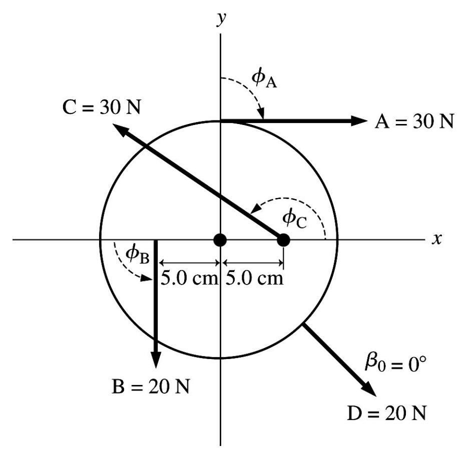
Solutions to HW21, Chapter 12

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

12.19. Visualize: The downward force that points at the axle provides no torque at all.
Solve: The line of action of the upward force is 10 cm away from the axis.

$$\tau = rF \Rightarrow F = \frac{\tau}{r} = \frac{5.0\ \text{N}\cdot\text{m}}{0.10\ \text{m}} = 50\ \text{N}$$

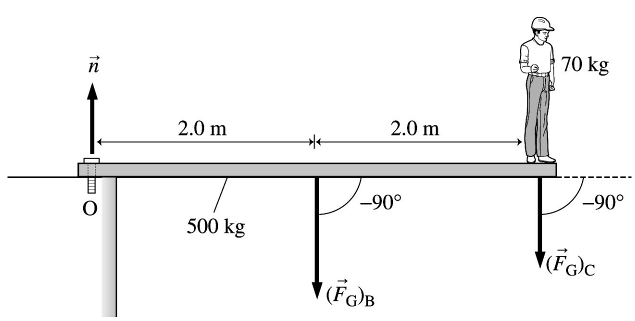
12.20. Model: The disk is a rotating rigid body.
Visualize:



The radius of the disk is 10 cm and the disk rotates on an axle through its center.
Solve: The net torque on the axle is

$$\begin{aligned}\tau &= F_A r_A \sin\phi_A + F_B r_B \sin\phi_B + F_C r_C \sin\phi_C + F_D r_D \sin\phi_D \\ &= (30\ \text{N})(0.10\ \text{m})\sin(-90°) + (20\ \text{N})(0.050\ \text{m})\sin 90° + (30\ \text{N})(0.050\ \text{m})\sin 135° + (20\ \text{N})(0.10\ \text{m})\sin 0° \\ &= -3\ \text{N m} + 1\ \text{N m} + 1.0607\ \text{N m} = -0.94\ \text{N m}\end{aligned}$$

Assess: A negative torque means a clockwise rotation of the disk.
12.22. Model: The beam is a solid rigid body.
Visualize:

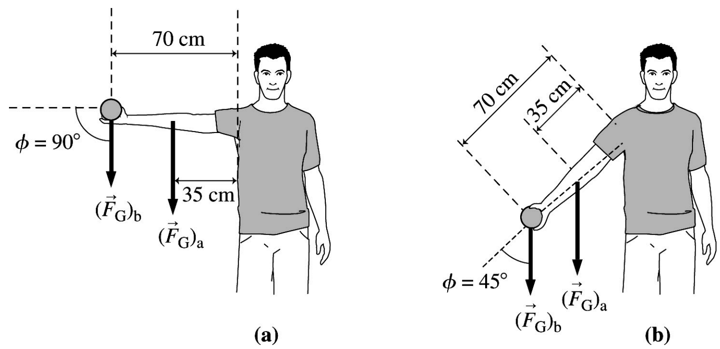


The steel beam experiences a torque due to the gravitational force on the construction worker $(\vec{F}_G)_C$ and the gravitational force on the beam $(\vec{F}_G)_B$. The normal force exerts no torque since the net torque is calculated about the point where the beam is bolted into place.
Solve: The net torque on the steel beam about point O is the sum of the torque due to $(\vec{F}_G)_C$ and the torque due to $(\vec{F}_G)_B$. The gravitational force on the beam acts at the center of mass.

$$\tau = ((F_G)_C)(4.0\text{ m})\sin(-90°) + ((F_G)_B)(2.0\text{ m})\sin(-90°)$$
$$= -(70\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m}) - (500\text{ kg})(9.80\text{ m/s}^2)(2.0\text{ m}) = -12.5\text{ kN m}$$

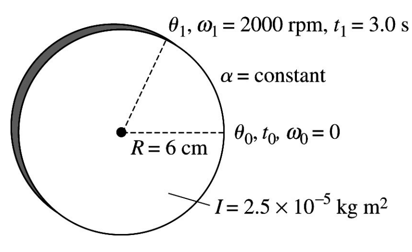
The negative torque means these forces would cause the beam to rotate clockwise. The magnitude of the torque is 12.5 kN m.

