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# N-Body Simulations of Late Stage Planetary Formation with a Simple Fragmentation Model

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**We present results of two-dimensional gravitational *N*-body simulations of the late stage of planetary formation. This stage is characterized by the direct accretion of hundreds of lunar-sized planetesimals into planetary bodies. Our simulation code is based on the Hermite Individual Timestep integration algorithm, and gravitational interactions among all bodies are included throughout the simulations. We compare our simulation with earlier works that do not include all interactions, and we find very good agreement. A previously published collisional fragmentation model is included in our simulation to study the effects of the production of fragments on the subsequent evolution of the larger planetary bodies. It is found that for realistic two-body collisions that, according to this model, both bodies will suffer fragmentation, and that the outcome of the collision will be a relatively large core containing most of the mass and a few small fragments. We present the results of simulations that include this simple fragmentation model. They indicate that the presence of small fragments have only a small effect on the growth or orbital evolution of the large planet-sized bodies.** © 1998 Academic Press

***Key Words:*** **orbits; planetary dynamics; planetary formation; planetesimals.**

## 1. INTRODUCTION

It is believed that the late stage of planetary formation is characterized by the accumulation of the final planets through direct accretion of lunar-sized protoplanets. Gravitational interactions between protoplanets that grew in relative isolation tend to perturb each other into crossing orbits so that close encounters and collisions occur. This process is thought to continue until the protoplanets have collided and accumulated into a few final planets which are gravitationally isolated (i.e., in non-crossing orbits).

As the protoplanets undergo close encounters their semi-major axes change substantially. This results in radial mixing; thus, it is unlikely that the final planets grew in localized feeding zones. Instead, it is more likely that they formed from protoplanets which originated at a wide variety of radial distances (Wetherill 1994). Hence, any modeling of late stage planet formation must include protoplanets that reflect this wide variety of initial semi-major axes and also account for a wide range of gravitational perturbations.

The outcomes of collisions between protoplanets ranging from lunar-sized to Mars-sized is not well understood. The physical characteristics of two colliding protoplanets may be as important as the dynamics of a collision in determining whether the two bodies will merge, fragment, or rebound from each other. The treatment of collisions may also have significant effects on the subsequent development of the planets. Thus improving our understanding of the role of fragmentation during the late stage of planet formation is of special importance.

The late stage of planetary formation has been modeled in both two and three dimensions using primarily two techniques, Monte Carlo calculations and *N*-body integrations. Most of the simulations of the final stage of planet formation have been performed using a Monte Carlo technique developed by Wetherill (1992, 1994, 1996). This technique is based on the earlier work of Arnold (1964, 1965) and uses the orbital elements of each body to determine the probabilities of encounters with other bodies. If two bodies have crossing orbits, the *dice are rolled* to determine if they pass within each other's sphere of influence (10 gravitational radii). When an encounter is predicted, the *dice are rolled* again to determine whether a collision has occurred or to determine the perturbations of the planetesi-



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mals' orbital elements resulting from the close encounter. Because the individual trajectories of each body are not integrated directly, this method is fast enough to follow the evolution of several hundred bodies in three dimensions over timescales of $10^8$ years. Wetherill (1992, 1994, 1996) has performed hundreds of such calculations in several studies. The main drawback of this method is that it only includes the perturbation between bodies in crossing or nearly crossing orbits.

Due to the relatively small number of protoplanets present in the late stage of formation, direct $N$-body simulations are possible. An $N$-body simulation of planetary formation directly integrates the equations of motion of $N$ gravitationally interacting bodies. This type of simulation of planetary formation must include the gravitational interactions between planetesimals and properly treat close encounters and collisions. Because the trajectories of the planetesimals are followed directly, an $N$-body simulation is free of the statistical assumptions used to model planetesimal interactions in the Monte Carlo approach. This is the main advantage of this method over statistical approaches. One of the disadvantages of $N$-body simulations is that they are computationally expensive. $N$-body simulations are only feasible for small numbers of bodies unless extremely fast computers are available. One compromise that has frequently been made to increase the feasibility of $N$-body simulations is to perform the simulation in two dimensions rather than three; however, there are significant differences between simulations in two and three dimensions that should be addressed. First, all crossing orbits in two dimensions intersect. In three dimensions, crossing orbits will most likely not intersect but be looped together. Second, the ratio of close encounters to collisions in three dimensions is greater than in two dimensions because the number of bodies passing within a distance $R$ of a body increases with $R^2$, whereas in two dimensions it increases only linearly with $R$ (Wetherill 1990). Thus these factors must be taken into consideration when interpreting the results of two-dimensional simulations in terms of real world phenomena.

