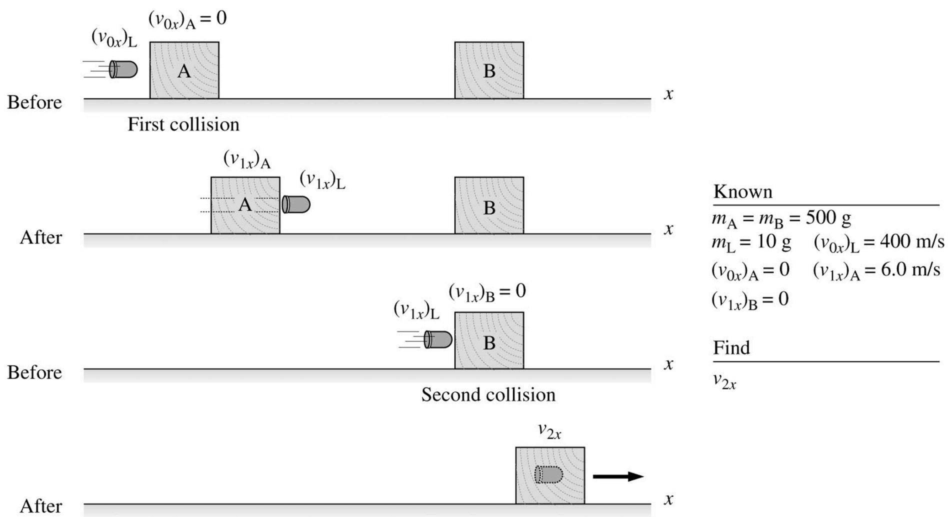
Solutions to HW19, Chapter 11

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

11.54. Model: Model the two blocks (A and B) and the bullet (L) as particles. This is a two-part problem. First, we have a collision between the bullet and the first block (A). Momentum is conserved since no external force acts on the system (bullet + block A). The second part of the problem involves a perfectly inelastic collision between the bullet and block B. Momentum is again conserved for this system (bullet + block B).
Visualize:



Solve: For the first collision the equation $p_{fx} = p_{ix}$ is

$$m_L(v_{1x})_L + m_A(v_{1x})_A = m_L(v_{0x})_L + m_A(v_{0x})_A$$

$$(0.010\text{ kg})(v_{1x})_L + (0.500\text{ kg})(6.0\text{ m/s}) = (0.010\text{ kg})(400\text{ m/s}) + 0\text{ kg m/s} \quad\Rightarrow\quad (v_{1x})_L = 100\text{ m/s}$$

The bullet emerges from the first block at 100 m/s. For the second collision the equation $p_{fx} = p_{ix}$ is

$$(m_L + m_B)v_{2x} = m_L(v_{1x})_L \quad\Rightarrow\quad (0.010\text{ kg} + 0.500\text{ kg})v_{2x} = (0.010\text{ kg})(100\text{ m/s}) \quad\Rightarrow\quad v_{2x} = 2.0\text{ m/s}$$

11.57. Model: The track is frictionless.
Visualize: $v_c = \sqrt{Rg}$
Solve: (a) First use conservation of momentum during the collision, then conservation of energy as the combined block goes to the top of the loop.

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$mv_m + 0 = (m+M)v_{tot}$$

$$v_m = \frac{m+M}{m}v_{tot}$$

Now use the conservation of energy. v_{tot} is the speed of the combined block just after the collision.

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}(m+M)v_{tot}^2 = (m+M)g(2R) + \frac{1}{2}(m+M)(v_c)^2$$

Cancel $(m+M)$ and replace v_c with $\sqrt{Rg}$.

$$v_{tot}^2 = 4Rg + Rg = 5Rg \Rightarrow v_{tot} = \sqrt{5Rg}$$

$$v_m = \frac{m+M}{m} v_{\text{tot}} = \frac{m+M}{m}\sqrt{5Rg}$$

(b) First use conservation of momentum and kinetic energy during the collision, then conservation of mechanical energy as the big block goes to the top of the loop. Call the speed of M just after the elastic collision V and the speed of m just after the collision v'_m.

$$\sum \vec{p}_{\text{i}} = \sum \vec{p}_{\text{f}}$$

We drop the vectors because this is one-dimensional motion, but v' may be negative.

$$mv_m + 0 = mv' + MV$$

$$v_m = v' + \frac{M}{m}V$$

Now use the conservation of energy as the block goes to the top of the loop.