12.23. Model: Model the arm as a uniform rigid rod. Its mass acts at the center of mass.
Visualize:



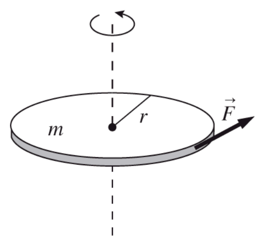
Solve: **(a)** The torque is due both to the gravitational force on the ball and the gravitational force on the arm:
$$\tau = \tau_{ball} + \tau_{arm} = (m_b g)r_b \sin 90° + (m_a g)r_a \sin 90°$$
$$= (3.0\text{ kg})(9.8\text{ m/s}^2)(0.70\text{ m}) + (4.0\text{ kg})(9.8\text{ m/s}^2)(0.35\text{ m}) = 34\text{ N m}$$
(b) The torque is reduced because the moment arms are reduced. Both forces act at $\phi = 45°$ from the radial line, so
$$\tau = \tau_{ball} + \tau_{arm} = (m_b g)r_b \sin 45° + (m_a g)r_a \sin 45°$$
$$= (3.0\text{ kg})(9.8\text{ m/s}^2)(0.70\text{ m})(0.707) + (4.0\text{ kg})(9.8\text{ m/s}^2)(0.35\text{ m})(0.707) = 24\text{ N m}$$

12.27. Model: The compact disk is a rigid body rotating about its center.
Visualize:



Solve: **(a)** The rotational kinematic equation $\omega_1 = \omega_0 + \alpha(t_1 - t_0)$ gives
$$(2000\text{ rpm})\left(\frac{2\pi}{60}\right)\text{rad/s} = 0\text{ rad} + \alpha(3.0\text{ s} - 0\text{ s}) \Rightarrow \alpha = \frac{200\pi}{9}\text{ rad/s}^2$$
The torque needed to obtain this operating angular velocity is
$$\tau = I\alpha = (2.5\times10^{-5}\text{ kg m}^2)\left(\frac{200\pi}{9}\text{ rad/s}^2\right) = 1.75\times10^{-3}\text{ N m}$$
(b) From the rotational kinematic equation,
$$\theta_1 = \theta_0 + \omega_0(t_1 - t_0) + \frac{1}{2}\alpha(t_1 - t_0)^2 = 0\text{ rad} + 0\text{ rad} + \frac{1}{2}\left(\frac{200\pi}{9}\text{ rad/s}^2\right)(3.0\text{ s} - 0\text{ s})^2$$
$$= 100\pi\text{ rad} = \frac{100\pi}{2\pi}\text{ revolutions} = 50\text{ rev}$$
Assess: Fifty revolutions in 3 seconds is a reasonable value.

12.28. Model: Model the disk as solid. The torque is constant so the angular acceleration is constant.
Visualize: The disk starts from rest, so $\omega_0 = 0$.



Known
$m = 4.0$ kg
$r = 0.18$ m
$F = 5.0$ N
$\Delta t = 4.0$ s
$\omega_0 = 0$

Find
ω_1

Solve:

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} \Rightarrow \Delta\omega = \omega_1 - \omega_0 = \omega_1 - 0 = \omega_1 = \frac{\tau\,\Delta t}{I} = \frac{rF\Delta t}{\frac{1}{2}mr^2} = \frac{F\Delta t}{\frac{1}{2}mr} = \frac{(5.0\text{ N})(4.0\text{ s})}{\frac{1}{2}(4.0\text{ kg})(0.18\text{ m})} = 55.6\text{ rad/s} = 530\text{ rpm}$$

Assess: 530 rpm is pretty fast but in the reasonable range.

12.37. Visualize: To determine angle α, put the *tails* of the vectors together.

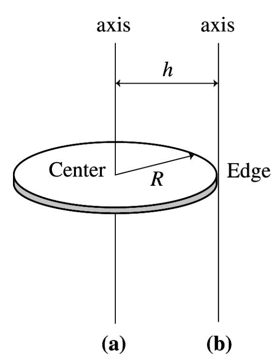
Solve: (a) The angle between the vectors when their tails coincide is 135°. The magnitude of $\vec{A}\times\vec{B}$ is $AB\sin\alpha = (6)(4)\sin 135° = 17$. The direction of $\vec{A}\times\vec{B}$, using the right-hand rule, is out of the page. Thus, $\vec{A}\times\vec{B} = (17,\text{ out of the page})$.

(b) The magnitude of $\vec{C}\times\vec{D}$ is $CD\sin\alpha = (6)(4)\sin 180° = 0$. Thus $\vec{C}\times\vec{D} = \vec{0}$.

12.40. Solve:

$$\vec{\tau} = \vec{r}\times\vec{F} = (5\hat{i} + 5\hat{j})\times(-10\hat{j})\text{ N m}$$
$$= [-50(\hat{i}\times\hat{j}) - 50(\hat{j}\times\hat{j})]\text{ N m} = [-50(+\hat{k}) - \vec{0}]\text{ N m} = -50\hat{k}\text{ N m}$$

12.52. Model: The disk is a rigid rotating body. The axis is perpendicular to the plane of the disk.
Visualize:



Solve: (a) From Table 12.2, the moment of inertia of a disk about its center is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0\text{ kg})(0.10\text{ m})^2 = 0.010\text{ kg m}^2$$

(b) To find the moment of inertia of the disk through the edge, we can make use of the parallel–axis theorem:

$$I = I_{\text{center}} + Mh^2 = (0.010\text{ kg m}^2) + (2.0\text{ kg})(0.10\text{ m})^2 = 0.030\text{ kg m}^2$$

Assess: The larger moment of inertia about the edge means there is more inertia to rotational motion about the edge than about the center.