N-Body Simulations of a Dwarf Spheroidal Galaxy Comparing Newtonian, Modified Newtonian, and Dark Matter Models.

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Abstract:

The goal of this paper is to compare the theories of Dark Matter and Modified Newtonian Dynamics, and then explain the results when we incorporated these theories into our simulation. We successfully modeled a Dwarf Spherical Galaxy using Newtonian gravity, Modified Newtonian gravity, and Dark Matter. The results of our project are in the form of distribution histograms to show various distributions, as well as in the animations we produced.

I: Introduction

The theory of Dark matter originated from observations of galactic clusters by Swiss Astronomer and Astrophysicist, Fritz Zwicky in 1933. When looking at galaxies in the Coma cluster, he observed that the galaxies were moving too fast for the amount of visible mass in the cluster. In order to gravitationally bind the galaxies in that cluster, the mass had to be much larger. He concluded that there was then a large amount of invisible mass, or Dark Matter, which would account for this cluster remaining bound (Sanders, 2010). Then in the 50's and 60's, Martin Schwarzschild began to look at the mass-to-light ratios of the individual galaxies, developing rotation curves. This led to Vera Rubin in the 70's and 80's to look at the rotation curves of spiral galaxies (Sanders, 2010). Her work showed the difference between the observed velocities of stars in spiral galaxies, versus the predicted velocities, as seen in Figure 1 (Hibbs, 2005). Curve A shows the velocities of stars some distance from the center of the galaxy as predicted by Newtonian gravity, where Curve B shows the actual observation.

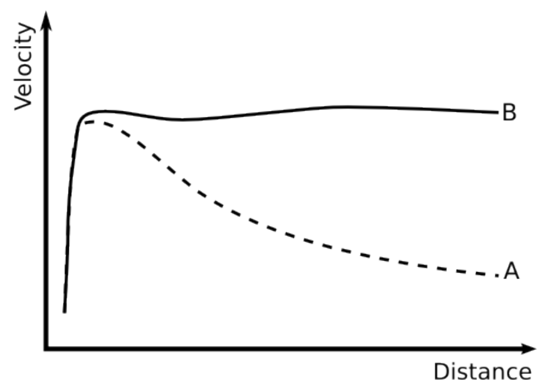

Figure 1: Rotation Curves of Spiral Galaxies. 'A' is the predicted curve; 'B' is the observed.

Dark Matter has never actually been seen, but its effects have been observed in galaxy rotations. It had become a well-accepted theory, until 1981 when Israeli physicist, Mordehai Milgrom, came up with a counter theory (Sanders and McGaugh, 2002). Milgrom proposes that Newton's theory of gravity breaks down at very low accelerations. His theory, now called, "Modified Newtonian Dynamics", or MoND, says that at very low accelerations, the force of gravity from a point mass would act more like a $^1/_r$ function rather than a $^1/_{r^2}$ (Milgrom and McGaugh, 2013).

One formulation of MoND (Sanders and McGaugh, 2002) rewrites Newton's Second Law as:

$$F_{net} = \boldsymbol{ma} \rightarrow \boldsymbol{ma\mu}(^a/_{a_o}) \tag{1}$$

where a_o is a constant and $\mu(x)$ is an unspecified function that must have the asymptotic form $\mu(x) = x$ when $x \ll 1$ and $\mu(x) = 1$ when $x \gg 1$. The constant a_o is 1.2 x 10^{-10} m/s^2 in order for the correction to prove true (McGaugh and Milgrom, 2013). This correction is then just an alternative calculation of the acceleration. Figure 2 shows Newtonian acceleration versus MoNDian acceleration in our units (which are solar masses, parsecs, and millions of years), for which a_o = 3.88pc/Myr^2 (dotted blue lines). It is evident that around 10 pc/Myr^2, the two accelerations begin to slightly diverge. By the time the Newtonian acceleration has dropped to 0.01 parsecs/Myr^2, the MoNDian acceleration is a factor of 10 greater. Therefore, the MoNDian calculation just replaces the Newtonian, but the effects are only seen within the MoNDian limits. This new calculation of acceleration would provide an alternative explanation as to

