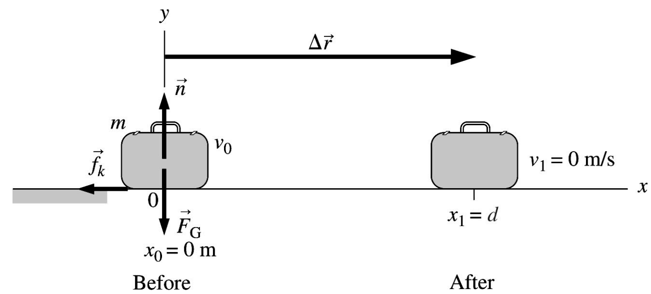
Solutions to HW17, Chapters 9 and 10

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

9.33. Model: Model the suitcase as a particle, use the model of kinetic friction, and use the work–kinetic energy theorem.

Visualize:



The net force on the suitcase is $\vec{F}_{\text{net}} = \vec{f}_{\text{k}}$.

Solve: The work–kinetic energy theorem gives

$$W_{\text{net}} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 \quad\Rightarrow\quad \vec{F}_{\text{net}} \cdot \Delta\vec{r} = \vec{f}_{\text{k}} \cdot \Delta\vec{r} = 0\text{ J} - \tfrac{1}{2}mv_0^2$$

$$(f_{\text{k}})d\cos(180°) = -\tfrac{1}{2}mv_0^2 - \mu_{\text{k}}mgd = -\tfrac{1}{2}mv_0^2 \quad\Rightarrow\quad \mu_{\text{k}} = \frac{v_0^2}{2gd}$$

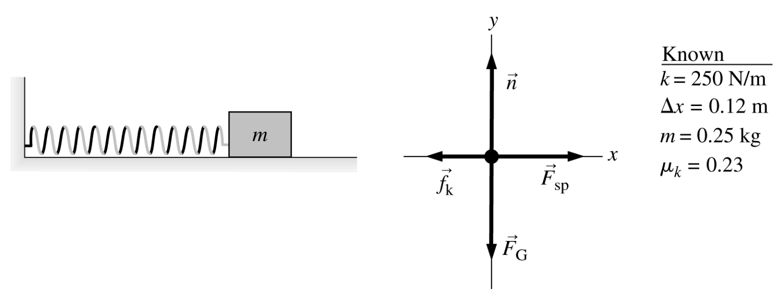
Inserting the given quantities into the expression for the coefficient of kinetic friction gives

$$\mu_{\text{k}} = \frac{v_0^2}{2gd} = \frac{(1.2\text{ m/s})^2}{2(9.8\text{ m/s}^2)(2.0\text{ m})} = 0.037$$

Assess: Friction transforms KE of the suitcase into thermal energy. In response, the suitcase slows down and comes to rest. Note that the coefficient of friction does not depend on the mass of the object, which is reasonable.

9.59. Model: The box starts from rest.

Visualize: Use the work–kinetic energy theorem



Solve: First compute the total work done on the box during the launch.

$$W = \int_{x_0}^{x_1} (F_{\text{sp}} - f_{\text{k}})dx = \int_{x_0}^{x_1} (kx - \mu_{\text{k}}n)dx = \left[\tfrac{1}{2}kx^2 - \mu_{\text{k}}mgx\right]_{x_0}^{x_1}$$

$$= \left[\tfrac{1}{2}(250\text{ N/m})x^2 - (0.23)(0.25\text{ kg})(9.8\text{ m/s}^2)x\right]_0^{0.12} = 1.73\text{ J}$$

Now use the work–kinetic energy theorem.

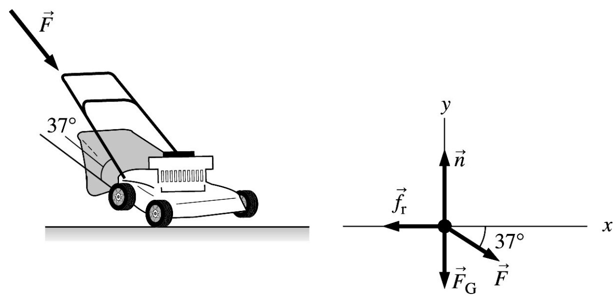
$$1.73\text{ J} = \tfrac{1}{2}mv_{\text{f}}^2 \Rightarrow v_{\text{f}} = \sqrt{\frac{2(1.73\text{ J})}{0.25\text{ kg}}} = 3.7\text{ m/s}$$

Assess: The friction decreased the launch speed only a bit.

