## Solutions to HW11, Chapter 7

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!
7.25. Model: The masses $m$ and $M$ are to be treated in the particle model. We will also assume a massless rope and frictionless pulley, and use the constant-acceleration kinematic equations for $m$ and $M$.
Visualize:

## Pictorial representation




Solve: Using $y_{1}=y_{0}+v_{0 y}\left(t_{1}-t_{0}\right)+\frac{1}{2} a_{\mathrm{M}}\left(t_{1}-t_{0}\right)^{2}$,

$$
(-1.0 \mathrm{~m})=0 \mathrm{~m}+0 \mathrm{~m}+\frac{1}{2} a_{\mathrm{M}}(6.0 \mathrm{~s}-0 \mathrm{~s})^{2} \Rightarrow a_{\mathrm{M}}=-0.0556 \mathrm{~m} / \mathrm{s}^{2}
$$

Newton's second law for $m$ and $M$ gives

$$
\sum\left(F_{\mathrm{on} \mathrm{~m}}\right)_{y}=T_{\mathrm{R} \text { on } \mathrm{m}}-\left(F_{\mathrm{G}}\right)_{\mathrm{m}}=m a_{\mathrm{m}} \quad \sum\left(F_{\mathrm{on} \mathrm{M}}\right)_{y}=T_{\mathrm{Ron} \mathrm{M}}-\left(F_{\mathrm{G}}\right)_{\mathrm{M}}=M a_{\mathrm{M}}
$$

The acceleration constraint is $a_{m}=-a_{M}$. Also, the tensions are an pseudo-action/reaction pair, so $T_{\mathrm{R} \text { on } \mathrm{m}}=T_{\mathrm{R} \text { on } \mathrm{M}}$. With these, the second-law equations become

$$
\begin{gathered}
T_{\mathrm{R} \text { on } \mathrm{M}}-M g=M a_{\mathrm{M}} \\
T_{\mathrm{R} \text { on } \mathrm{M}}-m g=-m a_{\mathrm{M}}
\end{gathered}
$$

Subtracting the second from the first gives

$$
\begin{aligned}
-M g+m g & =M a_{\mathrm{M}}+m a_{\mathrm{M}} \\
m & =M\left[\frac{g+a_{\mathrm{M}}}{g-a_{\mathrm{M}}}\right] \\
& =(100 \mathrm{~kg})\left[\frac{9.8 \mathrm{~m} / \mathrm{s}^{2}-0.556 \mathrm{~m} / \mathrm{s}^{2}}{9.8 \mathrm{~m} / \mathrm{s}^{2}+0.556 \mathrm{~m} / \mathrm{s}^{2}}\right]=99 \mathrm{~kg}
\end{aligned}
$$

Assess: Note that $a_{\mathrm{m}}=-a_{\mathrm{M}}=0.0556 \mathrm{~m} / \mathrm{s}^{2}$. For such a small acceleration, the $1 \%$ mass difference seems reasonable.
7.41. Model: Use the particle model for the book (B) and the coffee cup (C), the models of kinetic and static friction, and the constant-acceleration kinematic equations.
Visualize:


Solve: (a) Using $v_{1 x}^{2}=v_{0 x}^{2}+2 a\left(x_{1}-x_{0}\right)$, we find

$$
0 \mathrm{~m}^{2} / \mathrm{s}^{2}=(3.0 \mathrm{~m} / \mathrm{s})^{2}+2 a\left(x_{1}\right) \Rightarrow a x_{1}=-4.5 \mathrm{~m}^{2} / \mathrm{s}^{2}
$$

To find $x_{1}$, we must first find $a$. Newton's second law applied to the book and the coffee cup gives

$$
\begin{gathered}
\sum\left(F_{\mathrm{on} \mathrm{~B}}\right)_{y}=n_{\mathrm{B}}-\left(F_{\mathrm{G}}\right)_{\mathrm{B}} \cos \left(20^{\circ}\right)=0 \mathrm{~N} \Rightarrow n_{\mathrm{B}}=(1.0 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \cos \left(20^{\circ}\right)=9.21 \mathrm{~N} \\
\sum\left(F_{\mathrm{on} \mathrm{~B}}\right)_{x}=-T-f_{\mathrm{k}}-\left(F_{\mathrm{G}}\right)_{\mathrm{B}} \sin \left(20^{\circ}\right)=m_{\mathrm{B}} a_{\mathrm{B}} \quad \sum\left(F_{\mathrm{on} \mathrm{C}}\right)_{y}=T-\left(F_{\mathrm{G}}\right)_{\mathrm{C}}=m_{\mathrm{C}} a_{\mathrm{C}}
\end{gathered}
$$