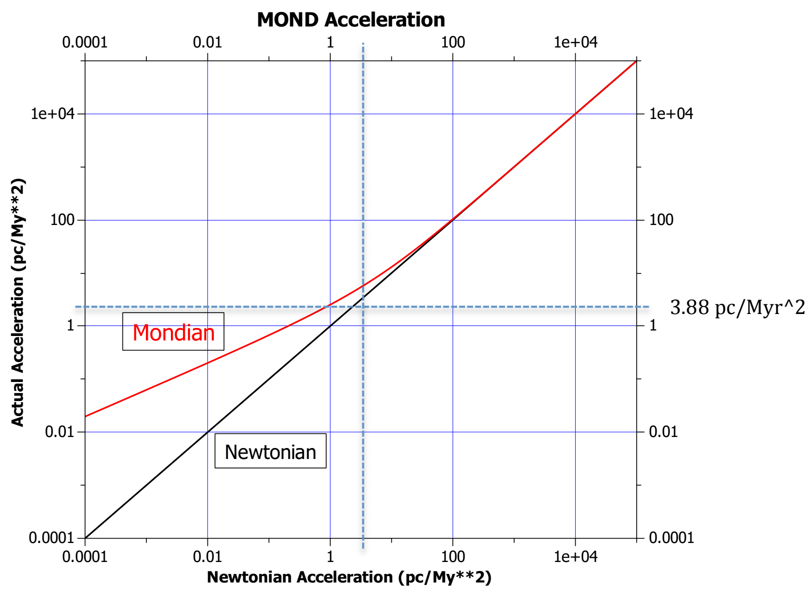

Figure 2: Newtonian vs. MoNDian Acceleration

why these galaxies were moving faster than the conventional Newtonian laws would predict.

II: Technique

We used an N-Body Simulation that uses an integrator called "Hermite Individual Time Steps", or "HITS", which has been used at Miami University for solar systems and modified to handle stellar systems (Alexander, Hesselbrock, Wu, Marshall, and Abel, 2011). In the N-body problem, the integrator steps through each of the bodies and updates its own position ($\vec{x_i}$), velocity ($\vec{v_i}$), acceleration ($\vec{a}_i$) and jerk ($\dot{\vec{a}}_i$) for each body's individual time step (Δt_i). The expression for the net Newtonian acceleration and jerk (the first time derivative of the acceleration) in the integrator is:

$$\vec{a} = -\sum_{i \neq j} \frac{Gm_j}{r^3} \vec{r} \tag{2}$$

$$\dot{\vec{a}}_n = -G \sum_{i \neq j} m_j \left\{ \frac{\vec{v}}{r^3} - \frac{\vec{r}}{r^5} (\vec{r} \bullet \vec{v}) \right\} \tag{3}$$

In order to accommodate the MoNDian acceleration within the integrator, we had to use the relationship between the Newtonian acceleration and the MoNDian to derive an expression.

Beginning with the relationship:

$$\vec{a}_n = \vec{a} \mu \left({|\vec{a}|} \big/ {a_o} \right) \tag{4}$$

(Sanders and McGaugh, 2002), where $\vec{a}_n$ is the net Newtonian acceleration of the body, and $\vec{a}$ is the actual net acceleration. We chose the simplest form for the μ function:

$$\mu(x) = \frac{x}{x+1} \tag{5}$$

Taking the magnitude of both sides, we obtain a quadratic and can solve for the magnitude of the actual acceleration:

$$\left|\vec{a}\right| = \frac{\left|\vec{a}_n\right|}{2} \pm \frac{\left|\vec{a}_n\right|}{2}\sqrt{1+4\frac{a_o}{\left|\vec{a}_n\right|}} \tag{6}$$