Since the mid-1980s, several $N$-body simulations have been performed using a variety of different techniques. Lecar and Aarseth (1986) developed a two-dimensional $N$-body simulation of the late stage of planetary formation using 200 lunar-size planetesimals initially in circular orbits distributed between 0.5 and 1.5 AU, with the aim that they would form Venus and Earth by inelastic collisions. They modeled the interactions between planetesimals using a perturbation scheme based on nearest neighbors. The perturbations to a planetesimal's orbit due to other planetesimals were included out to 300 gravitational sphere of influence radii. More distant interactions were not initially included; however, as the number of bodies decreased, the distance to which perturbations were calculated was steadily increased until all were included. After 50,000 years, six bodies remained, the largest of which had two-thirds the mass of the Earth. The simulation was carried out until the orbits of the remaining bodies were isolated from one another; however, two of the final six bodies were in crossing orbits. This configuration was shown to be stable by extending the simulation an additional 60,000 years with no further collisions.

Beaugé and Aarseth (1990) performed three $N$-body simulations of the late stage of planetary formation based on the Lecar and Aarseth model. They performed these simulations in two dimensions with 200 bodies of $1.15 \times 10^{23}$ kg mass in initially circular orbits between 0.6 and 1.6 AU. Each of these simulations had a different initial mass distribution. They improved the general model used by Lecar and Aarseth in two ways. They included a more realistic model for collision outcomes that allowed not only accretion, but inelastic bouncing and fragmentation of the bodies. They also added the perturbations of the most massive planetesimals to those of the nearest neighbors. Their results showed that protoplanets combine quickly in the early stages of the simulation but their growth is slowed by the fragmentation process later in the simulation. Each of their simulations produced four final bodies with characteristics in qualitative agreement with the terrestrial planets. This study illustrated the importance of including long-range interactions between protoplanets as the bodies continued to perturb each other into crossing orbits and no premature isolation of bodies occurred.

Recently, Makino (1991) developed an $N$-body algorithm based on a Hermite integrator with an Individual Timestep Scheme, and Makino and Aarseth (1992) used this algorithm with an Ahmad–Cohen scheme for gravitational problems. Kokubo and Ida (1995, 1996) have constructed a gravitational $N$-body simulation using this algorithm and have used it to model the middle stage of planetary formation. Their simulation used 2000 equal mass ($10^{21}$ kg) bodies distributed in a ring about 1 AU, incorporated perfectly inelastic collisions, and included the interactions between all planetesimals. To reduce the computational time, Kokubo and Ida increased the collision frequency in their simulation by scaling up the radius of each planetesimal by a factor of 5. They performed these simulations in both two and three dimensions. Their two-dimensional simulations showed orderly growth, while their three-dimensional simulations exhibited runaway growth.

In this paper, we present results of two-dimensional $N$-body simulations of the late stage of planetary formation that utilize the Hermite Individual Timestep Scheme as in Kokubo and Ida (1995, 1996). Using this technique and the faster computers that are available today, we are able to extend the calculations of Lecar and Aarseth (1986) to include interactions among all of the bodies. To demonstrate the viability of our code, we duplicate the simulation of Lecar and Aarseth, and we find very good agreement with their results. We also incorporated the simple fragmentation model of Beaugé and Aarseth (1990) into our simulation, and in attempting to duplicate their results, we noted some discrepancies. Instead of inelastic mergers dominating the collisions between bodies, we found a proliferation of small fragments that causes the number of bodies in the simulation to increase exponentially. To be practical, *N*-body simulations must keep the number of bodies at a manageable level; therefore, we present results of simulations that compare different techniques of including and excluding the small fragments.

## 2. *N*-BODY MODEL

### 2.1. *The HITS Scheme*

A collection of *N* bodies subject to their mutual gravitational interactions obeys the set of coupled equations of motion

$$\mathbf{a}_i = \frac{d\mathbf{v}_i}{dt} = -\sum_{j\neq i}^{N} Gm_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}, \qquad (1)$$

where $\mathbf{r}_i$, $\mathbf{v}_i$, $\mathbf{a}_i$, and $m_i$ are the position, velocity, acceleration, and mass of particle *i*, respectively. While it is quite easy to write this set of equations, obtaining a solution for the gravitational *N*-body problem can be quite difficult.

The Hermite Individual Timestep Scheme (HITS) used by Kokubo and Ida (1995, 1996) has been adopted for use in this study. In an *N*-body simulation of planetary formation, it is necessary to accurately follow the evolution of the orbits both when perturbations are small, and orbits are nearly Keplerian, and through close encounters and collisions when the perturbations are large. To integrate the orbits of a swarm of gravitationally interacting planetesimals over these widely varying timescales with a minimum of computational effort, an individual timestep scheme is used. In this scheme, each planetesimal has its own time and timestep. The individual timestep adapts to the timescale on which each individual planetesimal's trajectory is currently evolving. Thus, this scheme preserves the accuracy of the integration by properly treating the interactions of the planetesimals, but does not allow the small timestep of a single planetesimal to slow the integration of the entire swarm.