A couple weird fonts below where they try and indicate a vector symbol…ignore 'em.

9.71. Model: Model the lawnmower as a particle and use the model of kinetic friction.

Visualize:



We placed the origin of our coordinate system on the lawnmower and drew the free-body diagram of forces.

Solve: The normal force $\vec{n}$, which is related to the frictional force, is not equal to $\vec{F}_G$. This is due to the presence of $\vec{F}$. The rolling friction is $f_r = \mu_r n$, or $n = f_r/\mu_r$. The lawnmower moves at constant velocity, so $\vec{F}_{net} = \vec{0}$. The two components of Newton's second law are

$$(\Sigma F_y) = n - F_G - F\sin(37°) = ma_y = 0\text{ N} \Rightarrow f_r/\mu_r - mg - F\sin(37°) = 0\text{ N} \quad \Rightarrow \quad f_r = \mu_r mg + \mu_r F\sin 37°$$

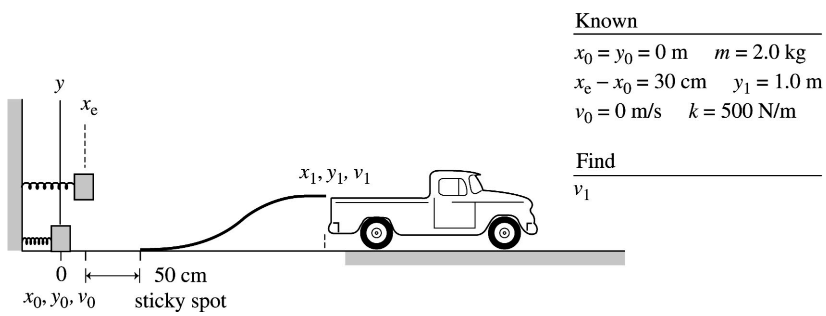
$$(\Sigma F_x) = F\cos(37°) - f_r = 0\text{ N} \quad \Rightarrow \quad F\cos(37°) - \mu_r mg - \mu_r F\sin(37°) = 0\text{ N}$$

$$F = \frac{\mu_r mg}{\cos(37°) - \mu_r \sin(37°)} = \frac{(0.15)(12\text{ kg})(9.8\text{ m/s}^2)}{0.7986 - (0.15)(0.6018)} = 24.9\text{ N}$$

Thus, the power supplied by the gardener in pushing the lawnmower at a constant speed of 1.2 m/s is $P = \vec{F}\cdot\vec{v} = Fv\cos\theta = (24.9\text{ N})(1.2\text{ m/s})\cos(37°) = 24\text{ W}$.

10.51. Model: We will use the spring, the package, and the ramp as the system. We will model the package as a particle.

Visualize:



We place the origin of our coordinate system on the end of the spring when it is compressed and is in contact with the package to be shot.

Model: (a) The energy conservation equation is

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_e - x_e)^2 + \Delta E_{th} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(\Delta x)^2 + W_{ext}$$

Using $y_1 = 1.0$ m, $\Delta E_{th} = 0$ J (the frictionless ramp), $v_0 = 0$ m/s, $y_0 = 0$ m, $\Delta x = 30$ cm, and $W_{ext} = 0$ J, we get

$$\tfrac{1}{2}mv_1^2 + mg(1.0\text{ m}) + 0\text{ J} + 0\text{ J} = 0\text{ J} + 0\text{ J} + \tfrac{1}{2}k(0.30\text{ m})^2 + 0\text{ J}$$

$$\tfrac{1}{2}(2.0\text{ kg})v_1^2 + (2.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = \tfrac{1}{2}(500\text{ N/m})(0.30\text{ m})^2$$

$$v_1 = 1.7\text{ m/s}$$

(b) How high can the package go after crossing the sticky spot? If the package can reach $y_1 \geq 1.0$ m before stopping $(v_1 = 0)$, then it makes it. But if $y_1 < 1.0$ m when $v_1 = 0$, the package does not make it. The friction of the sticky spot generates thermal energy

$$\Delta E_{\text{th}} = (\mu_k mg)\Delta x = (0.30)(2.0\text{ kg})(9.8\text{ m/s}^2)(0.50\text{ m}) = 2.94\text{ J}$$

