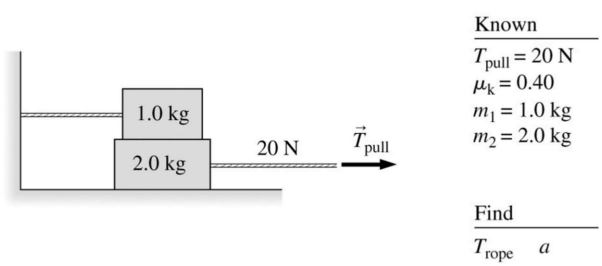
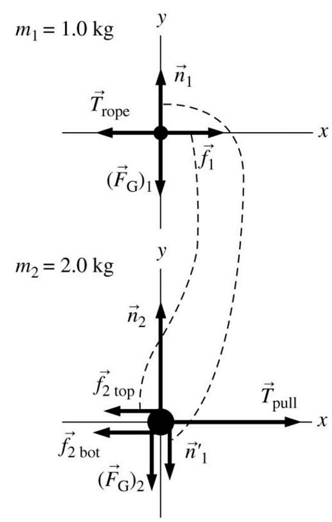
Solutions to HW10, Chapter 7

NOTE! The problems in masteringphysics.com had their numbers altered slightly for each individual student. The solutions below use the same numbers as those used in the book for that problem!

7.24. Model: The two blocks (1 and 2) form the system of interest and will be treated as particles. The ropes are assumed to be massless, and the model of kinetic friction will be used.
Visualize:

Pictorial representation





Solve: (a) The separate free-body diagrams for the two blocks show that there are two action/reaction pairs. Notice how block 1 both pushes down on block 2 (force $\vec{n}_1'$) and exerts a retarding friction force $\vec{f}_{2\text{ top}}$ on the top surface of block 2. Block 1 is in static equilibrium ($a_1 = 0$ m/s^2) but block 2 is accelerating to the right. Newton's second law for block 1 is

$$(F_{\text{net on 1}})_x = f_1 - T_{\text{rope}} = 0 \text{ N} \quad \Rightarrow \quad T_{\text{rope}} = f_1$$

$$(F_{\text{net on 1}})_y = n_1 - m_1 g = 0 \text{ N} \quad \Rightarrow \quad n_1 = m_1 g$$

Although block 1 is stationary, there is a *kinetic* force of friction because there is motion between blocks 1 and 2. The friction model means $f_1 = \mu_k n_1 = \mu_k m_1 g$. Substitute this result into the x-equation to get the tension in the rope:

$$T_{\text{rope}} = f_1 = \mu_k m_1 g = (0.40)(1.0 \text{ kg})(9.8 \text{ m/s}^2) = 3.9 \text{ N}$$

(b) Newton’s second law for block 2 is

$$a_x \equiv a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - f_{2\text{ top}} - f_{2\text{ bot}}}{m_2}$$

$$a_y = 0\ \text{m/s}^2 = \frac{(F_{\text{net on 2}})_y}{m_2} = \frac{n_2 - n_1' - m_2 g}{m_2}$$

Forces $\vec{n}_1$ and $\vec{n}_1'$ are an action/reaction pair, so $n_1' = n_1 = m_1 g$. Substituting into the y-equation gives $n_2 = (m_1 + m_2)g$. This is not surprising because the combined weight of both objects presses down on the surface. The kinetic friction on the bottom surface of block 2 is then

$$f_{2\text{ bot}} = \mu_k n_2 = \mu_k (m_1 + m_2) g$$

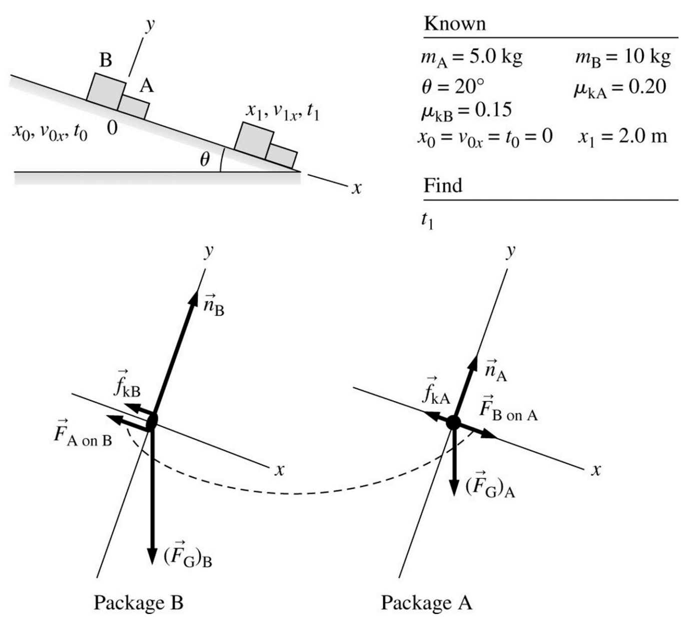
The forces $\vec{f}_1$ and $\vec{f}_{2\text{ top}}$ are an action/reaction pair, so $f_{2\text{ bot}} = f_1 = \mu_k m_1 g$. Inserting these friction results into the
x-equation gives