The details of the HITS algorithm are presented in Makino and Aarseth (1992), and we have adopted these steps with the exception that since we are including collisions, we do not require a softening parameter in the calculations of acceleration and its time derivative. In addition, we calculate each particle's timestep according to Eq. (7) in Makino and Aarseth (1992) with parameter $\eta$ that can be adjusted to control the accuracy of the simulation (Beaugé and Aarseth 1990). The accuracy is reflected by how well the integration scheme conserves the total energy. For $\eta = 0.001$, energy was conserved to within 0.002% per thousand years. With $\eta = 0.005$, energy was conserved to within 0.006% per thousand years. Both these values are similar to those reported by Kokubo and Ida (1995, 1996).

In the construction of our code, it was important that the trajectories of the planetesimals be followed accurately so that the detection of collisions was guaranteed. A collision between planetesimals was defined to have occurred if the separation between the two bodies was less than the sum of their radii (overlapping disks in two-dimensions). When two planetesimals approach one another their gravitational interaction makes each body's timestep smaller. In order to guarantee that this timestep was small enough that two planetesimals could not pass through each other undetected, a modified timestep was adopted for close encounters and collisions. The expression used to determine the value of the modified timestep is

$$\Delta t = \left| \frac{r_{ij}^2}{4\mathbf{r}_{ij} \cdot \mathbf{v}_{ij}} \right|, \qquad (2)$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $v_{ij} = |\mathbf{v}_i - \mathbf{v}_j|$. This expression, developed by Beaugé and Aarseth (1990), is equal to one quarter of the amount of time that is required for two bodies to reduce their separation to zero given that their velocities remain constant. This modified timestep was calculated for all planetesimals within 0.1 AU of the body being updated. If it was found to be less than the timestep determined by the general formula, the modified timestep was used. During initial testing of the program with $\eta = 0.001$, collisions were detected when two bodies overlapped by no more than 1.2% of the sum of their radii. Thus it seems unlikely that any collisions went undetected in the results presented here.

### 2.2. *Fragmentation Model*

As planetary bodies are generally assumed to form through the collision and accretion of planetesimals, understanding what happens during a collision and how it affects the subsequent development of the planets is an important key in understanding how planets may grow from a swarm of planetesimals. Collisions are a highly complex phenomenon. The outcomes of collisions between planetesimals and the subsequent development of the planets, may depend as much on the physical characteristics of the planetesimals as on the dynamics of each collision.

To investigate the role of fragmentation in the late stage of planet formation, the collision model of Beaugé and Aarseth (1990) has been adopted without modification. This model is based on the laboratory and analytic work of Greenberg *et al.* (1978) and Spaute *et al.* (1985) and allows for inelastic rebound, rebound with crater formation, and fragmentation of one or both of the colliding bodies. This model also assumes that the impact energy is distributed evenly between the colliding bodies. A brief description (a complete discussion with relevant parameter values is in Beaugé and Aarseth 1990 and references therein) of the model follows for two bodies with masses $m_1$ and $m_2$ ($m_1 \geq m_2$) and relative velocity $\mathbf{V}_r = \mathbf{v}_2 - \mathbf{v}_1$, where the total impact energy is

$$E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} V_r^2. \tag{3}$$

1. Inelastic Rebound. If the relative velocity of the two bodies is less than the critical value for fracturing, $V_\mathrm{r} < \mathrm{V_c}$, where $V_\mathrm{c} = 2S/c\rho \sim 55$ m/s ($S$, $c$, and $\rho$ are the crushing strength, sound speed, and density of the bodies, respectively), then the bodies rebound from each other with a loss of energy. The rebound velocity after the collision is $\mathbf{V}_\mathrm{reb} = -c_i\mathbf{V}_\mathrm{r}$, where $c_i$ is the coefficient of restitution, and the individual body velocities can be found from momentum conservation. If the rebound velocity is less than the two-body escape velocity, then the collision is treated as an inelastic merger.
2. Rebound with Cratering. For $V_\mathrm{r} \geq V_\mathrm{c}$, but not sufficiently large to shatter the bodies, each body will be locally damaged or cratered with a loss of mass. The fraction of ejecta with speeds greater than the parent body's escape speed is modeled as a power law (Greenberg *et al.* 1978), and this mass is added to the other body in the collision with final body velocities again determined from momentum conservation.
3. Fragmentation. Based on experiments performed by Greenberg *et al.* (1978), if the energy per unit volume absorbed by a body during a collision is greater than the impact strength, $S$, then the body will shatter into many fragments. For computational purposes, Beaugé and Aarseth used a scheme where each fragmenting collision would generate four fragments and a remaining core. The sizes of the fragments follows the theoretical distribution of Spaute *et al.* (1985) where in general, low energy fragmenting collisions (fragmenting parameter, $\alpha \geq 1$) create large fragments of unequal mass, and more energetic collisions ($\alpha < 1$) create four smaller equal mass fragments. In the computational model, the fragments are positioned at a distance of $4R$ ($R$ is the radius of the mass $m_1 + m_2$) from the core and assigned velocities equal to the escape velocity of $m_1 + m_2$.