The energy conservation equation is now

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \Delta E_{\text{th}} = \tfrac{1}{2}k(\Delta x)^2$$

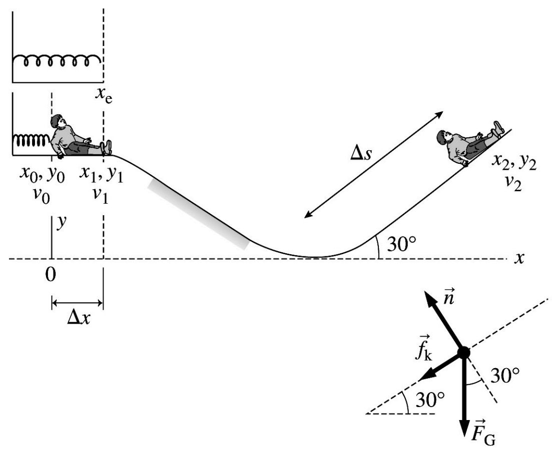
If we set $v_1 = 0$ m/s to find the highest point the package can reach, we get

$$y_1 = \left(\tfrac{1}{2}k\Delta x^2 - \Delta E_{\text{th}}\right)\big/(mg) = [\tfrac{1}{2}(500\text{ N/m})(0.30\text{ m})^2 - 2.94\text{ J}]/[(2.0\text{ kg})(9.8\text{ m/s}^2)] = 0.998\text{ m}$$

The package does not make it. It just barely misses.

10.54. Model: Assume an ideal spring, so Hooke's law is obeyed. Treat the physics student as a particle and apply the law of conservation of energy. Our system comprises the spring, the student, and the ground. We also use the model of kinetic friction.

Visualize: We place the origin of the coordinate system on the ground directly below the end of the compressed spring that is in contact with the student.



Known
$x_0 = 0$ m $v_0 = 0$ m/s
$x_1 - x_0 = 0.50$ m $k = 80{,}000$ N/m
$y_0 = y_1 = 10$ m $m = 100$ kg
$\mu_k = 0.15$ $v_2 = 0$ m/s

Find
v_1 $\Delta s = y_2/\sin 30°$

Solve: (a) The energy conservation equation gives

$$K_1 + U_{g1} + U_{s1} + \Delta E_{\text{th}} = K_0 + U_{g0} + U_{s0} + W_{\text{ext}}$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_1 - x_e)^2 + 0\text{ J} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

Since $y_1 = y_0 = 10$ m, $x_1 = x_e$, $v_0 = 0$ m/s, $k = 80{,}000$ N/m, $m = 100$ kg, and $(x_1 - x_0) = 0.5$ m,

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}k(x_1 - x_0)^2 \quad \Rightarrow \quad v_1 = \sqrt{\frac{k}{m}}(x_1 - x_0) = \sqrt{\frac{80{,}000\text{ N}/m}{100\text{ kg}}}(0.50\text{ m}) = 14\text{ m/s}$$

(b) Friction creates thermal energy. Applying the conservation of energy equation once again:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{\text{th}} = K_0 + U_{g0} + U_{s0} + W_{\text{ext}}$$

$$\tfrac{1}{2}mv_2^2 + mgy_2 + 0\text{ J} + f_k\Delta s = 0\text{ J} + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

With $v_2 = 0$ m/s and $y_2 = \Delta s \sin(30°)$, the above equation is simplified to

$$mg\Delta s\sin(30°) + \mu_k n\Delta s = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

From the free-body diagram for the physics student, we see that $n = F_G\cos(30°) = mg\cos(30°)$. Thus, the conservation of energy equation gives

$$\Delta s[mg\sin(30°) + \mu_k mg\cos(30°)] = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