$$U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}}$$

$$0 + \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2 = Mg(2R) + \frac{1}{2}mv_m'^2 + \frac{1}{2}M\left(\sqrt{Rg}\right)^2$$

Subtract $\frac{1}{2}mv_m'^2$ from both sides, cancel M, and solve for V.

$$V = \sqrt{4Rg + Rg} = \sqrt{5Rg}$$

Now go back to the conservation of kinetic energy in the elastic collision.

$$\sum K_{\text{i}} = \sum K_{\text{f}}$$

$$\frac{1}{2}mv_m^2 = \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2$$

Cancel $\frac{1}{2}$ and divide by m.

$$v_m^2 = v_m'^2 + \frac{M}{m}5Rg$$

Find $v_m'^2$ from the momentum equation.

$$v_m^2 = \left(v_m - \frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

$$v_m^2 = v_m^2 - 2\frac{M}{m}v_m V + \left(\frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

Subtract v_m^2 from both sides, cancel $\frac{M}{m}$, and solve for v_m.

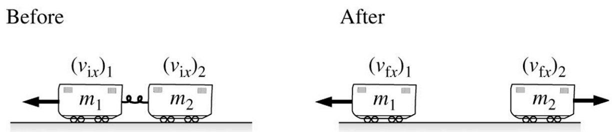
$$v_m = \frac{1}{2}\frac{M}{m}V + \frac{1}{2}\frac{5Rg}{\sqrt{5Rg}} = \frac{1}{2}\frac{M}{m}\sqrt{5Rg} + \frac{1}{2}\sqrt{5Rg} = \frac{1}{2}\frac{m+M}{m}\sqrt{5Rg}$$

Assess: We expected the initial speed needed to be greater for the inelastic case because kinetic energy isn't conserved in the collision.

11.62. Model: The two railcars make up a system. The impulse approximation is used while the spring is expanding, so friction can be ignored.

Visualize:

Pictorial representation



Known
$(v_{ix})_1 = (v_{ix})_2 = 0$
$m_1 = 30$ tons $m_2 = 90$ tons
$(v_{fx})_2 - (v_{fx})_1 = 4.0$ m/s

Find
$(v_{fx})_1$

Solve: Since the cars are at rest initially, the total momentum of the system is zero. Conservation of momentum gives

$$0 = m_1(v_{fx})_1 + m_2(v_{fx})_2$$

We are only told that the relative velocity of the two cars after the spring expands is 4.0 m/s, so

$$(v_{fx})_2 - (v_{fx})_1 = 4.0 \text{ m/s}$$

Substitute $(v_{fx})_2 = (v_{fx})_1 + 4.0$ m/s into the conservation of momentum equation, then solve for $(v_{fx})_1$:

$$0 = m_1(v_{fx})_1 + m_2[(v_{fx})_1 + 4.0 \text{ m/s}]$$

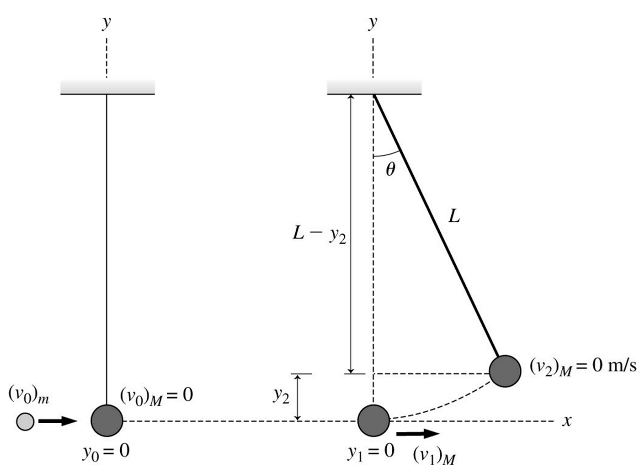
$$(v_{fx})_1 = -\frac{m_2(4.0 \text{ m/s})}{(m_1 + m_2)} = -\frac{(90 \text{ tons})(4.0 \text{ m/s})}{(30 \text{ tons} + 90 \text{ tons})} = -3.0 \text{ m/s}$$

so the speed of the 30 ton car relative to the ground is 3.0 m/s.

Assess: The other more massive railcar has a velocity $(v_{fx})_2 = (v_{fx})_1 + 4.0 \text{ m/s} = 1.0 \text{ m/s}$. A slower speed for the more massive car makes sense.