After incorporating this fragmentation model into our simulation, we found that, in almost all cases, the collisions were energetic enough to fragment both bodies. This disagrees with the results presented in Beaugé and Aarseth where they show that inelastic mergers dominate early in their simulation. Before presenting our results, we discuss the application of this model. First, consider the case of inelastic bouncing. When $V_\mathrm{r} \leq V_\mathrm{c}$ during a collision, where $V_\mathrm{c} = 55$ m s$^{-1}$, inelastic bouncing will occur. Consider two bodies colliding with $V_\mathrm{r} = V_\mathrm{c}$. Using Beaugé and Aarseth's coefficient of restitution, the rebound velocity of this collision is 39 m s$^{-1}$. If this rebound velocity is less than the two-body escape velocity the two bodies will be merged. Now the initial mass in Beaugé and Aarseth's simulations was $1.15 \times 10^{23}$ kg. A collision between two of these initial masses has a two-body escape velocity of about 2400 m s$^{-1}$. Therefore, for bodies of this size, inelastic rebound always means inelastic merger. Only for bodies of sufficiently small mass ($M < 4.2 \times 10^{19}$ kg) would one expect true rebound where the bodies escape after the collision.

Next consider the case of rebound with crater formation. This collision outcome occurs when $V_\mathrm{r} > V_\mathrm{c}$, but the impact energy of the collision is less than that needed to fragment the bodies. For two colliding bodies of equal mass, the parameters given in Beaugé and Aarseth give a relative velocity at the fragmentation threshold of $\sim$282 m s$^{-1}$. Consider two bodies colliding with a relative velocity slightly less than this; using the modified coefficient of restitution, their rebound velocity would be less than $\sim$141 m s$^{-1}$. As in the case of inelastic rebound, the masses of two colliding bodies would have to be low ($M < 1.4 \times 10^{22}$ kg) for the rebound velocity to be greater than the two body escape velocity. Thus for practical purposes, rebound with crater formation results in the inelastic merger between two colliding bodies of this initial mass.

During the initial testing of the collision model, it was found that collisions between lunar-sized planetesimals had relative velocities sufficient to cause the fragmentation of both colliding bodies without exception. For a sample of 100 collisions of lunar-sized bodies in the Lecar and Aarseth case simulation (presented in Section 3.1), the average relative velocity at impact was $\sim$3000 m s$^{-1}$ and $\alpha$ was never greater than 0.41. In all of the simulations performed using the Beaugé and Aarseth collision model, every collision resulted in the fragmentation of the colliding bodies with $\alpha < 1$. Thus the collision of two planetesimals results in the creation of four fragments with equal mass and a remaining core. For colliding bodies of roughly lunar mass the fragments carry away about 8% of the mass present in the initial two-body system. The remaining 92% of the mass is present in the remaining core. As the mass of the colliding bodies is increased, the fraction of the total mass carried away by the fragments decreases and collisions more closely resemble inelastic mergers than the destruction of the colliding bodies.

As the computational time required by a gravitational $N$-body simulation increases with $N^2$, a fragmentation model which increases the number of bodies in the simulation with every collision is not practical. For this reason, the focus of this study has been to investigate how the proliferation of small fragments that are produced in the fragmentation model described above influences the development of the largest planetesimals.

## 3. SIMULATION RESULTS

### *3.1. Comparison with Lecar and Aarseth (1986)*

In order to demonstrate the validity of our *N*-body program, a simulation was performed which is compared with the results of Lecar and Aarseth (1986). This simulation began with 200 lunar-sized bodies in initially circular orbits between 0.5 and 1.5 AU. The bodies had an initial mass of $7.35 \times 10^{22}$ kg with a mass density, $\rho = 3.34 \times 10^3$ kg m$^{-3}$, and they were distributed evenly in the orbital plane (i.e., uniform surface density). When two planetesimals collided they were merged and followed the trajectory of their center of mass.

In Fig. 1, the mean and standard deviation of the orbital eccentricities are plotted as functions of time for the first $2.5 \times 10^4$ years of the simulation. Both quantities vary smoothly until after $\sim 10^4$ years, when the number of bodies left in the simulation is small enough so that individual close encounters can significantly alter the mean and standard deviation of the eccentricity. These results are in excellent agreement with those reported by Lecar and Aarseth, and thus show that our treatment of the interactions between planetesimals is consistent with theirs. Because Lecar and Aarseth did not include long-range interactions between planetesimals during the early part of their simulation, this close agreement also indicates that distant interactions do not have a large influence on the mean and standard deviation of the eccentricity of a large number of planetesimals of roughly equal mass.

