

Also write name on back of exam in top right-hand corner

Exam 3, PHY 191B 11/29/16, 100pts

Name ROBERT H. GODDARD

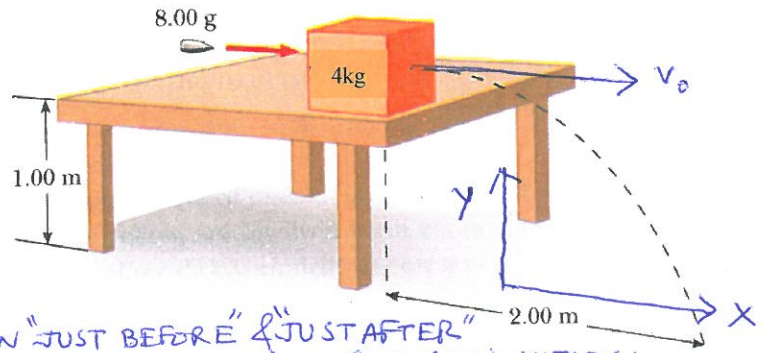
The use of a calculator and *provided* cheat-sheet is allowed.

Put your cellphone, laptop, and Apple Watch in your backpack and place the backpack at the front of the hall.

Anyone found in possession of these items, or any written information other than the provided cheat-sheet, will be expelled from the exam.

Part A (48 points) : 12 multiple-choice numerical questions. Each question is worth 4 points. SHOW REASONING CLEARLY! Correct answer w/o clear reasoning = ZERO credit!

1. An 8.00 gram bullet is fired into a 4 kg block initially at rest at the edge of a frictionless table of height 1.00 m, as shown below. The bullet remains in the block, and after impact the block lands 2.00 m from the bottom of the table. The initial speed of the bullet is



- a) 0.6 km/s b) 1.1 km/s
 c) 1.7 km/s d) 2.0 km/s
 e) 2.2 km/s

MOMENTUM CONSERVATION "JUST BEFORE" & "JUST AFTER" COLLISION YIELDS:

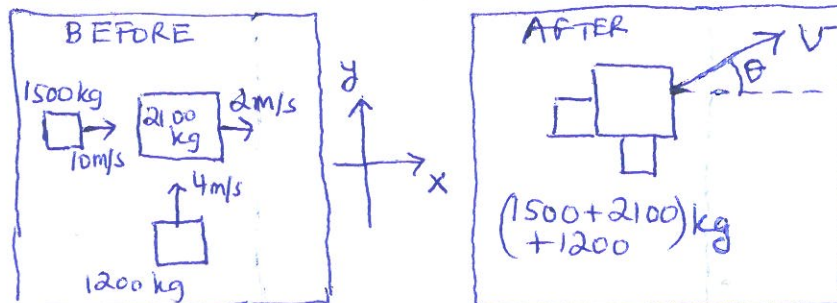
$$(8 \times 10^{-3}) v = (8 \times 10^{-3} + 4) v_0 \quad \text{--- (i)}$$

FIND!

To find v_0 , use projectile dynamics: Horizontal motion $\rightarrow v_0 \Delta t = 2$ --- (ii)
 & for vertical motion, $-1 = v_{oy} \Delta t + \frac{1}{2} (-9.8) (\Delta t)^2 \Rightarrow \Delta t = 0.45 \text{ sec}$.
 Substitute in eqn (ii) to find $v_0 = 2/0.45 = 4.44 \text{ m/s}$, which we substitute in (i) to find $v = (4.008)(4.44)/0.008 = 2224 \text{ m/s}$ or **2.2 km/s**

2. A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously from the side and the rear (!). One car is a 1200 kg compact traveling north at 4.0 m/s. the other is a 1500 kg midsize traveling east at 10 m/s. The three vehicles become entangled and slide as one body with a common final velocity. The direction in which the three entangled vehicles are sliding is

- a) 7° north of east b) 14° north of east c) 21° north of east d) 23° north of east
 e) 28° north of east



