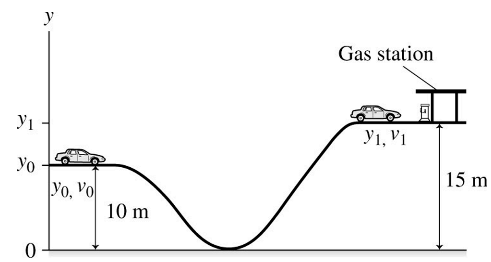
Solutions to HW16, Chapter 10

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

10.11. Model: Model the car as a particle with zero rolling friction and no air resistance. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion.
Visualize:



Solve: The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500\text{ kg})(10.0\text{ m/s})^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 2.22\times10^5\text{ J}$$

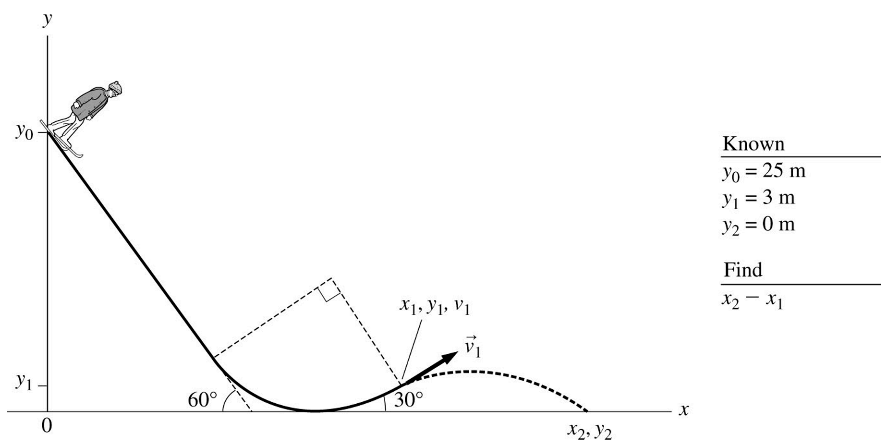
The car increases its height to 15 m at the gas station. The conservation of energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ is

$$2.22\times10^5\text{ J} = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow 2.22\times10^5\text{ J} = \frac{1}{2}(1500\text{ kg})v_1^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})$$

$$\Rightarrow v_1 = 1.4\text{ m/s}$$

Assess: A lower speed at the gas station is reasonable because the car has decreased its kinetic energy and increased its potential energy compared to its starting values.

10.44. Model: Since there is no friction, the sum of the kinetic and gravitational potential energy does not change. Model Julie as a particle.
Visualize:



We place the coordinate system at the bottom of the ramp directly below Julie's starting position. From geometry, Julie launches off the end of the ramp at a 30º angle.

Solve: Energy conservation: $K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

Using $v_0 = 0$ m/s, $y_0 = 25$ m, and $y_1 = 3$ m, the above equation simplifies to

$$\frac{1}{2}mv_1^2 + mgy_1 = mgy_0 \Rightarrow v_1 = \sqrt{2g(y_0 - y_1)} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m} - 3\text{ m})} = 20.77\text{ m/s}$$

We can now use kinematic equations to find the touchdown point from the base of the ramp. First we'll consider the vertical motion:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \frac{1}{2}a_y(t_2 - t_1)^2 \quad 0\text{ m} = 3\text{ m} + (v_1\sin 30°)(t_2 - t_1) + \frac{1}{2}(-9.8\text{ m/s}^2)(t_2 - t_1)^2$$

$$\Rightarrow (t_2 - t_1)^2 - \frac{(20.77\text{ m/s})\sin 30°}{(4.9\text{ m/s}^2)}(t_2 - t_1) - \frac{(3\text{ m})}{(4.9\text{ m/s}^2)} = 0$$

$$(t_2 - t_1)^2 - (2.119\text{ s})(t_2 - t_1) - (0.6122\text{ s}^2) = 0 \Rightarrow (t_2 - t_1) = 2.377\text{ s}$$

For the horizontal motion:

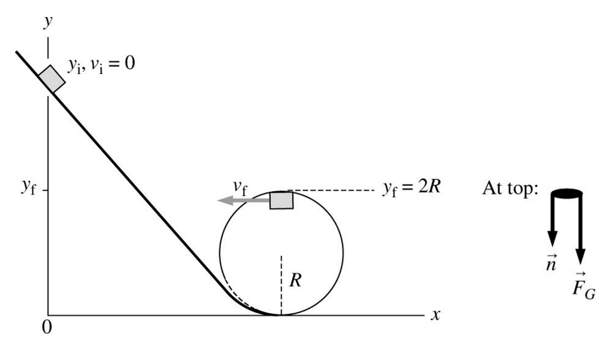
$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

Error! Objects cannot be created from editing field codes.

