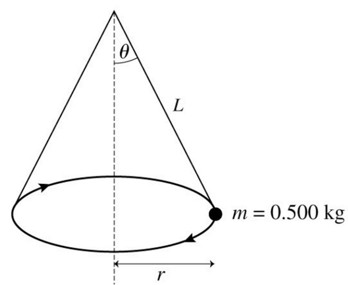
Solutions to HW12, Chapter 8

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

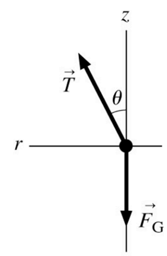
8.47. Use the particle model for the ball, which is undergoing uniform circular motion.
Visualize: We are given L, r, and m, so our answers must be in terms of those variables. L is the hypotenuse of the right triangle. The ball moves in a *horizontal* circle of radius $r = L\cos\theta$. The acceleration and net force point toward the center of the circle, not along the string.

Pictorial representation



Known
$L = 1.0$ m $\quad r = 0.20$ m
$\theta = \sin^{-1}(r/L) = 11.54°$
$m = 0.500$ kg

Find
T, ω



Solve:
(a) Apply Newton's second law in the z-direction.

$$\sum F_z = T\cos\theta - mg = 0 \Rightarrow T = \frac{mg}{\cos\theta}$$

From the right triangle $\cos\theta = \sqrt{L^2 - r^2}/L$.

$$T = \frac{mg}{\cos\theta} = \frac{mgL}{\sqrt{L^2 - r^2}}$$

(b) Apply Newton's second law in the r-direction.

$$\sum F_r = T\sin\theta = m\omega^2 r = m\omega^2(L\sin\theta) \Rightarrow T = m\omega^2 L.$$

Set the two expressions for T equal to each other, cancel m and one L, and solve for ω.

$$\frac{mgL}{\sqrt{L^2 - r^2}} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$$

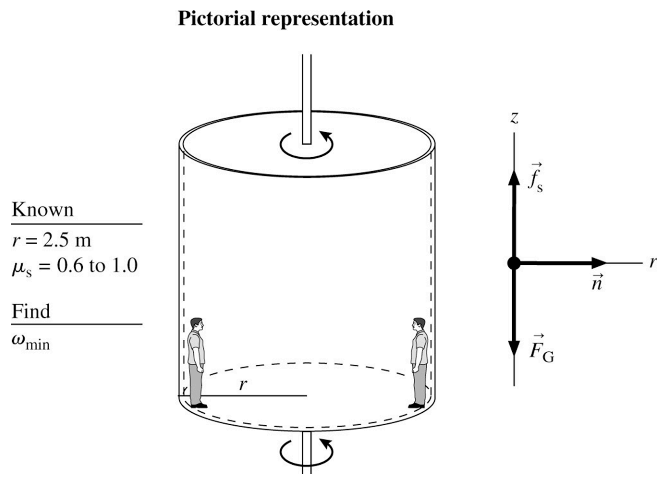
(c) Insert $L = 1.0$ m, $r = 0.20$ m and $m = 0.50$ kg.

$$T = \frac{mgL}{\sqrt{L^2 - r^2}} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}} = 5.0\text{ N}$$

$$\omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}} = \sqrt{\frac{9.8\text{ m/s}^2}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}}} = 3.163\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 30\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation for ω, but not for T, so the 500 g was necessary information.

8.49. Model: Consider the passenger to be a particle and use the model of static friction.
Visualize:



Solve: The passengers stick to the wall if the static friction force is sufficient to support the gravitational force on them: $f_s = F_G$. The minimum angular velocity occurs when static friction reaches its maximum possible value $(f_s)_{max} = \mu_s n$. Although clothing has a range of coefficients of friction, it is the clothing with the smallest coefficient $(\mu_s = 0.60)$ that will slip first, so this is the case we need to examine. Assuming that the person is stuck to the wall, Newton's second law is

$$\Sigma F_r = n = m\omega^2 r \qquad \Sigma F_z = f_s - w = 0 \Rightarrow f_s = mg$$

The minimum frequency occurs when

$$f_s = (f_s)_{max} = \mu_s n = \mu_s m r \omega_{min}^2$$

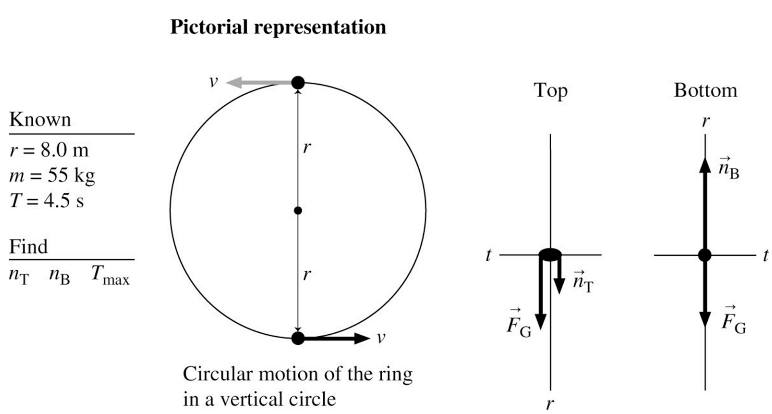
Using this expression for f_s in the z-equation gives

$$f_s = \mu_s m r \omega_{min}^2 = mg$$

$$\Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.60(2.5\text{ m})}} = 2.56\text{ rad/s} = 2.56\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 24\text{ rpm}$$

Assess: Note the velocity does not depend on the mass of the individual. Therefore, the minimum mass sign is not necessary.

8.51. Model: Model a passenger as a particle rotating in a vertical circle.
Visualize:



Solve: **(a)** Newton's second law at the top is

$$\sum F_r = n_\text{T} + F_\text{G} = ma_r = \frac{mv^2}{r} \Rightarrow n_\text{T} + mg = \frac{mv^2}{r}$$

The speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(8.0 \text{ m})}{4.5 \text{ s}} = 11.17 \text{ m/s}$$

$$\Rightarrow n_\text{T} = m\left(\frac{v^2}{r} - g\right) = (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} - 9.8 \text{ m/s}^2\right] = 319 \text{ N}$$

That is, the ring pushes on the passenger with a force of 3.2×10^2 N at the top of the ride. Newton's second law at the bottom:

$$\sum F_r = n_\text{B} - F_\text{G} = ma_r = \frac{mv^2}{r} \Rightarrow n_\text{B} = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

$$= (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} + 9.8 \text{ m/s}^2\right] = 1397 \text{ N}$$

Thus the force with which the ring pushes on the rider when she is at the bottom of the ring is 1.4 kN.
(b) To just stay on at the top, $n_\text{T} = 0$ N in the r-equation at the top in part **(a)**. Thus,

$$mg = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T_\text{max}}\right)^2 \Rightarrow T_\text{max} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{8.0 \text{ m}}{9.8 \text{ m/s}^2}} = 5.7 \text{ s}$$