## Solutions to HW13, Chapter 8

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!
8.25. Model: Model the ball as a particle that is moving in a vertical circle.

Visualize:

## Pictorial representation

Known

| $m=500 \mathrm{~g}$ |
| :--- |
| $r=102 \mathrm{~cm}$ |
| $v($ Top $)=4.0 \mathrm{~m} / \mathrm{s}$ |
| $v($ Bottom $)=7.5 \mathrm{~m} / \mathrm{s}$ |
| Find |
| $F_{\mathrm{G}} T_{1} T_{2}$ |





Bottom

Solve: (a) The ball's gravitational force $F_{\mathrm{G}}=m g=(0.500 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=4.9 \mathrm{~N}$.
(b) Newton's second law at the top is

$$
\begin{gathered}
\sum F_{r}=T_{1}+F_{\mathrm{G}}=m a_{r}=m \frac{v^{2}}{r} \\
\Rightarrow T_{1}=m\left(\frac{v^{2}}{r}-g\right)=(0.500 \mathrm{~kg})\left[\frac{(4.0 \mathrm{~m} / \mathrm{s})^{2}}{1.02 \mathrm{~m}}-9.8 \mathrm{~m} / \mathrm{s}^{2}\right]=2.9 \mathrm{~N}
\end{gathered}
$$

(c) Newton's second law at the bottom is

$$
\begin{gathered}
\sum F_{r}=T_{2}-F_{\mathrm{G}}=\frac{m v^{2}}{r} \\
\Rightarrow T_{2}=m\left(g+\frac{v^{2}}{r}\right)=(0.500 \mathrm{~kg})\left[9.8 \mathrm{~m} / \mathrm{s}^{2}+\frac{(7.5 \mathrm{~m} / \mathrm{s})^{2}}{1.02 \mathrm{~m}}\right]=32 \mathrm{~N}
\end{gathered}
$$

8.46. Model: Use the particle model for a sphere revolving in a horizontal circle. Visualize:

Pictorial representation


Solve: Newton's second law in the $r$ - and $z$-directions is

$$
\sum(F)_{r}=T_{1} \cos 30^{\circ}+T_{2} \cos 30^{\circ}=\frac{m v_{t}^{2}}{r} \quad \sum(F)_{z}=T_{1} \sin 30^{\circ}-T_{2} \sin 30^{\circ}-F_{\mathrm{G}}=0 \mathrm{~N}
$$

Using $r=(1.0 \mathrm{~m}) \cos 30^{\circ}=0.886 \mathrm{~m}$, these equations become

$$
\begin{aligned}
& T_{1}+T_{2}=\frac{m v_{t}^{2}}{r \cos 30^{\circ}}=\frac{(0.300 \mathrm{~kg})(7.5 \mathrm{~m} / \mathrm{s})^{2}}{(0.866 \mathrm{~m})(0.866)}=22.5 \mathrm{~N} \\
& T_{1}-T_{2}=\frac{m g}{\sin 30^{\circ}}=\frac{(0.300 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}{(0.5)}=5.88 \mathrm{~N}
\end{aligned}
$$

Solving for $T_{1}$ and $T_{2}$ yields $T_{1}=14.2 \mathrm{~N} \approx 14 \mathrm{~N}$ and $T_{2}=8.3 \mathrm{~N}$.
8.53. Model: Model the ball as a particle in uniform circular motion. Rolling friction is ignored. Visualize:

## Pictorial representation



Solve: The track exerts both an upward normal force and an inward normal force. From Newton's second law, $n_{1}=m g=(0.030 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=0.294 \mathrm{~N}$, up

$$
n_{2}=m r \omega^{2}=(0.030 \mathrm{~kg})(0.20 \mathrm{~m})\left[\frac{60 \mathrm{rev}}{\mathrm{~min}} \times \frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}} \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right]^{2}=0.2369 \mathrm{~N}, \text { in }
$$

$$
F_{\mathrm{net}}=\sqrt{n_{1}^{2}+n_{2}^{2}}=\sqrt{(0.294 \mathrm{~N})^{2}+(0.2369 \mathrm{~N})^{2}}=0.38 \mathrm{~N}
$$

8.57. Model: Use the particle model for a ball in motion in a vertical circle and then as a projectile. Visualize:

Pictorial representation



At top of circle

Solve: For the circular motion, Newton's second law along the $r$-direction is

$$
\sum F_{r}=T+F_{\mathrm{G}}=\frac{m v_{t}^{2}}{r}
$$

Since the string goes slack as the particle makes it over the top, $T=0 \mathrm{~N}$. That is,

$$
F_{\mathrm{G}}=m g=\frac{m v_{t}^{2}}{r} \Rightarrow v_{t}=\sqrt{g r}=\sqrt{\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.5 \mathrm{~m})}=2.21 \mathrm{~m} / \mathrm{s}
$$

The ball begins projectile motion as the string is released. The time it takes for the ball to hit the floor can be found as follows:

$$
y_{1}=y_{0}+v_{0 y}\left(t_{1}-t_{0}\right)+\frac{1}{2} a_{y}\left(t_{1}-t_{0}\right)^{2} \Rightarrow 0 \mathrm{~m}=2.0 \mathrm{~m}+0 \mathrm{~m}+\frac{1}{2}\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left(t_{1}-0 \mathrm{~s}\right)^{2} \Rightarrow t_{1}=0.639 \mathrm{~s}
$$

The place where the ball hits the ground is

$$
x_{1}=x_{0}+v_{0 x}\left(t_{1}-t_{0}\right)=0 \mathrm{~m}+(+2.21 \mathrm{~m} / \mathrm{s})(0.639 \mathrm{~s}-0 \mathrm{~s})=+1.41 \mathrm{~m}
$$

The ball hits the ground 1.4 m to the right of the point beneath the center of the circle.
8.69. Model: Use the particle model for the ball, which is in uniform circular motion. Visualize:



Solve: From Newton's second law along $r$ and $z$ directions,

$$
\sum F_{r}=n \cos \theta=\frac{m v^{2}}{r} \quad \sum F_{z}=n \sin \theta-m g=0 \Rightarrow n \sin \theta=m g
$$

Dividing the two force equations gives

$$
\tan \theta=\frac{g r}{v^{2}}
$$

From the geometry of the cone, $\tan \theta=r / y$. Thus

$$
\frac{r}{y}=\frac{g r}{v^{2}} \Rightarrow v=\sqrt{g y}
$$