Assess: Note that we did not have to make use of the information about the circular arc at the bottom that carries Julie through a $90°$ turn.

10.45. Model: This is a two-part problem. In the first part, we will find the critical velocity for the block to go over the top of the loop without falling off. Since there is no friction, the sum of the kinetic and gravitational potential energy is conserved during the block's motion. We will use this conservation equation in the second part to find the minimum height the block must start from to make it around the loop.

Visualize:



We place the origin of our coordinate system directly below the block's starting position on the frictionless track.

Solve: The free-body diagram on the block implies

$$F_G + n = \frac{mv_c^2}{R}$$

For the block to just stay on track, $n = 0$. Thus the critical velocity v_c is

$$F_G = mg = \frac{mv_c^2}{R} \Rightarrow v_c^2 = gR$$

The block needs kinetic energy $\frac{1}{2}mv_c^2 = \frac{1}{2}mgR$ to go over the top of the loop. We can now use the conservation of mechanical energy equation to find the minimum height *h*.

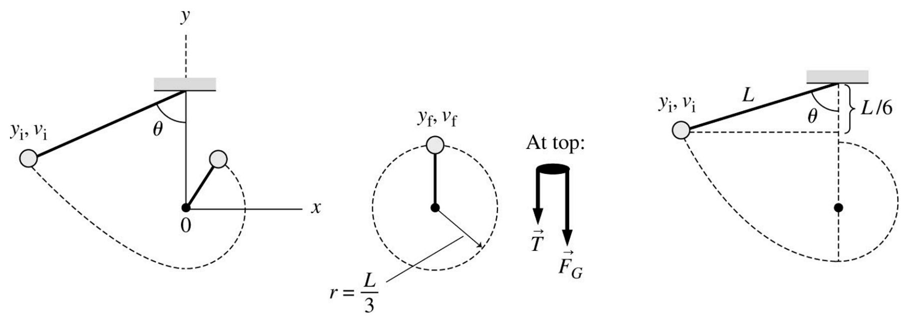
$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c = \sqrt{gR}$, $y_f = 2R$, $v_i = 0$ m/s, and $y_i = h$, we obtain

$$\frac{1}{2}gR + g(2R) = 0 + gh \Rightarrow h = 2.5R$$

10.69. Model: This is a two-part problem. In the first part, we will find the critical velocity for the ball to go over the top of the peg without the string going slack. Using the energy conservation equation, we will then obtain the gravitational potential energy that gets transformed into the critical kinetic energy of the ball, thus determining the angle θ.

Visualize:



We place the origin of our coordinate system on the peg. This choice will provide a reference to measure gravitational potential energy. For θ to be minimum, the ball will just go over the top of the peg.

Solve: The two forces in the free-body force diagram provide the centripetal acceleration at the top of the circle. Newton's second law at this point is

$$F_G + T = \frac{mv^2}{r}$$

where T is the tension in the string. The critical speed v_c at which the string goes slack is found when $T \to 0$. In this case,

$$mg = \frac{mv_C^2}{r} \Rightarrow v_C^2 = gr = gL/3$$

The ball should have kinetic energy at least equal to

$$\frac{1}{2}mv_C^2 = \frac{1}{2}mg\left(\frac{L}{3}\right)$$

for the ball to go over the top of the peg. We will now use the conservation of mechanical energy equation to get the minimum angle θ. The equation for the conservation of energy is

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c, y_f = \frac{1}{3}L, v_i = 0,$ and the above value for v_C^2, we get

$$\frac{1}{2}mg\frac{L}{3} + mg\frac{L}{3} = mgy_i \Rightarrow y_i = \frac{L}{2}$$

That is, the ball is a vertical distance $\frac{1}{2}L$ above the peg's location or a distance of

$$\left(\frac{2L}{3} - \frac{L}{2}\right) = \frac{L}{6}$$

below the point of suspension of the pendulum, as shown in the figure on the right. Thus,

$$\cos\theta = \frac{L/6}{L} = \frac{1}{6} \Rightarrow \theta = 80.4°$$