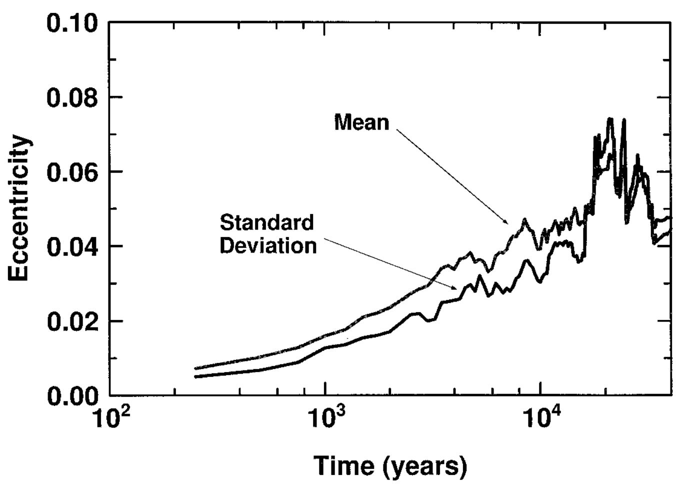


**FIG. 1.** The time evolution of the average orbital eccentricity and standard deviation calculated for all bodies in the simulation for comparison with Lecar and Aarseth (1986).

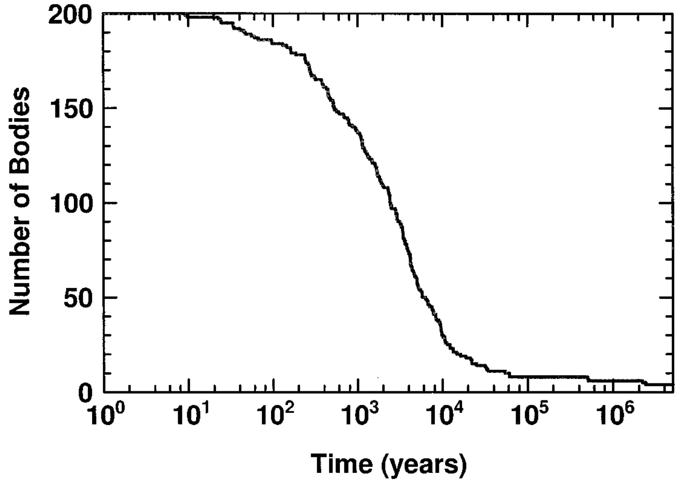


**FIG. 2.** The number of bodies in the simulation as a function of time for comparison with Lecar and Aarseth (1986).

Fig. 2 shows the number of bodies in the simulation plotted as a function of time. Here there is also very solid agreement with Lecar and Aarseth. Our simulation produces its first collision at just under 10 yr, and a total of 16 collisions by the first 100 yr. After this time, the number of bodies decreases smoothly with a half life of $\sim 3000$ yr until fewer than 30 bodies remain and collisions occur less frequently. This behavior is also in excellent agreement with Lecar and Aarseth.

The one difference between our simulation and that in Lecar and Aarseth is that they reached 6 bodies by 50,000 yr, whereas our simulation was at 10 bodies at the same time. We ran our simulation for a total time of $5 \times 10^6$ yr, and 6 more collisions resulted to bring the final number of bodies to 4. Figs. 3 and 4 show the configuration and orbits of the final bodies in our simulation after $5 \times 10^6$ years. The four final bodies are in qualitative agreement with the terrestrial planets in terms of their masses and orbits.

### *3.2. Fragmentation Simulations*

As discussed in Section 2, for simulations utilizing this fragmentation model with lunar sized bodies, we expect that every collision will result in the fragmentation of both bodies. Most of the mass will remain in the core, while a smaller fraction will escape in the form of four smaller bodies. The net result is that the number of bodies in the simulation increases exponentially as subsequent generations of collisional fragments of smaller and smaller size are produced. In a practical *N*-body simulation, there must be some lower bound to the size of the fragments that can

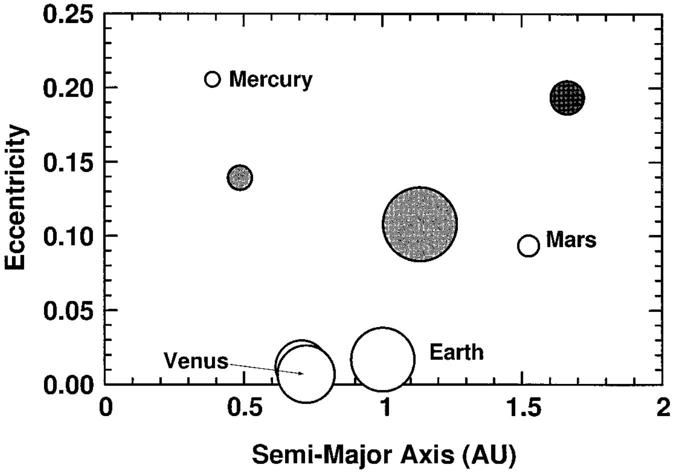


**FIG. 3.** The eccentricity vs semimajor axis at 5 Myr of the final bodies (shaded circles) for comparison with Lecar and Aarseth. The area of the circle is proportional to the planetary mass. Also shown for comparison are the terrestrial planets (white circles).