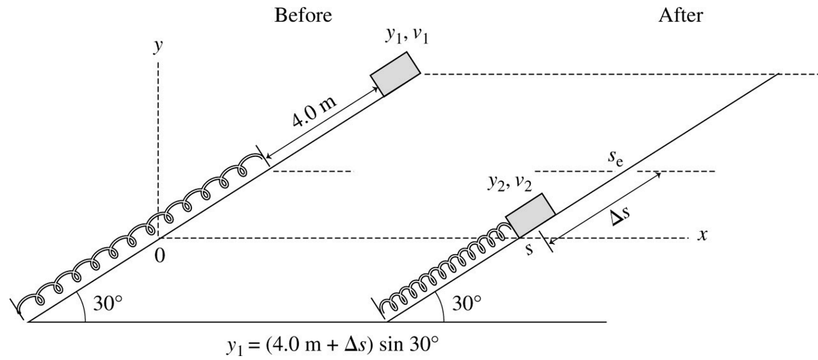
Using $m = 100$ kg, $k = 80{,}000$ N/m, $(x_1 - x_0) = 0.50$ m, $y_0 = 10$ m, and $\mu_k = 0.15$, we get

$$\Delta s = \frac{mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2}{mg[\sin(30°) + \mu_k\cos(30°)]} = 32\text{ m}$$

Assess: $y_2 = \Delta s \sin(30°) = 16$ m, which is greater than $y_0 = 10$ m. The higher value is due to the transformation of the spring energy into gravitational potential energy.

10.72. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, hence the mechanical energy $K + U_g + U_s$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system at the point of maximum compression. We will use lengths along the ramp with the variable *s* rather than *x*.

Solve: (a) The conservation of energy equation $K_2 + U_{g2} + U_{s2} = K_1 + U_{g1} + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + k(0 \text{ m})^2$$

$$\frac{1}{2}m(0 \text{ m/s})^2 + mg(0 \text{ m}) + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}m(0 \text{ m/s})^2 + mg(4.0 \text{ m} + \Delta s)\sin 30° + 0 \text{ J}$$

$$\frac{1}{2}(250 \text{ N/m})(\Delta s)^2 = (10 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m} + \Delta s)\left(\frac{1}{2}\right)$$

This gives the quadratic equation:

$$(125 \text{ N/m})(\Delta s)^2 - (49 \text{ kg}\cdot\text{m/s}^2)\Delta s - 196 \text{ kg}\cdot\text{m}^2/\text{s}^2 = 0$$

$$\Rightarrow \Delta s = 1.46 \text{ m and } -1.07 \text{ m (unphysical)}$$

The maximum compression is 1.46 m which rounds to 1.5 m.

(b) [See next page for simpler solution by Samir!!] We will now apply the conservation of mechanical energy to a point where the vertical position is *y* and the block's velocity is *v*. We place the origin of our coordinate system on the free end of the spring when the spring is neither compressed nor stretched.

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}k(0 \text{ m})^2$$

$$\frac{1}{2}mv^2 + mg(-\Delta s \sin 30°) + \frac{1}{2}k(\Delta s)^2 = 0 \text{ J} + mg(4.0 \text{ m} \sin 30°) + 0 \text{ J}$$

$$\frac{1}{2}k(\Delta s)^2 - (mg \sin 30°)\Delta s + \frac{1}{2}mv^2 - mg \sin 30° (4.0 \text{ m}) = 0$$

To find the compression where *v* is maximum, take the derivative of this equation with respect to Δs:

$$\frac{1}{2}k\, 2(\Delta s) - (mg \sin 30°) + \frac{1}{2}m\, 2v\frac{dv}{d\Delta s} - 0 = 0$$