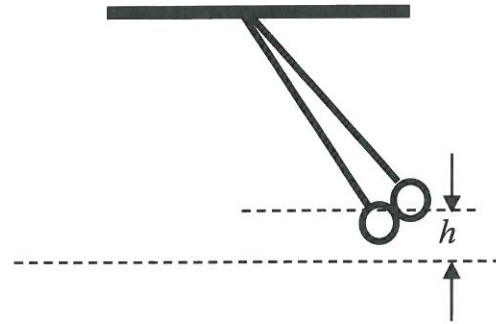
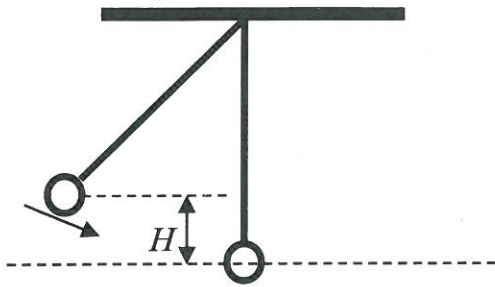
C.O.L.M. in 2D yields:

$$(P_i)_x = (P_f)_x \Rightarrow 1500(10) + 2100(2) = (1500 + 2100 + 1200)v \cos \theta \quad \text{--- (i)}$$

$$(P_i)_y = (P_f)_y \Rightarrow 1200(4) = (1500 + 2100 + 1200)v \sin \theta \quad \text{--- (ii)}$$

DIVIDE EQN (ii) by (i): $\frac{4800 v \sin \theta}{4800 v \cos \theta} = \frac{1200(4)}{1500(10) + 2100(2)} \Rightarrow \tan \theta = 0.25 \Rightarrow \theta = 14^\circ \text{ N of E}$ 1

3. Two identical masses are hung on strings of the same length. One mass is released from a height h above its free-hanging position and strikes the second mass; the two stick together and move off. What is the height h to which the stuck masses rise?



a) $H/4$

b) $H/2$

c) $3H/4$

d) H

IMMEDIATELY BEFORE COLLISION

Apply C.O.M.E.

$$\Delta U + \Delta K = W_{nc}$$

$$-mgH + \frac{1}{2}mv_{\text{before}}^2 = 0$$

$\Rightarrow v_{\text{before}} = \text{speed of left-hand mass just before striking stationary mass} = \sqrt{2gH}$

COLLISION

Apply C.O.L.M.

$$P_{\text{before}} = P_{\text{after}}$$

$$\Rightarrow m\sqrt{2gH} = (2m)v_{\text{after}}$$

$\Rightarrow v_{\text{after}} = \text{speed of stuck masses just after collision} = \frac{\sqrt{2gH}}{2}$

At height 'h'

Apply C.O.M.E.

$$\Delta K + \Delta U = W_{nc}$$

$$0 - \frac{1}{2}(2m)v_{\text{after}}^2 + (2m)gh = 0$$

$$\Rightarrow \frac{1}{2} \cdot \frac{2gH}{4} = gh$$

$$\Rightarrow \boxed{h = H/4}$$

4. A small block of mass $m_1 = 0.500 \text{ kg}$ is released from rest at the top of a curve-shaped frictionless wedge of mass $m_2 = 3.00 \text{ kg}$, which sits on a frictionless horizontal surface as in the figure (a) below. When the block leaves the wedge, its velocity is measured to be 4.00 m/s to the right, as in figure (b) (note that the wedge also has a velocity v_2 at that moment, as shown!). Use C. O. L. M. & C. O. M. E. considerations to find the height h of the wedge.

TIP: First find v_2 !

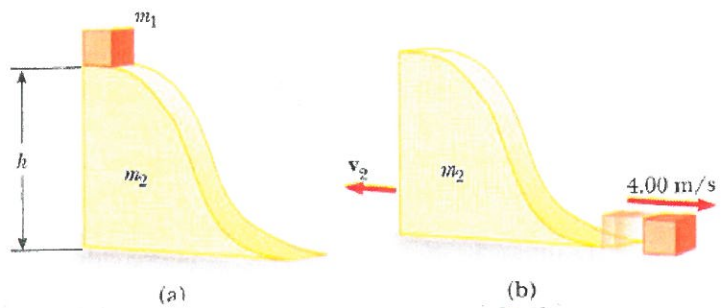
a) 0.70 m

b) 0.95 m

c) 1.20 m

d) 1.45 m

e) 1.70 m



C.O.L.M. : $p_i = p_f$

$$\Rightarrow 0 = 0.5(4) - 3(v_2)$$

$$\Rightarrow v_2 = \frac{2}{3} \text{ m/s}$$

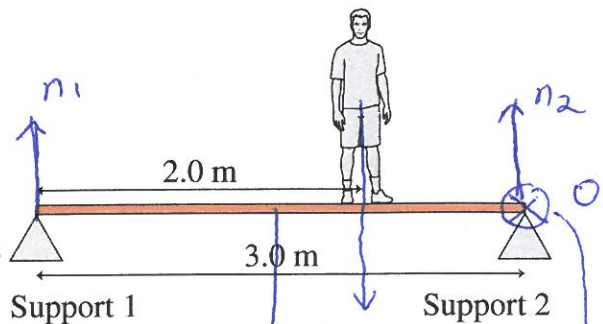
C.O.M.E. : $\Delta K + \Delta U = W_{nc} \Rightarrow \frac{1}{2}(0.5)4^2 + \frac{1}{2}(3)\left(\frac{2}{3}\right)^2 - (0.5)(9.8)h = 0$

$$\Rightarrow h = \frac{4.67}{4.9} = 0.95 \text{ m}$$

5. The figure shows a 3.0-m-long, 100 kg rigid beam supported at each end. An 80 kg student stands 2.0 m from support 1 in the figure. How much upward force does support 1 exert on the beam?