Checking this equation with the MoNDian limits shows us that it will fulfill the necessary requirements. When $\left|\vec{a}_n\right| >> a_o$:

$$\left|\vec{a}\right| = \frac{\left|\vec{a}_n\right|}{2} \pm \frac{\left|\vec{a}_n\right|}{2}\sqrt{1} \rightarrow \left|\vec{a}\right| = \left|\vec{a}_n\right| \tag{7}$$

Which holds true with Newton's Laws. When $\left|\vec{a}_n\right| << a_o$:

$$\left|\vec{a}\right| = \frac{\left|\vec{a}_n\right|}{2} \pm \frac{\left|\vec{a}_n\right|}{2}\sqrt{4\frac{a_o}{\left|\vec{a}_n\right|}} \rightarrow \sqrt{\left|\vec{a}_n\right| a_o} \tag{8}$$

which is the relationship derived by Milgrom and McGaugh in their 2013 paper. The HITS integrator requires for the vector components of the acceleration, so having the just the magnitude will not be enough. We assumed that the direction of the MoNDian acceleration would be in the same direction as the Newtonian, therefore we were able to accept this equation and move forward. The integrator requires an analytical expression for the jerk, $(\dot{\vec{a}}_i)$. Differentiating equation (6) with respect to time gives:

$$\dot{\vec{a}} = \frac{1}{2}\left\{ \dot{\vec{a}}_n \left[2\sqrt{a_o / \left|\vec{a}_n\right|} \right] - \vec{a}_n \frac{\vec{a}_n \bullet \dot{\vec{a}}_n}{\left|\vec{a}_n\right|^2} \left[\sqrt{a_o / \left|\vec{a}_n\right|} \right] \right\} \qquad (9)$$

These equations were added into the integrator's acceleration subroutine. The code steps through the subroutine, which calculates the total acceleration on each body. When it arrives at the MoNDian correction to the acceleration, there is a "switch" that would allow us to turn the correction on or off. There are also switches that allow us to turn off/on the body-body interactions, a black hole, and the Dark Matter and Baryon clouds. The MoND correction is the last calculation in the acceleration subroutine, because the correction only occurs when the net acceleration of the body is within the limit.

To best model the effects of Dark Matter and MOND, we chose to model a Dwarf Spherical Galaxy. These galaxies are much smaller than the typical Spiral galaxy, usually within the range of 10^7 and 10^8 SM. Because of their small and spherical size, Dwarf Spherical Galaxies are easier to simulate in order to visibly see the effects. Also, Dwarf Spherical Galaxies show signs of containing a significant amount of Dark Matter, making them prime candidates for MoND simulations. We modeled the mass distribution of a Dwarf Spherical by using a Baryonic Cloud of mass 10^7 SM with an exponential density profile:

$$\rho_B(r) = \rho_o e^{-r/d} \qquad (10)$$

Where d is an arbitrary constant we chose to be 50pc, and $\rho_o = \frac{M_B}{8\pi d^2}$. This was to model the mass of the galaxy without having to use 10^7 different bodies. Instead, we randomly scattered 500 bodies of mass .1 – 10 SM within a 500 parsec radius sphere, giving them random initial velocity components. We found that 500 bodies are much easier to calculate and animate than 10^7. In order to decide the initial velocities, we ran several different trials with initial velocity ranges beginning with 0 pc/Myr, all the way up to ±60 pc/Myr. We gave the bodies initial velocities, ranging between -15 and 15 pc/Myr (1 pc/Myr ≈ 1 km/sec), because the MoND trial remained intact, while the Newtonian trial did not (these initial conditions remained constant throughout all of our trials). This would allow us to then adjust our Dark Matter model accordingly in order to produce results similar to that of MoND. For the model Dark Matter, we started with the density profile (Carroll and Ostlie, 2007):

$$\rho_D(r) = \frac{\rho_o}{r/a\left(1 + r/a\right)^2} \tag{11}$$

where a is an effective radius, and ρ_o is the central density. Integrating over a radius and initial parameters, we found ρ_o to be about $\frac{M_d}{4\pi a^3}$, where M_d is the total mass of the Dark Matter cloud. We chose M_d to be 10^8 SM. We chose the effective radius a to be 200 because it produced similar results to that of the MoND trial.