11.63. Model: We can divide this problem into two parts. First, we have an elastic collision between the 20 g ball (m) and the 100 g ball (M). Second, the 100 g ball swings up as a pendulum.

Visualize:



The figure shows three distinct moments of time: the time before the collision, the time after the collision but before the two balls move, and the time the 100 g ball reaches its highest point. We place the origin of our coordinate system on the 100 g ball when it is hanging motionless.

Solve: For a perfectly elastic collision, the ball moves forward with speed

$$(v_1)_M = \frac{2m_m}{m_m + m_M}(v_0)_m = \frac{1}{3}(v_0)_m$$

In the second part, the sum of the kinetic and gravitational potential energy is conserved as the 100 g ball swings up after the collision. That is, $K_2 + U_{g2} = K_1 + U_{g1}$. We have

$$\frac{1}{2}M(v_2)_M^2 + Mgy_2 = \frac{1}{2}M(v_1)_M^2 + Mgy_1$$

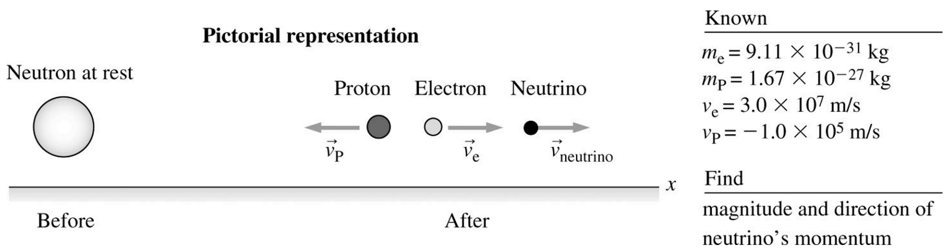
Using $(v_2)_M = 0$ J, $(v_1)_M = \frac{(v_0)_m}{3}$, $y_1 = 0$ m, and $y_2 = L - L\cos\theta$, the energy equation simplifies to

$$g(L - L\cos\theta) = \frac{1}{2}\frac{(v_0)_m^2}{9}$$

$$\Rightarrow (v_0)_m = \sqrt{18\text{ g }L(1-\cos\theta)} = \sqrt{18(9.8\text{ m/s}^2)(1.0\text{ m})(1-\cos 50°)} = 7.9\text{ m/s}$$

11.69. Model: The neutron's decay is an "explosion" of the neutron into several pieces. The neutron is an isolated system, so its momentum should be conserved. The observed decay products, the electron and proton, move in opposite directions.

Visualize:



Solve: **(a)** The initial momentum is $p_{ix} = 0$ kg m/s. The final momentum $p_{fx} = m_e v_e + m_p v_p$ is

$$p_{fx} = 2.73\times10^{-23}\text{ kg m/s} - 1.67\times10^{-22}\text{ kg m/s} = -1.4\times10^{-22}\text{ kg m/s}$$

No, momentum does not seem to be conserved.

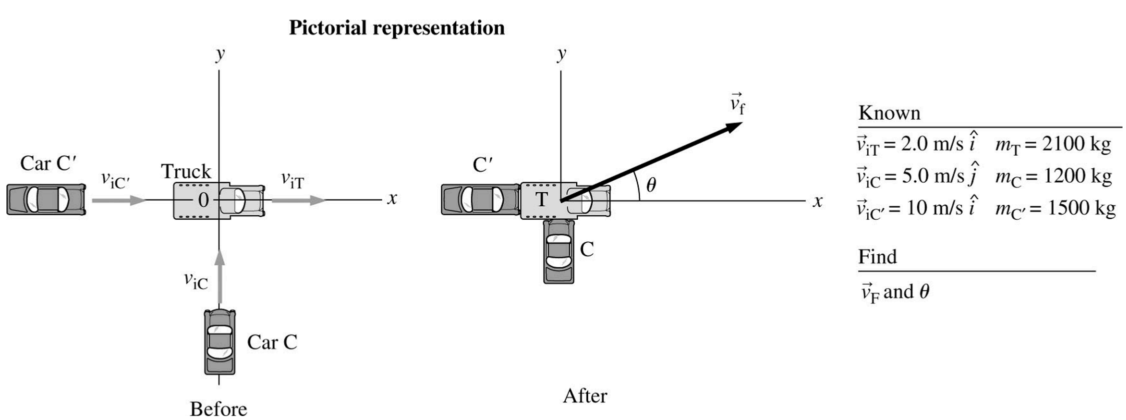
(b) and **(c)** If the neutrino is needed to conserve momentum, then $p_e + p_P + p_{neutrino} = 0$ kg m/s. This requires

$$p_{neutrino} = -(p_e + p_P) = +1.4\times10^{-22}\text{ kg m/s}$$

The neutrino must "carry away" 1.4×10^{-22} kg m/s of momentum in the same direction as the electron.