- a) 250 N b) 500 N
 c) 750 N d) 1000 N
 e) 1250 N

The student & beam are in static equilibrium. $\Rightarrow T_{net}$ about ANY PIVOT PT. is zero.



n_1 & n_2 are the unknown upward (normal) forces @ support 1 & support 2. We're

interested in calculating n_1 , but don't care about n_2 . So let's treat support 2 as a pivot point about which we will calculate the net torque and set it equal to zero.

$$\tau_{\text{about } O} = 0 \Rightarrow n_1(3) - 100g(1.5) - 80g(1) = 0$$

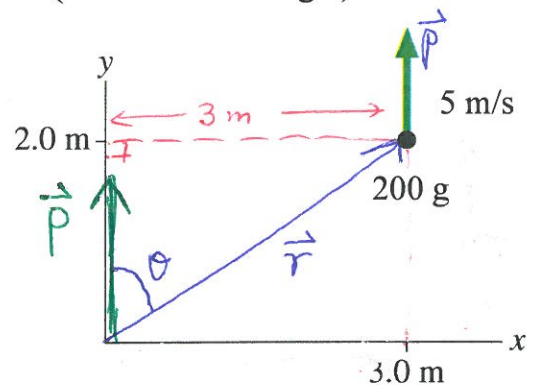
$$\Rightarrow n_1 = 751 \text{ N}$$

(STATIC EQUILIBRIUM also implies $F_{net} = 0$ but we did not need to use that here).

6. What are the magnitude and direction of the angular momentum (relative to the origin) of the 200 gram particle in the figure?

(Hint: use one of the definitions of the angular momentum)

- a) 1.4 kgm^2/s , out of the page
 b) 1.4 kgm^2/s , in to the page
 c) 1.8 kgm^2/s , out of the page
 d) 1.8 kgm^2/s , in to the page
 e) 3.0 kgm^2/s , out of the page
 f) 3.0 kgm^2/s , in to the page



$\vec{L} = \vec{r} \times \vec{p} \Rightarrow$ To take cross-product, first join the butts of the vectors together, by translating one of them, say \vec{p} , over.

$\vec{r} \times \vec{p}$ is \perp to both \vec{r} & \vec{p} and, by RHR, points out-of-the page.

The magnitude is $|\vec{r} \times \vec{p}| = r p \sin \theta = (\sqrt{2^2 + 3^2})(5) \left(\frac{3}{\sqrt{2^2 + 3^2}}\right)$
 $mv = (0.2)(5) = 1$
 $= 3 \text{ kgm}^2, \text{ out of page}$

7. The two blocks ($m_1 > m_2$) in the figure are connected by a massless rope that passes over a pulley. The pulley is in the shape of a **solid sphere** of mass $m = 3.5$ kg and has radius R . The blocks are initially held at rest, then are released. Mass m_1 is observed to descend with a constant acceleration of 2.5 m/s². The difference in tensions between the two strings on either side of the spherical pulley is

- a) 2.0 N
 b) 2.6 N
 c) 3.0 N
 d) 3.5 N
 e) 3.8 N

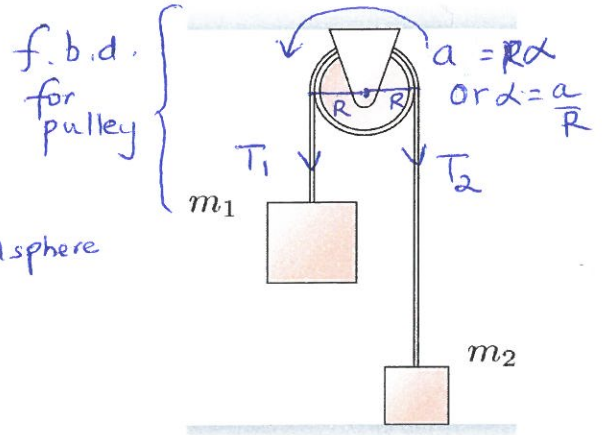
$$T_{\text{net}} = I\alpha$$

$$\Rightarrow T_1 R - T_2 R = \left(\frac{2m_p R^2}{5} \right) \left(\frac{a}{R} \right)$$