III: Results

We used the data to produce animations, seen at the following link http://www.cas.miamioh.edu/~alexansg/tristan/tr_animations.htm,

and the histograms in Figures 3, 4, and 5 to show the results of our research.

i) Animations:

The 3-D animation displays a side-by-side comparison of all three trials, Newtonian, MoNDian, and Dark Matter respectfully. The animation was written in python computer language, coded to read the data produced by the integrator every frame step (0.5Myr). The code created an array of visual spheres to represent the bodies. With every frame step, the code would update each body's position, moving it's respective sphere. Python was used because it allowed us to control the animation, such as scaling the size of the spheres and adding the 500pc ring. Python also allows the user to freely zoom in/out and rotate the visual screen, allowing us to view our galaxy from all distances and angles.

The animations run for 1 billion years each, with a frame step of 500,000 years in the animation. One can see that the Newtonian trial dissolves rather quickly, with very few of the stars remaining within the initial 500pc radius sphere, marked by the white ring. Meanwhile, the MoND and Dark Matter trials are almost identical. All of the stars remain in bound orbits about the center of mass, rarely getting much farther out past the 500pc ring. If they were unmarked, it would be almost impossible to identify each trial. The animations successfully show that Dark Matter can produce results similar to MoND.

ii) Histograms:

Figure 3 is a set of histograms of the radial distances of the bodies at 300Myr and 750Myr for all three trials. As expected and shown through the animations, the Newtonian run dissolved very quickly, with stars that seemed to escape completely from the galaxy. The MoND and Dark Matter trials, as expected, are also very similar with only slight differences. These differences are not clear enough to pull any conclusions from.

