}{2} m_p a + \frac{0.50 \text{ Nm}}{0.060 \text{ m}}$$

Adding these three equations,

$$(m_2 - m_1) g = a\left(m_1 + m_2 + \frac{1}{2} m_p\right) + 8.333 \text{ N}$$

$$\Rightarrow a = \frac{(m_2 - m_1) g - 8.333 \text{ N}}{m_1 + m_2 + \frac{1}{2} m_p} = \frac{(4.0 \text{ kg} - 2.0 \text{ kg})(9.8 \text{ m/s}^2) - 8.333 \text{ N}}{2.0 \text{ kg} + 4.0 \text{ kg} + (2.0 \text{ kg}/2)} = 1.610 \text{ m/s}^2$$

We can now use kinematics to find the time taken by the 4.0 kg block to reach the floor:

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2} a_2 (t_1 - t_0)^2 \Rightarrow 0 = 1.0 \text{ m} + 0 + \frac{1}{2}(-1.610 \text{ m/s}^2)(t_1 - 0 \text{ s})^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2(1.0 \text{ m})}{(1.610 \text{ m/s}^2)}} = 1.1 \text{ s}$$