$$\Rightarrow T_1 - T_2 = \frac{2}{5} m_p a$$

$$= \frac{2}{5} (3.5) (2.5)$$

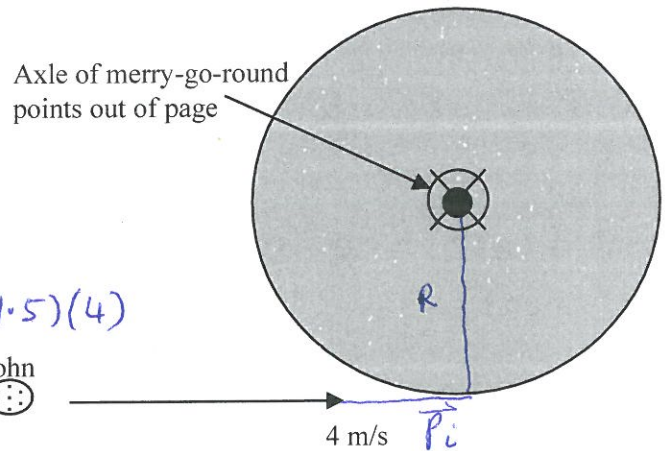
$$= 3.5 \text{ N}$$



8. A 3.0 m diameter disk-shaped playground merry-go-round has a mass 200 kg and is initially at rest. John (mass 40 kg) runs tangent to the merry-go-round at 4 m/s and jumps on to the outer edge. The kinetic energy lost in the collision between John and the merry-go-round is

- a) 75 J
 b) 144 J
 c) 229 J
 d) 302 J
 e) 332 J

View from above of merry-go-round & John



By C.O.A.M., $L_i = L_f$

$$L_i = m_{\text{John}} R v \quad (\text{from } \vec{r} \times \vec{p}_i) = 40(1.5)(4)$$

$$L_f = \left(\frac{m_{\text{disk}} R^2}{2} + m_{\text{John}} R^2 \right) \omega_f$$

$$I_f = \frac{200(1.5^2)}{2} + 40(1.5)^2 = 315 \text{ kgm}^2$$

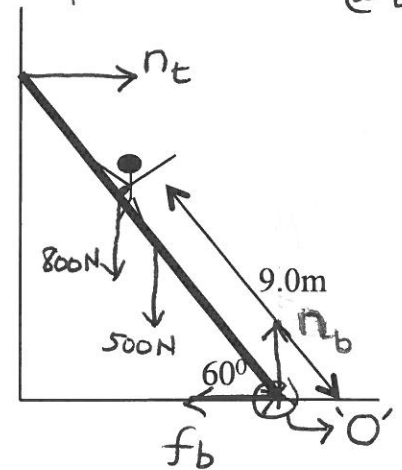
$$\therefore \omega_f = \frac{40(1.5)(4)}{315} = 0.76 \text{ rad s}^{-1}$$

$$\therefore KE_{\text{lost}} = KE_{\text{initial}} - KE_{\text{final}} = \frac{1}{2} m_{\text{John}} v^2 - \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2} (40) 4^2 - \frac{1}{2} (315) (0.76)^2 = 229 \text{ J}$$

f_b, n_b are frictional force & normal force @ bottom & n_t is normal force @ top.

9. A 15.0m, 500N uniform ladder rests against a frictionless wall, making an angle of 60° with the horizontal. A firefighter is standing on the ladder. If the ladder is on the verge of slipping when the firefighter (assumed to be a particle of mass 800 N) is located 9.0 m up along the ladder, what is the coefficient of static friction between ladder and ground?
[Hint: Consider the equilibrium condition for the net force, and the equilibrium condition for the net torque about any pivot axis]



- a) 0.30 **b) 0.32** c) 0.34
d) 0.36 e) 0.38 f) 0.40

- net $F_x = 0 \Rightarrow f_b = n_t$ — (i)
- net $F_y = 0 \Rightarrow n_b - 800 - 500 \Rightarrow 0 \Rightarrow n_b = 1300 \text{ N}$ — (ii)
- Also, $f_b = \mu_s n_b = 1300 \mu_s$ — (iii)
- Choose pivot point located at O (WHY? B/c 2 forces pass thru it, hence 2 less torques to calculate), calculate T_{net} about this 'O', and set equal to zero.

$$800(9 \cos 60) + 500(7.5 \cos 60) - n_t(15 \sin 60) = 0$$

$$\Rightarrow n_t = 421.47 \text{ N, Plug into Eqns. (iii) \& (i),}$$

$$421.47 = 1300 \mu_s \Rightarrow \mu_s = 0.32$$

10. A 400 g oscillator has a speed of 30 cm/s when its displacement is 1.0 cm and 20 cm/s when its displacement is 2.0 cm. What is the oscillator's amplitude?