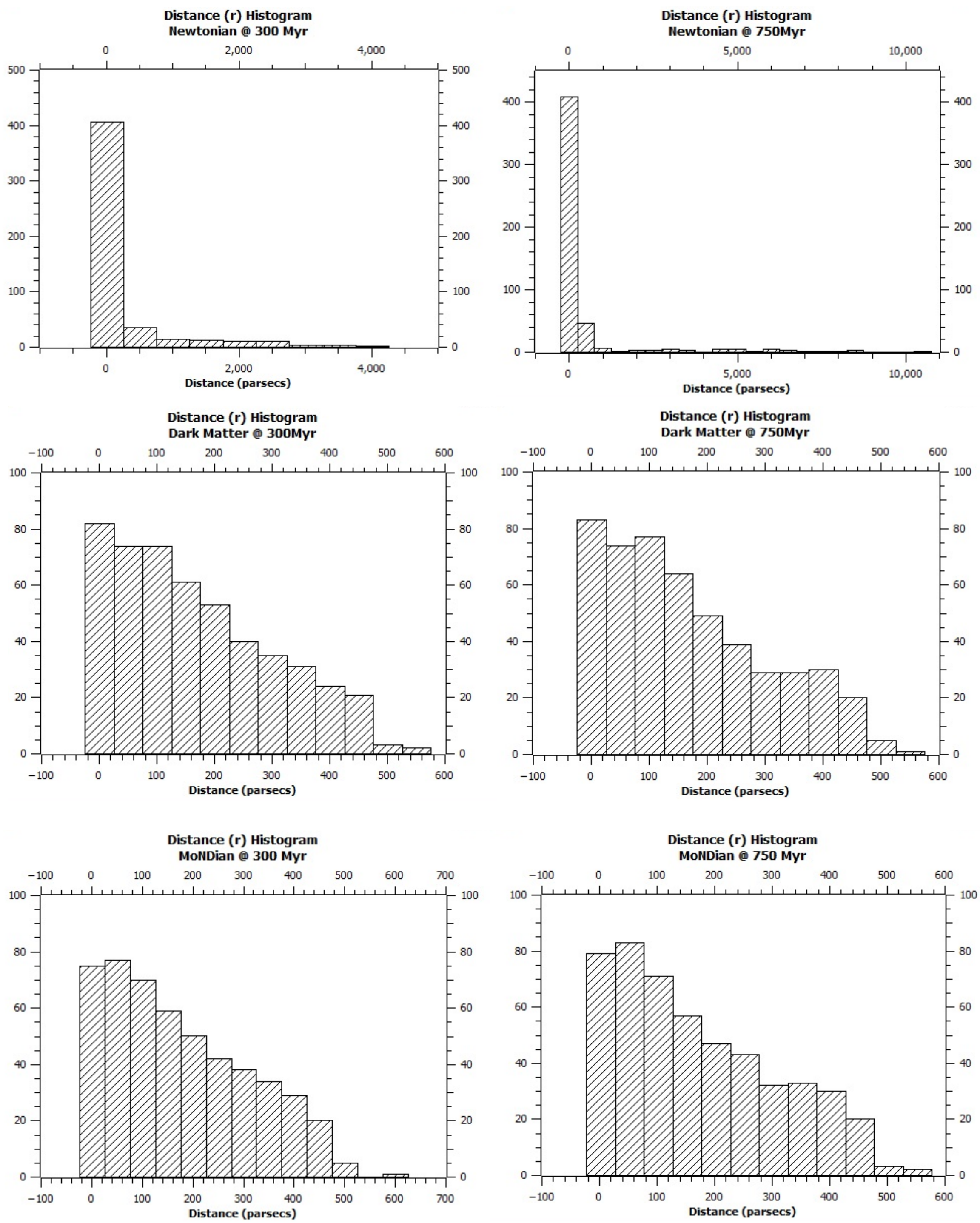

Figure 3: Histograms of the Distance (r) of the bodies from the origin. Histograms for Newtonian, Dark Matter, and MoNDian trials at 300 Myr and 750 Myr

Figure 4 is the set of histograms from the actual accelerations of the trials. These histograms give us a much better look into the differences between the MoND and Dark Matter trials. The Newtonian produces results again as expected, with very low accelerations. What is interesting about the Newtonian histograms is that the accelerations are all under 3.88 pc/Myr^2, and therefore would all be recalculated using the MoND correction. The MoND histograms then show what this correction looks like. The Dark Matter on the other hand has accelerations much higher than both the MoND and the Newtonian trials. This was not as expected, as the Dark Matter and MoND were supposed to produce similar results. This gives us a glimpse into possible research of the differences between the two theories and the outcomes that they produce.