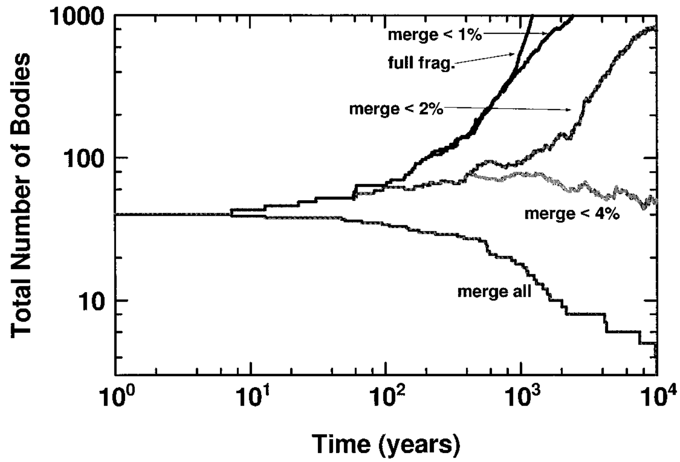


**FIG. 5.** The number of bodies in the simulation vs time on a logarithmic scale for the five different fragmentation cases as indicated.

be included in the simulation. Hence, the question that we wish to address here is: within the constraints of this fragmentation model, what effect does the inclusion or exclusion of fragments of various sizes have on the production and evolution of the larger bodies in the simulation?

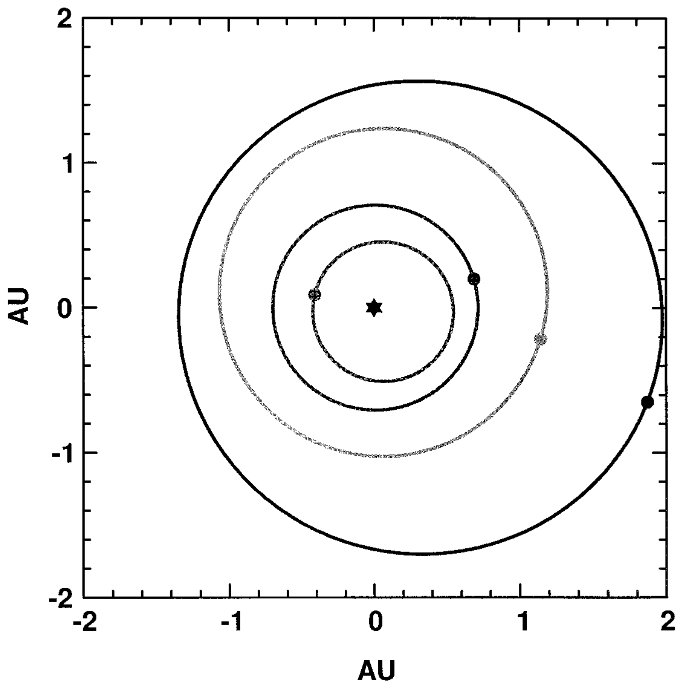


**FIG. 4.** The orbits of the final bodies at 5 Myr for comparison with Lecar and Aarseth (1986).

To determine whether the small fragments are important, we have performed five simulations, each with a different method of including the small fragments. These simulations all began with 40 identical bodies of initial mass $1.15 \times 10^{23}$ kg and mass density $\rho = 3 \times 10^3$ kg m$^{-3}$. They were placed in a 0.10 AU wide ring of uniform surface density centered on 1 AU and given circular velocities with random phase. In each of these simulations the 40 bodies were given identical initial positions and velocities. For practical reasons, a simulation was stopped when the number of bodies exceeded 1,000.

In the first simulation, labeled *full fragmentation* in the figures, all bodies are allowed to fully fragment on collisions, and the trajectories of all fragments are followed throughout the simulation. The most straightforward method of accounting for the small fragments is to simply discard them when they reach a limiting size; however, we found that doing this can remove a significant amount of the total mass from the simulation. Instead, fragments that reach a chosen minimum size are forced to merge inelastically on their next collision. In this way, mass is not lost from the simulation, but the effects of the small fragments can still be studied. In the simulations labeled merge <1%, merge <2%, and merge <4%, fragments of all masses are created; however, fragments with masses less than 1%, 2%, and 4% of the initial mass ($1.15 \times 10^{23}$ kg) respectively are not permitted to fragment further, and must suffer an inelastic merger on their next collision. The last simulation, labeled *merge all* in the figures, treats all collisions as completely inelastic mergers.