- a) 2.5 cm** b) 2.8 cm c) 3.2 cm
d) 3.5 cm e) 3.8 cm

$$E_{\text{Total}} = \frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2$$

$$\Rightarrow \frac{1}{2} (0.4)(0.3^2) + \frac{1}{2} k (0.01^2) = \frac{1}{2} (0.4)(0.2^2) + \frac{1}{2} k (0.02^2)$$

$$\text{Solve for 'k': } k (0.02^2 - 0.01^2) = 0.4 (0.3^2 - 0.2^2)$$

$$\Rightarrow k = \frac{0.02}{0.0003} = 66.7 \text{ N/m}$$

$$\text{Use } E_{\text{Total}} = \frac{1}{2} k A^2 \text{ to find } \frac{1}{2} (0.4)(0.3^2) + \frac{1}{2} (66.7)(0.01^2) = \frac{1}{2} (66.7) A^2 \text{ \& now solve for A.}$$

$$\text{So... } A = 2.5 \text{ cm.}$$

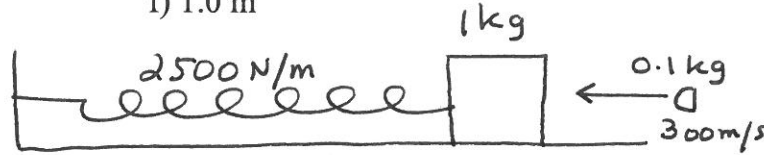
11. A 1.00 kg block is attached to a horizontal spring with spring constant 2500 N/m. The block is at rest on a frictionless surface. A 100 gram bullet is fired into the block with a speed of 300 m/s, in the face opposite the spring; the bullet embeds itself inside the block. What is the amplitude of the subsequent oscillations undergone by the block?

- a) 0.1 m
e) 0.8 m

- b) 0.2 m
f) 1.0 m

- c) 0.4 m

- (d) 0.6 m**



By C. O. L. M. during collision,

$$P_i = P_f$$

$$\Rightarrow (0.1)(300) = (1 + 0.1)v \Rightarrow v = 27.27 \text{ m/s}$$

this is v_{max} b/c the block starts w/ this velocity & is only slowed thereafter.

$$E_{\text{Total}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2$$

$$\Rightarrow \frac{1}{2} (1.1)(27.27^2) = \frac{1}{2} (2500) A^2$$

$$\& A = \sqrt{\frac{1.1 \times 27.27^2}{2500}} = 0.57 \text{ m}$$

12. Planet X has a value of $g = 3 \text{ m/s}^2$. You suspend an unknown mass by a spring (unstretched length 45 cm) and find that, in equilibrium, the extension of the spring is 20 cm. You pull the mass further down by 5 cm and then let go, setting the mass in SHM. The time period of the SHM is approximately

- a) 2.7 s

- b) 2.4 s

- c) 2 s

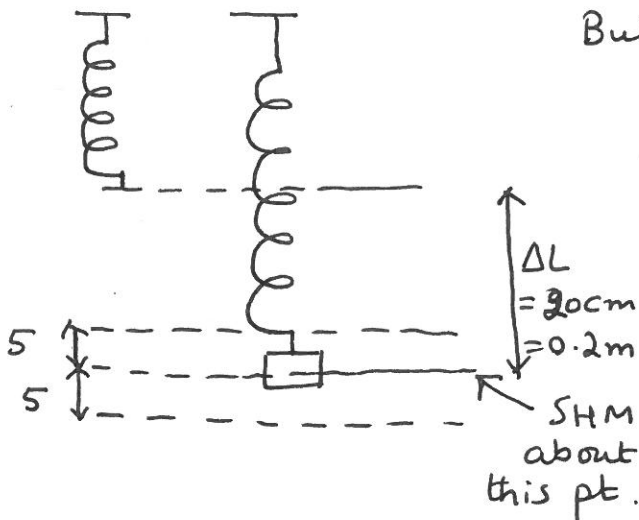
- (d) 1.6 s**

- e) 1.3 s

In equilibrium, $mg = k \Delta L \Rightarrow \frac{k}{m} = \frac{g}{\Delta L} = \frac{3}{0.2} = 15$

But $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega = \sqrt{15} = 3.87 \text{ rad/s}^{-1}$

$\& T = \frac{2\pi}{\omega} = \frac{2\pi}{3.87} = 1.6 \text{ s}$



16. The sequence of figures on the right show a mass-spring system oscillating in simple harmonic motion. If we model the displacement of the mass as $x(t) = A \cos(\omega t + \phi)$, what is the value of ϕ if the oscillating mass happens to be at $t = 0$ in the state shown by figure #3?

