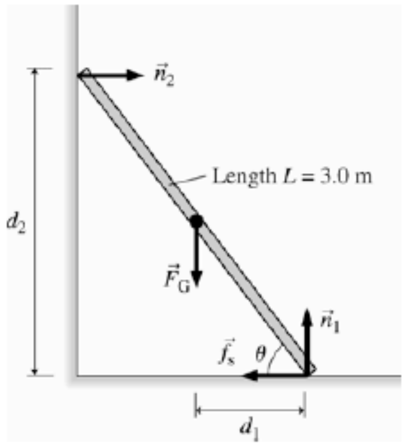
Solutions to HW22, Chapter 12

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

12.58. Model: The ladder is a rigid rod of length L. To not slip, it must be in both translational equilibrium ($\vec{F}_{net} = \vec{0}$ N) and rotational equilibrium ($\tau_{net} = 0$ N m). We also apply the model of static friction.

Visualize:



Since the wall is frictionless, the only force from the wall on the ladder is the normal force $\vec{n}_2$. On the other hand, the floor exerts both the normal force $\vec{n}_1$ and the static frictional force $\vec{f}_s$. The gravitational force $\vec{F}_G$ on the ladder acts through the center of mass of the ladder.

Solve: The x- and y-components of $\vec{F}_{net} = \vec{0}$ N are

$$\Sigma F_x = n_2 - f_s = 0 \text{ N} \Rightarrow f_s = n_2 \qquad \Sigma F_y = n_1 - F_G = 0 \text{ N} \Rightarrow n_1 = F_G$$

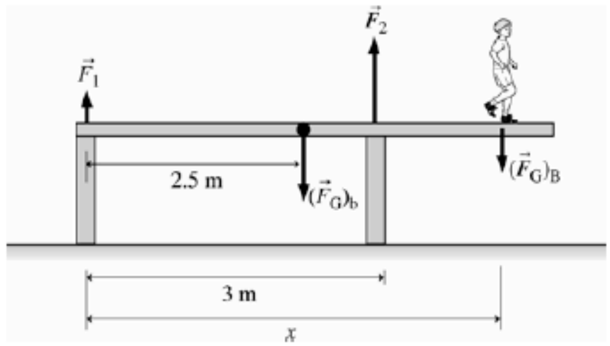
The minimum angle occurs when the static friction is at its maximum value $f_{s\,max} = \mu_s n_1$. Thus we have $n_2 = f_s = \mu_s n_1 = \mu_s mg$. We choose the bottom corner of the ladder as a pivot point to obtain τ_{net}, because two forces pass through this point and have no torque about it. The net torque about the bottom corner is

$$\tau_{net} = d_1 mg - d_2 n_2 = (0.5L\cos\theta_{min})mg - (L\sin\theta_{min})\mu_s mg = 0 \text{ N m}$$

$$\Rightarrow 0.5\cos\theta_{min} = \mu_s \sin\theta_{min} \Rightarrow \tan\theta_{min} = \frac{0.5}{\mu_s} = \frac{0.5}{0.4} = 1.25 \Rightarrow \theta_{min} = 51°$$

12.60. Model: Model the beam as a rigid body. For the beam not to fall over, it must be both in translational equilibrium ($\vec{F}_{net} = \vec{0}$ N) and rotational equilibrium ($\tau_{net} = 0$ Nm).

Visualize:



The boy walks along the beam a distance x, measured from the left end of the beam. There are four forces acting on the beam. F_1 and F_2 are from the two supports, $(\vec{F}_G)_b$ is the gravitational force on the beam, and $(\vec{F}_G)_B$ is the gravitational force on the boy.

Solve: We pick our pivot point on the left end through the first support. The equation for rotational equilibrium is

$$-(F_G)_b(2.5\text{ m}) + F_2(3.0\text{ m}) - (F_G)_B x = 0\text{ N m}$$

$$-(40\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) + F_2(3.0\text{ m}) - (20\text{ kg})(9.80\text{ m/s}^2)x = 0\text{ N m}$$

The equation for translation equilibrium is

$$\sum F_y = 0\text{ N} = F_1 + F_2 - (F_G)_b - (F_G)_B$$

$$\Rightarrow F_1 + F_2 = (F_G)_b + (F_G)_B = (40\text{ kg} + 20\text{ kg})(9.8\text{ m/s}^2) = 588\text{ N}$$

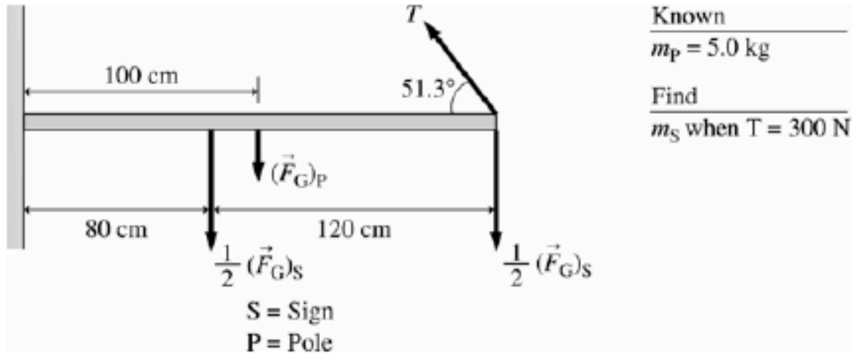
Just when the boy is at the point where the beam tips, $F_1 = 0$ N. Thus $F_2 = 588$ N. With this value of F_2, we can simplify the torque equation to:

$$-(40\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) + (588\text{ N})(3.0\text{ m}) - (20\text{ kg})(9.80\text{ m/s}^2)x = 0\text{ N m}$$

$$\Rightarrow x = 4.0\text{ m}$$

Thus, the distance from the right end is $5.0\text{ m} - 4.0\text{ m} = 1.0\text{ m}$.

12.62. Model: The pole is a uniform rod. The sign is also uniform.
Visualize:



Solve: The geometry of the rod and cable give the angle that the cable makes with the rod.

$$\theta = \tan^{-1}\left(\frac{250}{200}\right) = 51.3°$$

The rod is in rotational equilibrium about its left-hand end.

$$\tau_{\text{net}} = 0 = -(100\text{ cm})(F_G)_P - (80\text{ cm})\left(\frac{1}{2}\right)(F_G)_S - (200\text{ cm})\left(\frac{1}{2}\right)(F_G)_S + (200\text{ cm})T\sin 51.3°$$

$$= -(100\text{ cm})(5.0\text{ kg})(9.8\text{ m/s}^2) - m_S(9.8\text{ m/s}^2)(140\text{ cm}) + (156\text{ cm})T$$

With $T = 300\,N$, $m_S = 30.6\text{ kg} \approx 31\text{ kg}$.

Assess: A mass of 30.6 kg is reasonable for a sign.