$$a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - \mu_k m_1 g - \mu_k (m_1 + m_2) g}{m_2}$$

$$= \frac{20\ \text{N} - (0.40)(1.0\ \text{kg})(9.8\ \text{m/s}^2) - (0.40)(1.0\ \text{kg} + 2.0\ \text{kg})(9.8\ \text{m/s}^2)}{2.0\ \text{kg}} = 2.2\ \text{m/s}^2$$

7.33. Model: Assume package A and package B are particles. Use the model of kinetic friction and the constant-acceleration kinematic equations.
Visualize:

Pictorial representation



Solve: Package B has a smaller coefficient of friction, so its acceleration down the ramp is greater than that of package A. It will therefore push against package A and, by Newton’s third law, package A will push back on B. The acceleration constraint is $a_A = a_B \equiv a$.
Newton’s second law applied to each package gives

$$\Sigma(F_{\text{on A}})_x = F_{\text{B on A}} + (F_G)_A \sin\theta - f_{kA} = m_A a$$

$$F_{\text{B on A}} + m_A g \sin\theta - \mu_{kA}(m_A g \cos\theta) = m_A a$$

$$\Sigma(F_{\text{on B}})_x = -F_{\text{A on B}} - f_{\text{kB}} + (F_G)_B \sin\theta = m_B a$$
$$-F_{\text{A on B}} - \mu_{\text{kB}}(m_B g\cos\theta) + m_B g\sin\theta = m_B a$$

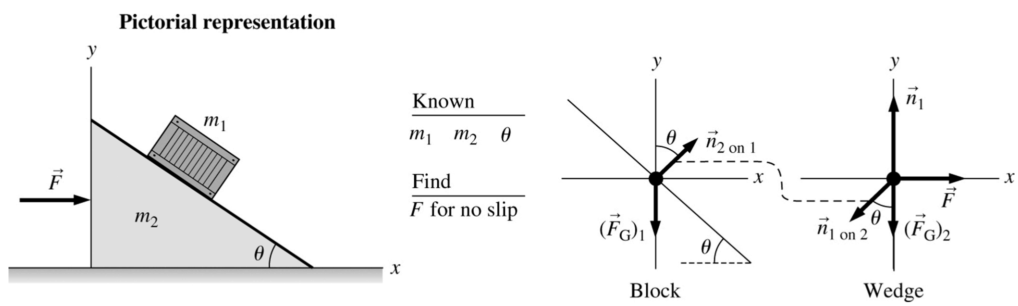
where we have used $n_A = m_A \cos\theta g$ and $n_B = m_B \cos\theta g$. Adding the two force equations, and using $F_{\text{A on B}} = F_{\text{B on A}}$ because they are an action/reaction pair, we get

$$a = g\sin\theta - \frac{(\mu_{\text{kA}} m_A + \mu_{\text{kB}} m_B)(g\cos\theta)}{m_A + m_B} = \frac{[(020)(5.0\text{ kg}) + (0.15)(10\text{ kg})](9.8\text{ m/s}^2)\cos(20°)}{5.0\text{ kg}+10\text{ kg}} = 1.82\text{ m/s}^2$$

Finally, using $x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2$, we find

$$2.0\text{ m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(1.82\text{ m/s}^2)(t_1 - 0\text{ s})^2 \quad\Rightarrow\quad t_1 = \sqrt{2(2.0\text{ m})/(1.82\text{ m/s}^2)} = 1.5\text{ s}$$

7.49. Model: Use the particle model for the wedge and the block.
Visualize:



The block will not slip relative to the wedge if they both have the same horizontal acceleration a. Note that $n_{1\text{ on }2}$ and $n_{2\text{ on }1}$ form a third-law pair, so $n_{1\text{ on }2} = n_{2\text{ on }1}$.

Solve: Newton's second law applied to block m_1 in the y-direction gives

$$\Sigma(F_{\text{on }1})_y = n_{2\text{ on }1}\cos\theta - (F_G)_1 = 0\text{ N} \quad\Rightarrow\quad n_{2\text{ on }1} = \frac{m_1 g}{\cos\theta}$$

Combining this equation with the x-component of Newton's second law yields:

$$\Sigma(F_{\text{on }1})_x = n_{2\text{ on }1}\sin\theta = m_1 a \quad\Rightarrow\quad a = \frac{n_{2\text{ on }1}\sin\theta}{m_1} = g\tan\theta$$

Newton's second law applied to the wedge-block system gives

$$F = m_1 a + m_2 a = (m_1 + m_2)a = (m_1 + m_2)g\tan\theta$$