- a) 0
- b) $7\pi/4$
- c) $3\pi/2$
- d) $5\pi/4$
- e) $\pi/2$

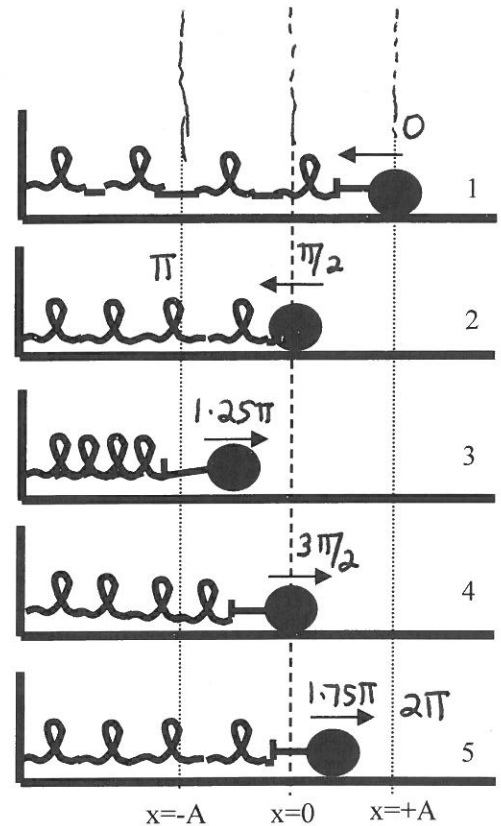
$$X(t) = A \cos(\omega t + \phi)$$

If $x = A$ @ $t = 0$, $\phi = 0$
@ $v = -A/2$

If @ $t = 0$, x lies between

REASON: 0 & $-A$ (as shown in fig #3)

then ϕ must correspond to a value between π & $3\pi/2$; the only choice is $5\pi/4$



17. A penny rides on top of a piston as it undergoes vertical simple harmonic motion. If the frequency is low, the penny rides up and down without difficulty. If the frequency is steadily increased, there come a point at which the penny leaves the surface. At what point in the cycle is the penny most likely to first lose contact with the piston?

- a) At the equilibrium point (i.e., midpoint) of the oscillation
- b) In between the equilibrium point (i.e., midpoint) and the top extremity of the oscillation while going down
- c) In between the equilibrium point (i.e., midpoint) and the top extremity of the oscillation while going up.
- d) At the bottom extremity of the oscillation
- e) At the top extremity of the oscillation

REASON:

acceleration downward of piston is maximum @ the top of the extremity, hence most likely to first exceed 'g' there, causing the "bottom to drop out" from under the penny which can only move as fast as free fall would allow.

18. Observe the demo shown. One of two possibilities listed below will happen.

Pick the correct one. You do not need to show your reasoning.

Only your answer will be graded. You get 2 points if you choose correctly, and -2 points if you choose incorrectly....so no random guesses please!

- a) The wheel will veer to your left (not Samir's left, your left).
- b) The wheel will veer to your right (not Samir's right, your right).

Part C (40 points) Questions 19 – 21: 3 Numerical Problems.

SHOW WORK CLEARLY.

Correct answer w/o clear show of work = ZERO credit!

19. Skiers Don and Mike, though ridiculously unskilled, are quite the crowd-pleaser in rural Wisconsin. In one routine, Mike (90 kg), wearing wood skis (μ_k for wood and snow is 0.06), starts at the top of a 200-m-long, 20° slope. Don (115 kg) waits for him halfway down. As Mike skis past, Don jumps into his arms and Mike carries Don the rest of the way down. **What is their speed as they both crash into a tree at the bottom of the slope?** (12 pts)

Hint: It is simplest to use C. O. M. E. and C. O. L. M.!

[TIPS: 1) First find Mike's speed immediately before Don jumps into his lap. (4 pts)

2) Next, find their joint speed immediately after Don has settled in Mike's lap (happens very quickly). (4 pts)

3) Finally, figure their joint speed at the bottom. (4pts)]

- 1) For Mike, from (A) to (B), apply C.O.M.E. to find his speed @ v_1 just before his "collision" w/ Don.

$$\Delta K + \Delta U = W_{nc}$$

$$\Rightarrow \left(\frac{1}{2} (90) v_1^2 - 0 \right) - 90(9.8)(100 \sin 20^\circ) = - \underbrace{\mu_k}_{0.06} (90)(9.8) \cos 20^\circ (100)$$

$f_k = \mu_k n = \mu_k mg \cos \theta$

$$\Rightarrow \frac{1}{2} (90) v_1^2 = 90(9.8)(100) \sin 20^\circ - 90(9.8)(100) \cos 20^\circ (0.06)$$

$$\Rightarrow v_1 = 23.66 \text{ m/s}$$

- 2) Now Don has a perfectly inelastic collision w/ Mike. Their joint speed v_2 , just after collision is given by C.O.L.M.