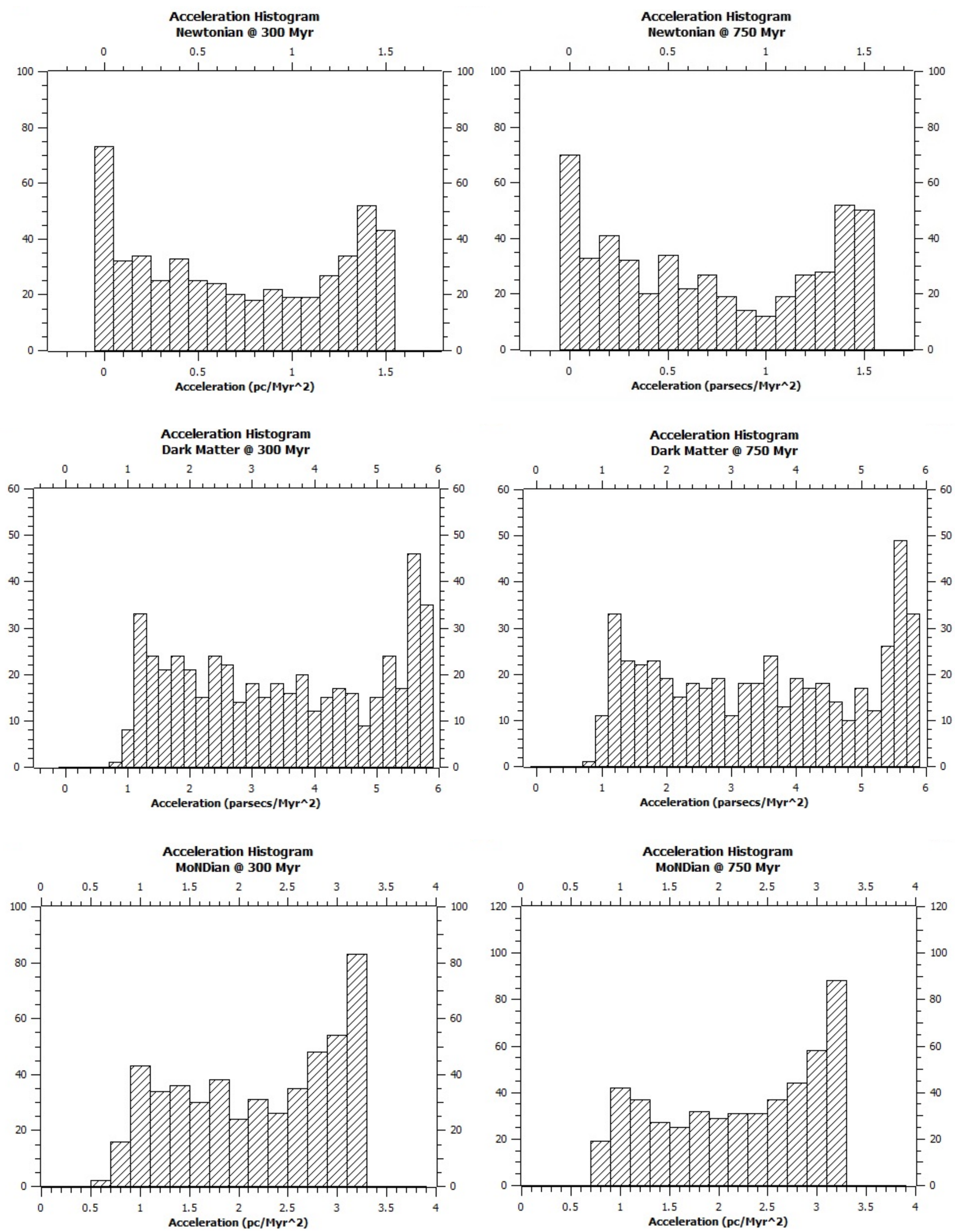

Figure 4: Histograms of the Actual Acceleration of the bodies. Histograms for Newtonian, Dark Matter, and MoNDian Trials at 300 Myr and 750Mry.

Figure 5 shows scatter plots of Distances versus the x-component of Velocity in the trials. The spread of the velocity in the Newtonian trial decreases with time, as the distance spread increases. The MoND and Dark Matter trials are similar, where the only difference can really be seen in the velocity. The MoND seems to have a slightly tighter spread than the Dark Matter does, however, like the differences with Figure 3, it is difficult to pull any solid conclusions from these differences.