The last two equations can be rewritten, using $a_{\mathrm{C}}=a_{\mathrm{B}}=a$, as

$$
-T-\mu_{\mathrm{k}} n_{\mathrm{B}}-m_{\mathrm{B}} g \sin \left(20^{\circ}\right)=m_{\mathrm{B}} a \quad T-m_{\mathrm{C}} g=m_{\mathrm{C}} a
$$

Adding the two equations gives

$$
a\left(m_{\mathrm{C}}+m_{\mathrm{B}}\right)=-g\left[m_{\mathrm{C}}+m_{\mathrm{B}} \sin \left(20^{\circ}\right)\right]-\mu_{\mathrm{k}}(9.21 \mathrm{~N})
$$

$$
(1.5 \mathrm{~kg}) a=-\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left[0.500 \mathrm{~kg}+(1.0 \mathrm{~kg}) \sin 20^{\circ}\right]-(0.20)(9.21 \mathrm{~N}) \Rightarrow a=-6.73 \mathrm{~m} / \mathrm{s}^{2}
$$

Using this value for $a$, we can now find $x_{1}$ as follows:

$$
x_{1}=\frac{-4.5 \mathrm{~m}^{2} / \mathrm{s}^{2}}{a}=\frac{-4.5 \mathrm{~m}^{2} / \mathrm{s}^{2}}{-6.73 \mathrm{~m} / \mathrm{s}^{2}}=0.67 \mathrm{~m}
$$

(b) The maximum static friction force is $\left(f_{\mathrm{s}}\right)_{\max }=\mu_{\mathrm{S}} n_{\mathrm{B}}=(0.50)(9.21 \mathrm{~N})=4.60 \mathrm{~N}$. We'll see if the force $f_{\mathrm{s}}$ needed to keep the book in place is larger or smaller than $\left(f_{\mathrm{s}}\right)_{\max }$. When the cup is at rest, the string tension is $T=m_{\mathrm{C}} g$. Newton's first law for the book is

$$
\begin{aligned}
\sum\left(F_{\mathrm{on} \mathrm{~B}}\right)_{x} & =f_{\mathrm{s}}-T-w_{\mathrm{B}} \sin \left(20^{\circ}\right)=f_{\mathrm{s}}-m_{\mathrm{C}} g-m_{\mathrm{B}} g \sin \left(20^{\circ}\right)=0 \\
f_{\mathrm{s}} & =\left(M_{\mathrm{C}}+M_{\mathrm{B}} \sin 20^{\circ}\right) g=8.25 \mathrm{~N}
\end{aligned}
$$

Because $f_{\mathrm{s}}>\left(f_{\mathrm{s}}\right)_{\max }$, the book slides back down.
7.45. Model: The painter and the chair are treated as a single object and represented as a particle. We assume that the rope is massless and that the pulley is massless and frictionless.
Visualize:
Pictorial representation


Solve: If the painter pulls down on the rope with force $F$, Newton's third law requires the rope to pull up on the painter with force $F$. This is just the tension in the rope. With our model of the rope and pulley, the same tension force $F$ also pulls up on the painter's chair. Newton's second law for (painter + chair) gives

$$
\begin{aligned}
2 F-F_{\mathrm{G}} & =\left(m_{\mathrm{P}}+m_{\mathrm{C}}\right) a \\
F & =\left(\frac{1}{2}\right)\left[\left(m_{\mathrm{P}}+m_{\mathrm{C}}\right) a+\left(m_{\mathrm{P}}+m_{\mathrm{C}}\right) g\right]=\frac{1}{2}\left(m_{\mathrm{P}}+m_{\mathrm{C}}\right)(a+g) \\
& =\left(\frac{1}{2}\right)(70 \mathrm{~kg}+10 \mathrm{~kg})\left(0.20 \mathrm{~m} / \mathrm{s}^{2}+9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=4.0 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

Assess: A force of 400 N , which is approximately one-half the total gravitational force, is reasonable since the upward acceleration is small.
CAN YOU ALSO FIGURE OUT THE FORCE WITH WHICH THE CHAIR PUSHES ON THE PAINTER? (in other words, the normal force between the chair and painter)
7.53. Model: Blocks 1 and 2 make up the system of interest and will be treated as particles. Assume a massless rope and frictionless pulley.
Visualize:

## Pictorial representation



Solve: The blocks accelerate with the same magnitude but in opposite directions. Thus the acceleration constraint is $a_{2}=a=-a_{1}$, where a will have a positive value. There are two real action/reaction pairs. The two
tension forces will act as if they are action/reaction pairs because we are assuming a massless rope and a frictionless pulley. Make sure you understand why the friction forces point in the directions shown in the freebody diagrams, especially force $\hat{f}_{1}^{\prime}$ exerted on block 2 by block 1 . We have quite a few pieces of information to include. First, Newton's second law applied to blocks 1 and 2 gives

$$
\begin{gathered}
\left({\stackrel{F}{\text { net on } 1})_{x}=f_{1}-T_{1}=\mu_{\mathrm{k}} n_{1}-T_{1}=m_{1} a_{1}=-m_{1} a}_{\left(F_{\text {net on 1 }}\right)_{y}=n_{1}-m_{1} g=0 \mathrm{~N} \Rightarrow n_{1}=m_{1} g}^{\left(F_{\text {net on 2 }}\right)_{x}=T_{\text {pull }}-f_{1}^{\prime}-f_{2}-T_{2}=T_{\text {pull }}-f_{1}^{\prime}-\mu_{\mathrm{k}} n_{2}-T_{2}=m_{2} a_{2}=m_{2} a}\right. \\
\left(F_{\text {net on 2 } 2}\right)_{y}=n_{2}-n_{1}^{\prime}-m_{2} g=0 \mathrm{~N} \Rightarrow n_{2}=n_{1}^{\prime}+m_{2} g
\end{gathered}
$$

We've already used the kinetic friction model in both $x$-equations. Next, Newton's third law gives

$$
n_{1}^{\prime}=n_{1}=m_{1} g \quad f_{1}^{\prime}=f_{1}=\mu_{\mathrm{k}} n_{1}=\mu_{\mathrm{k}} m_{1} g \quad T_{1}=T_{2}=T
$$

Knowing $n_{1}^{\prime}$, we can now use the $y$-equation of block 2 to find $n_{2}$. Substitute all these pieces into the two $x$-equations, and we end up with two equations with two unknowns:

$$
\mu_{\mathrm{k}} m_{1} g-T=-m_{1} a \quad T_{\text {pull }}-T-\mu_{\mathrm{k}} m_{1} g-\mu_{\mathrm{k}}\left(m_{1}+m_{2}\right) g=m_{2} a
$$

Subtract the first equation from the second to get

$$
\begin{aligned}
& T_{\text {pull }}-\mu_{\mathrm{k}}\left(3 m_{1}+m_{2}\right) g=\left(m_{1}+m_{2}\right) a \\
& a=\frac{T_{\text {pull }}-\mu_{\mathrm{k}}\left(3 m_{1}+m_{2}\right) g}{m_{1}+m_{2}}=\frac{20 \mathrm{~N}-(0.30)[3(1.0 \mathrm{~kg})+2.0 \mathrm{~kg}]\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}{1.0 \mathrm{~kg}+2.0 \mathrm{~kg}}=1.8 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

7.54. Model: Use the particle model for the two blocks. Assume a massless rope and massless, frictionless pulleys.
Visualize:

## Pictorial representation



Note that for every meter block 1 moves forward, one meter is provided to block 2 . So each rope on $m_{2}$ has to be lengthened by one-half meter. Thus, the acceleration constraint is $a_{2}=-\frac{1}{2} a_{1}$.
Solve: Newton's second law applied to block 1 gives $T=m_{1} a_{1}$. Newton's second law applied to block 2 gives $2 T-\left(F_{\mathrm{G}}\right)_{2}=m_{2} a_{2}$. Combining these two equations gives

$$
2\left(m_{1} a_{1}\right)-m_{2} g=m_{2}\left(-\frac{1}{2} a_{1}\right) \Rightarrow a_{1}\left(4 m_{1}+m_{2}\right)=2 m_{2} g \Rightarrow a_{1}=\frac{2 m_{2} g}{4 m_{1}+m_{2}}
$$

where we have used $a_{2}=-\frac{1}{2} a_{1}$.
Assess: If $m_{1}=0 \mathrm{~kg}$, then $a_{2}=-g$. This is what is expected for a freely falling object.