Since $\frac{dv}{d\Delta s} = 0$ at the maximum, we have

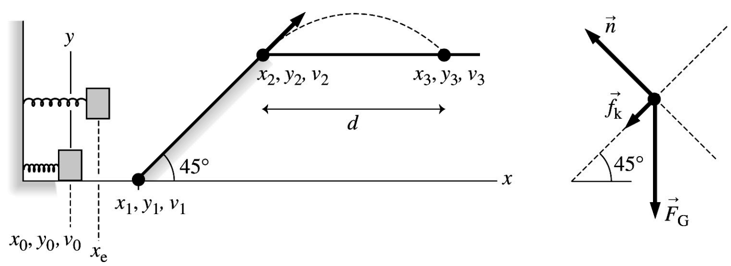
$$\Delta s = (mg \sin 30°)/k = (10 \text{ kg})(9.8 \text{ m/s}^2)(0.5)/(250 \text{ N/m}) = 19.6 \text{ cm}$$

The last line of the solution to part (b) above can be written down by physical reasoning (same reasoning we used to solve Whiteboard problem 09/10-11 in class (in Lecture9_10_2 posted on Oct 21, slides #10 and 11) see course schedule:

The block continues to slide down with an increasing speed even after striking the spring, just that now the increase in speed is not as rapid. The acceleration of the block down the slope, before striking the spring, was $g\sin 30^0$, and the force causing this acceleration was $mg\sin 30^0$. After striking the spring, the net force down the slope is $F_{net} = mg\sin 30^0 - kx$, where x is the compression of the spring, and the net acceleration down the slope, using $F = ma$, is $a = F_{net}/m = g\sin 30^0 - kx/m$. The block continues to speed up, even though a decreases (because of the increasing magnitude of x and therefore kx), and only begins to slow down when a becomes negative. Thus, the maximum speed is attained precisely when $a = 0$! That happens when $g\sin 30^0 - kx/m = 0$, i.e., when $x = mg\sin 30^0 / k = 19.6$ cm (same as long-winded math soln on previous pg!)

10.73. Model: Assume the spring to be ideal that obeys Hooke's law, and model the block as a particle.

Visualize: We place the origin of the coordinate system on the free end of the compressed spring which is in contact with the block. Because the horizontal surface at the bottom of the ramp is frictionless, the spring energy appears as kinetic energy of the block until the block begins to climb up the incline.



Solve: Although we could find the speed v_1 of the block as it leaves the spring, we don't need to. We can use energy conservation to relate the initial potential energy of the spring to the energy of the block as it begins projectile motion at point 2. However, friction requires us to calculate the increase in thermal energy. The energy equation is

$$K_2 + U_{g2} + \Delta E_{th} = K_0 + U_{g0} + W_{ext} \quad \Rightarrow \quad \tfrac{1}{2}mv_2^2 + mgy_2 + f_k \Delta s = \tfrac{1}{2}k(x_0 - x_e)^2$$

The distance along the slope is $\Delta s = y_2/\sin(45°)$. The friction force is $f_k = \mu_k n$, and we can see from the free-body diagram that $n = mg\cos(45°)$. Thus

$$v_2 = \sqrt{\frac{k}{m}(x_0 - x_e)^2 - 2gy_2 - 2\mu_k g y_2 \cot(45°)}$$

$$= \left[\frac{1000 \text{ N/m}}{0.20 \text{ kg}}(0.15 \text{ m})^2 - 2(9.8 \text{ m/s}^2)(2.0 \text{ m}) - 2(0.20)(9.8 \text{ m/s}^2)(2.0 \text{ m})\cot(45°)\right]^{1/2} = 8.091 \text{ m/s}$$

Having found the velocity v_2, we can now find $(x_3 - x_2) = d$ using the kinematic equations of projectile motion:

$$y_3 = y_2 + v_{2y}(t_3 - t_2) + \frac{1}{2}a_{2y}(t_3 - t_2)^2$$

$$2.0 \text{ m} = 2.0 \text{ m} + v_2 \sin(45°)(t_3 - t_2) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_3 - t_2)^2$$

$$t_3 - t_2 = 0 \text{ s and } 1.168 \text{ s}$$

And…finally,

$$x_3 = x_2 + v_{2x}(t_3 - t_2) + \frac{1}{2}a_{2x}(t_3 - t_2)^2$$

$$d = (x_3 - x_2) = v_2 \cos(45°)(1.168 \text{ s}) + 0 \text{ m} = 6.7 \text{ m}$$