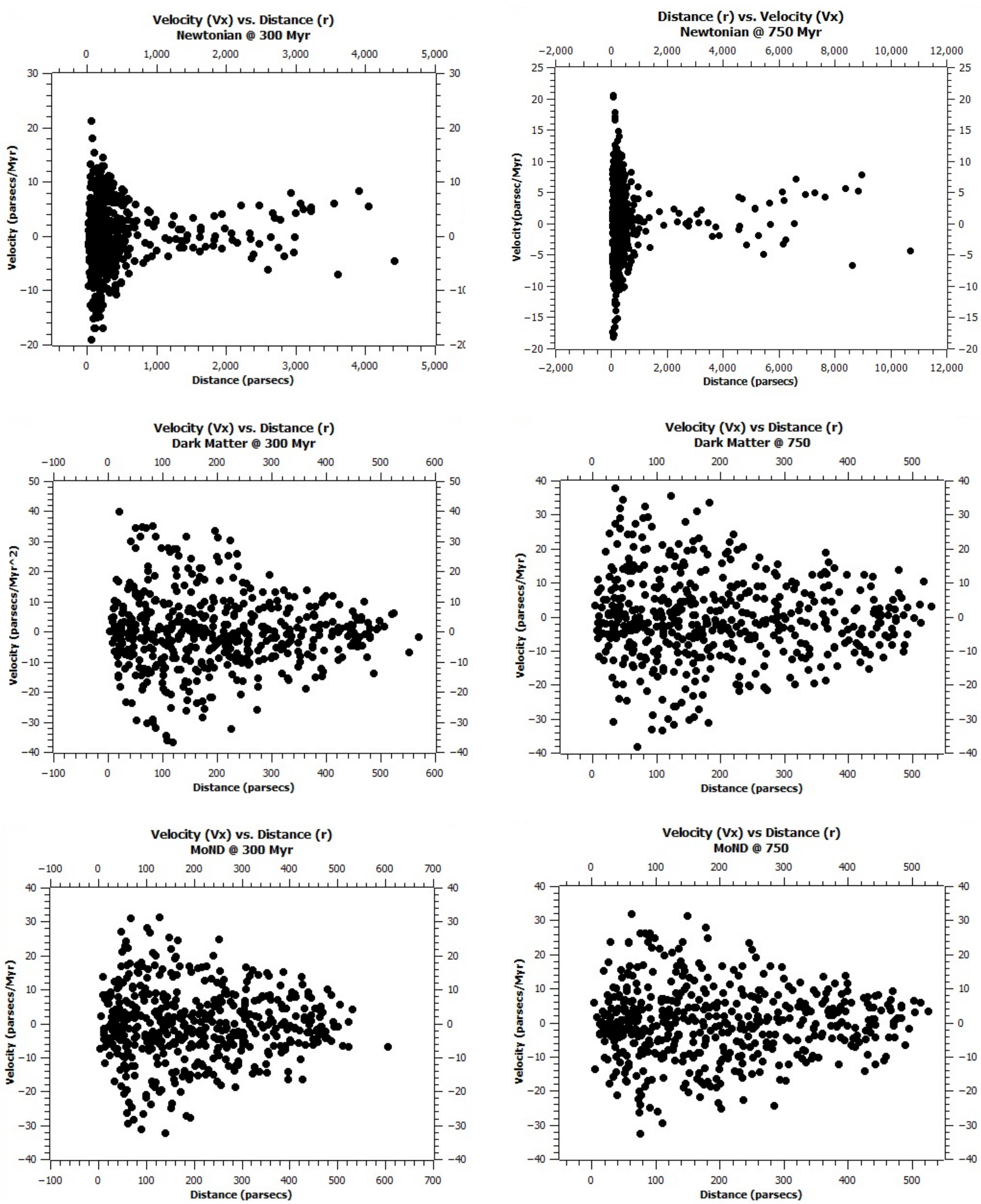

Figure 5: Scatter Plots of Distance (r) vs. Velocity for Newtonian, Dark Matter, and MoNDian trials at 300Myr and 750Myr

IV: Conclusion/Discussion

We successfully simulated a basic Dwarf Spherical Galaxy, showing the effects of a Dark Matter Cloud, Modified Newtonian Dynamics, and classic Newtonian Dynamics. The simulation showed us that Modified Newtonian Dynamics and Dark Matter produce similar results. While visually similar, there are still differences between the two galaxies. These differences are to be expected, because while very similar in the animations and figures, the MoND and Dark Matter trials are still calculating different. Therefore, we should see some differences in the outcomes, as in the acceleration histograms. Further research could be done to examine these differences closer in order to better understand the two theories and their effects on our universe.

V: References

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