Figures 5 and 6 plot the number of bodies in these five simulations as functions of time. Figure 5 shows the data on a logarithmic time scale to emphasize the behavior at early times, while Fig. 6 plots the same data on a linear time scale to enhance the later times. For the full fragmentation

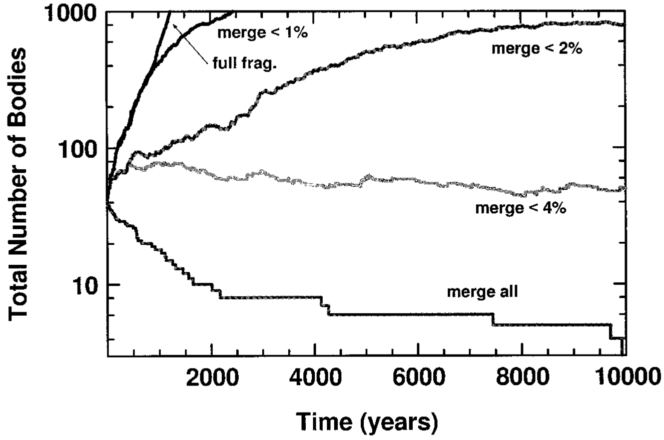


**FIG. 6.** The number of bodies in the simulation vs time on a linear scale for the five different fragmentation cases as indicated.

case, the number of bodies steadily grows to 1,000 in about 1,230 yr. The merge <1% case follows the full fragmentation case, but begins to diverge at about 800 yr when small fragments begin to be accreted. This case reaches 1,000 bodies in about 2,430 yr. The merge <4% case reaches a maximum number of about 80 bodies at ~1,500 yr and then steadily decreases to about 50 bodies at 10,000 years. The merge <2% case diverges from the merge <4% case at about 400 years as smaller fragments are created. It then climbs to about 800 bodies, where a quasi steady-state is reached as the number of fragments being created is balanced by the number being accreted. Finally, the merge all case monotonically decreases to three bodies by 10,000 yr.

The necessary computation time required for these simulations is a crucial practical factor. While the absolute time will vary depending on the machine being used, we can quote the relative computation times for the simulations that ran to completion. The merge <4% case required about 18 times the CPU time of the merge all case, and the merge <2% case required approximately 1,000 times the computation time than the merge all case. This, of course, reflects the general increase in computation time as $N^2$ (in two dimensions) and the additional effect that for a large number of bodies, the number of collisions and close encounters increases so that the average timestep is reduced.

Figure 7 shows the mass distributions of the bodies for all five simulations after 1,000 years. The full fragmentation case is shown in Fig. 7a, where there are 550 total bodies in the simulation with 18 of these having masses greater than 80% of the initial mass. These bodies make up more than 88% of the total mass in the simulation. Several groups of fragments can be seen in Fig. 7a. The first generation of fragments result from collisions between the initial masses and have masses of roughly $10^{21}$ kg. The second generation of fragments resulted from the collision between first generation fragments and initial masses and have masses of about half that of first generation fragments. The third generation of fragments were produced by collisions between fragments and have masses as small as $10^{13}$ kg.

Figures 7b through 7e show the mass and semi-major axes of the bodies after 1,000 yr for each of the simulations with differing treatments of planetesimal fragments. In each of these, there are roughly 15 to 20 bodies with a mass greater than 80% of the initial planetesimal mass. Among these large bodies, there are also between six and nine bodies whose masses are greater than about two initial masses and one to three bodies whose mass is about three initial masses.

Figures 8a through 8e show the masses and semi-major axes of the bodies at the end of each simulation, which was 10,000 years if the number of bodies never reached 1,000. In each of the simulations that reached 10,000 years, there are between 3 and 6 bodies with masses greater than 80% of the initial mass, and there are two bodies in each simulation with masses approximately 10 times the initial mass. Furthermore, the orbital semimajor axes of these large bodies are similar in each of the simulations. This can be seen more clearly in Fig. 9 where we plot the semimajor axis vs the eccentricity for all of the bodies at the end of the simulations that reached 10,000 yr. Here, we see that each simulation produces two large bodies in similar orbits. As would be expected, the large bodies in the merge all simulation are slightly more massive than those in the other simulations because none of their mass has been lost to the creation of small fragments during collisions. From these results it appears that the inclusion of Beaugé and Aarseth's fragmentation model in late stage planetary formation simulations has only a small effect on the evolution of the number or mass of the largest planetesimals.

Figure 10 plots the time evolution of the fraction of the total mass in the simulation that is contained in large bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg). For the cases where fragments less than some minimum are elastically merged, this fraction initially decreases steadily as mass is transferred to smaller fragments; however, as can be seen in the merge <4% case, the mass must be returned to the large bodies as fragments less than the minimum are accreted. At 10,000 yr, the merge <2% case is just beginning to reabsorb the smaller fragments, and the slope of the curve in Fig. 10 is beginning to reach a minimum.