11.72. Model: Model the truck (T) and the two cars (C and C′) as particles. The three forming our system stick together during their collision, which is perfectly inelastic. Since no significant external forces act on the system during the brief collision time, the momentum of the system is conserved.

Visualize:



Solve: The three momenta are

$$\vec{p}_{iT} = m_T\vec{v}_{iT} = (2100 \text{ kg})(2.0 \text{ m/s})\hat{i} = 4200\hat{i} \text{ kg m/s}$$

$$\vec{p}_{iC} = m_C\vec{v}_{iC} = (1200 \text{ kg})(5.0 \text{ m/s})\hat{j} = 6000\hat{j} \text{ kg m/s}$$

$$\vec{p}_{iC'} = m_{C'}\vec{v}_{iC'} = (1500 \text{ kg})(10 \text{ m/s})\hat{i} = 15{,}000\hat{i} \text{ kg m/s}$$

$$\vec{p}_f = \vec{p}_i = \vec{p}_{iT} + \vec{p}_{iC} + \vec{p}_{iC'} = (19{,}200\hat{i} + 6000\hat{j}) \text{ kg m/s}$$

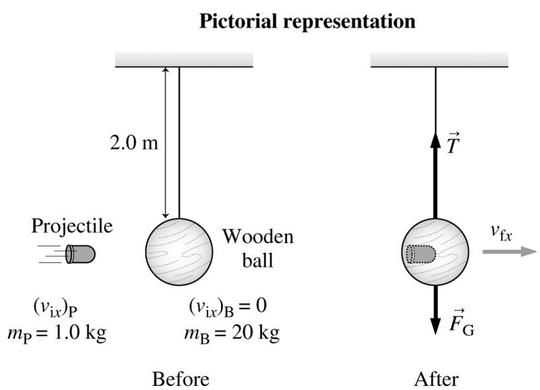
$$p_f = (m_T + m_C + m_{C'})v_f = \sqrt{(19{,}200 \text{ kg m/s})^2 + (6000 \text{ kg m/s})^2}$$

$$v_f = 4.2 \text{ m/s}, \ \theta = \tan^{-1}\frac{p_y}{p_x} = \tan^{-1}\frac{6000}{19{,}200} = 17^\circ \text{ above the } +x\text{-axis}$$

Assess: A speed of 4.2 m/s for the entangled three vehicles is reasonable since the individual speeds of the cars and the truck before entanglement were of the same order of magnitude.

11.81. Model: The projectile + wood ball are our system. In the collision, momentum is conserved.

Visualize:



Solve: The momentum conservation equation $p_{fx} = p_{ix}$ is

$$(m_P + m_B)v_{fx} = m_P(v_{ix})_P + m_B(v_{ix})_B \ \Rightarrow \ (1.0 \text{ kg} + 20 \text{ kg})v_{fx} = (1.0 \text{ kg})(v_{ix})_P + 0 \text{ kg m/s}$$

$$(v_{ix})_P = 21v_{fx}$$

We therefore need to determine v_{fx}. Newton's second law for circular motion is

$$T - F_G = T - (m_P + m_B)g = \frac{(m_P + m_B)v_{fx}^2}{r}$$

Using $T_{max} = 400$ N, this equation gives

$$400 \text{ N} - (1.0 \text{ kg} + 20 \text{ kg})(9.8 \text{ m/s}) = \frac{(1.0 \text{ kg} + 20 \text{ kg})v_{fx}^2}{2.0 \text{ m}} \ \Rightarrow \ (v_{fx})_{max} = 4.3 \text{ m/s}$$

Going back to the momentum conservation equation,

$$(v_{ix})_P = 21v_{fx} = (21)(4.3 \text{ m/s}) = 90 \text{ m/s}$$

That is, the largest speed this projectile can have without causing the cable to break is 90 m/s.