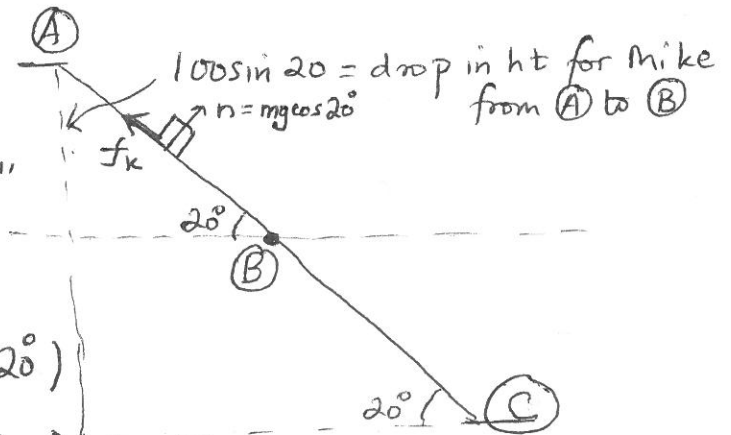
$$90(23.66) = (90 + 115) v_2 \Rightarrow v_2 = 10.39 \text{ m/s}$$

- 3) Now use C.O.M.E. again, just as in (1) above, to figure their joint speed @ (C). $\Delta K + \Delta U = W_{nc}$

$$\Rightarrow \frac{1}{2} (205) v^2 - \frac{1}{2} (205) (10.39)^2 - (205)(9.8)(100) \sin 20^\circ (0.06) = - \underbrace{(0.06)(205)(9.8) \cos 20^\circ (100)}_{f_k d}$$

$$\Rightarrow \frac{1}{2} v^2 - \frac{1}{2} (10.39)^2 = 279.9$$

$$\Rightarrow \boxed{v = 25.8 \text{ m/s}}$$



Part C (40 points) Questions 19 – 21 : 3 Numerical Problems.

SHOW WORK CLEARLY.

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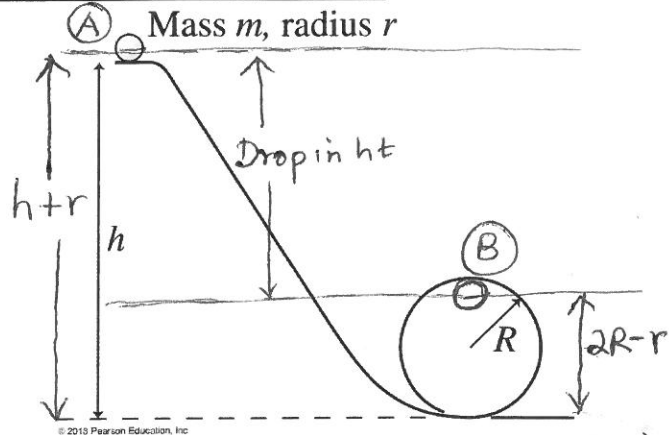
20. A disk of mass m and radius r rolls without slipping down the track around a loop-the-loop of radius R , starting from height h , as shown in the figure. Here, $h = 7R$ and $r = 0.1R$ and $R = 1.5$ m.

What is the normal force n on the disk from the track as the disk rolls around the top of the hoop? (14pts)

NOTE: n is not zero, because the disk starts at a height h much higher than the minimum height required to barely loop-the-loop.

[TIP: First, find the speed v of the disk at the top of the loop (what conservation law should you use?) (8pts)

Once you know v , use your knowledge of centripetal force to compute n . (6 pts)]



- Apply C.O.M.E. between (A) & (B) to find disk's speed @ B.

$$\Delta K + \Delta U = W_{nc}^{\rightarrow 0}$$

$$\left(\frac{1}{2} m v_B^2 + \frac{1}{2} I \omega_B^2 - 0 \right) + mg \left[(2R-r) - (h+r) \right] = 0$$

$$\frac{1}{2} \cdot \frac{1}{2} m r^2 \cdot \frac{v_B^2}{r^2}$$

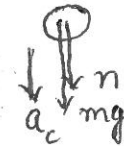
$$\therefore \frac{3}{4} m v_B^2 = mg (h+r - 2R+r)$$

$$\Rightarrow v_B^2 = \frac{4}{3} g (7R + 0.1R - 2R + 0.1R) = \frac{4}{3} g (5.2)R$$

$$\text{i.e. } v_B^2 = 6.93gR$$

- Now, at (B), the f.b.d. looks like

$$n + mg = \frac{m v_B^2}{R-r}$$



$$\Rightarrow n = \frac{m(6.93gR)}{R-0.1R} - mg = mg \left(\frac{6.93R}{0.9R} - 1 \right)$$

$$= mg (7.7 - 1)$$

or $\boxed{6.7 mg}$

Part C (40 points) Questions 19 – 21 : 3 Numerical Problems.

SHOW WORK CLEARLY.

Correct answer w/o clear show of work = ZERO credit!

21. A 150 gram, 25-cm-long rod hangs vertically on a frictionless, horizontal axle passing through its top end. A 12 gram lump of clay traveling horizontally hits and sticks instantaneously to the center of the rod causing the rod to subsequently deflect from the vertical by a maximum of 19° .

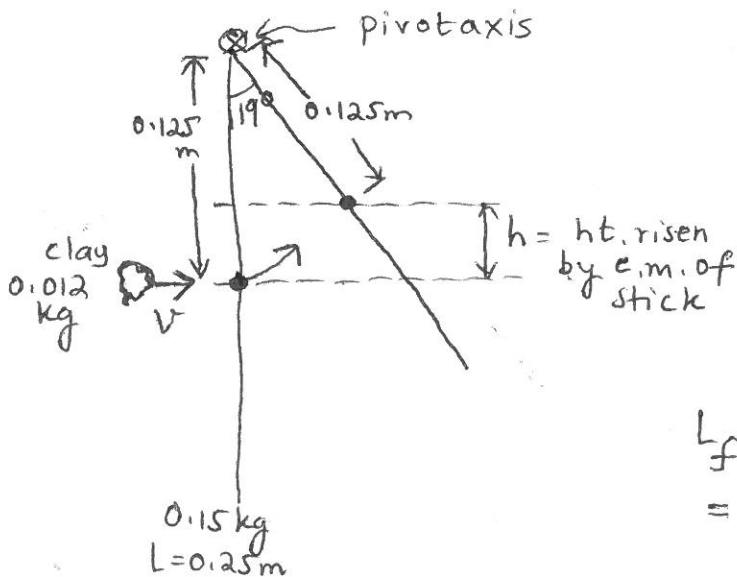
What was the initial velocity of the lump of clay? (14pts)

(14pts)

[Hint: The moment of inertia of a thin rod of mass M and length L about an axis passing through its end is $\frac{1}{3} ML^2$. The lump of clay can be assumed to be a particle.]

Hint: What two conservation laws should you utilize here?

Work backwards. i.e., first find the angular speed needed by the rod + clay after impact in order to rise by 19° (8 pts). Next, find the initial speed of the clay before impact (6 pts).



• Apply C.O. to collision

$$L_i = \text{initial angular momentum of clay} \\ = \underbrace{(0.0125)}_r \underbrace{(0.012v)}_p$$

$$\text{b/c } |L| = r p \sin 90^\circ$$

$$L_f = \text{final angular momentum of clay + rod} \\ = \left[\underbrace{(0.012)(0.125^2)}_{I_{\text{clay}} \text{ (for point particle)}} + \frac{1}{3} \underbrace{(0.15)(0.25^2)}_{I_{\text{rod}} \text{ about end}} \right] \omega$$

Apply C.O.A.M., i.e. $L_i = L_f$

$$\Rightarrow 0.0015v = 0.0033125\omega, \text{ or } v = 2.2\omega \quad \text{--- (i)}$$

• Now, after collision, apply C.O.M.E. to rising stick w/ clay stuck on it.

$$\Delta K + \Delta U = W_{nc} \text{ as stick rises by } 19^\circ$$

$$\left(0 - \frac{1}{2} (I_{\text{clay}} + I_{\text{rod}}) \omega^2 \right) + (m_{\text{rod}} + m_{\text{clay}})gh = 0$$

$$\Rightarrow \frac{1}{2} (0.0033125) \omega^2 = (0.15 + 0.012)(9.8)(0.00681) \leftarrow h = 0.125(1 - \cos 19^\circ) = 0.00681$$

$$\Rightarrow \omega = 2.55 \text{ rad s}^{-1} \quad \text{--- (ii)}$$

Plugging into (i), $v = (2.2)(2.55)$

$$\boxed{v = 5.6 \text{ m/s}}$$