In planetary formation simulations, the major concern is with the large bodies that have the potential to evolve into planets. In the five different fragmentation simulations, we wanted to determine if there were any significant differences in the number of large bodies produced and their orbits. Figures 11 (logarithmic time scale) and 12

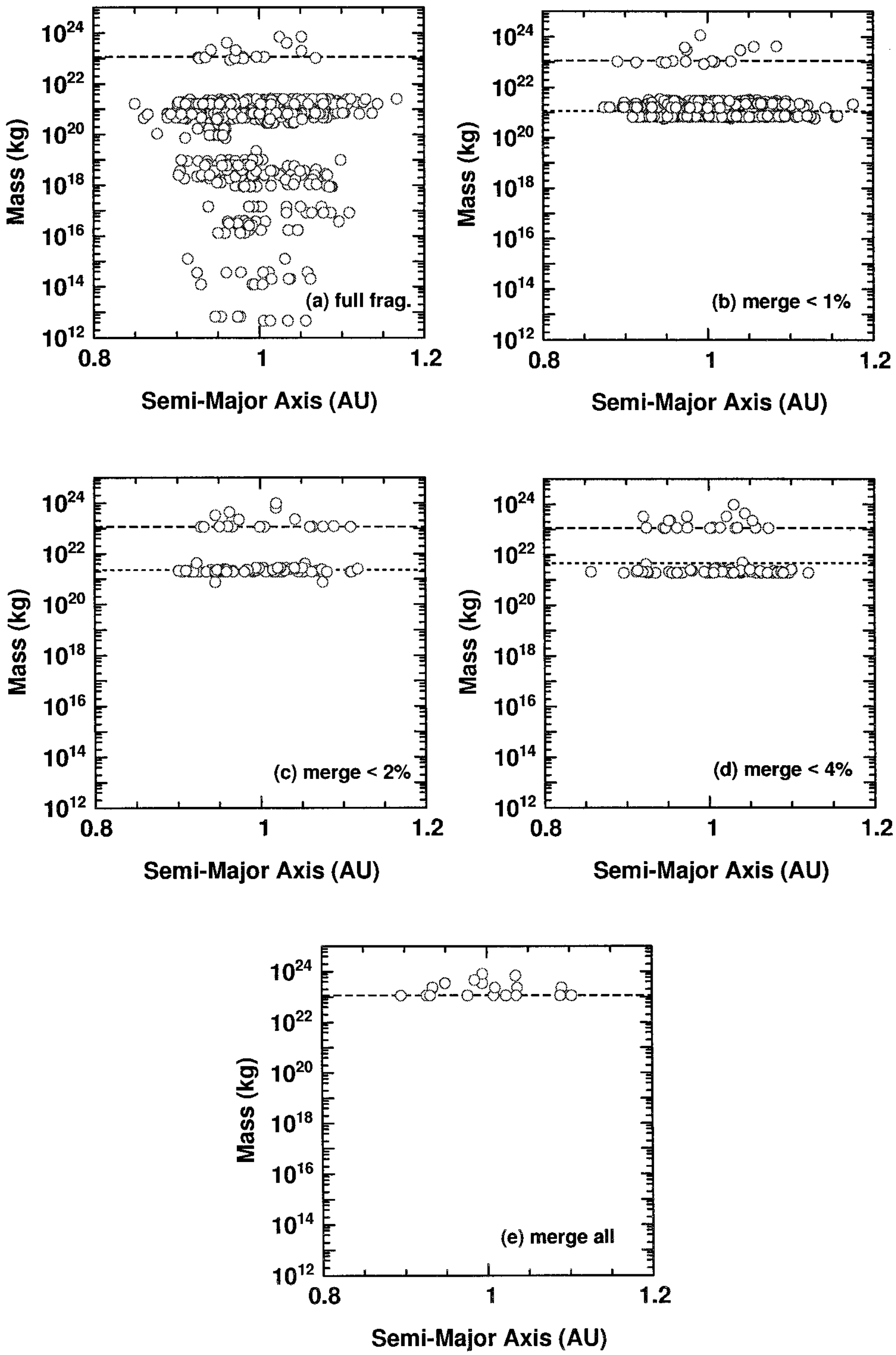


**FIG. 7.** Mass distribution in semimajor axis after 1,000 yr for the five fragmentation simulations: (a) full fragmentation; (b) merge masses less than 1% of the initial mass on their next collision; (c) merge masses less than 2% of the initial mass on their next collision; (d) merge masses less than 4% of the initial mass on their next collision; and (e) all collisions are treated as inelastic mergers. The dashed lines indicate the initial mass and the minimum fragment mass where appropriate.

(linear time scale) plot the number of bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg) as functions of time. These plots illustrate that the number of large bodies in all five simulations is roughly the same whether or not small fragments are created. To investigate the effect that the inclusion of small fragments may have on the orbits of the larger bodies, a plot of the average eccentricity of the larger bodies as a function of time was

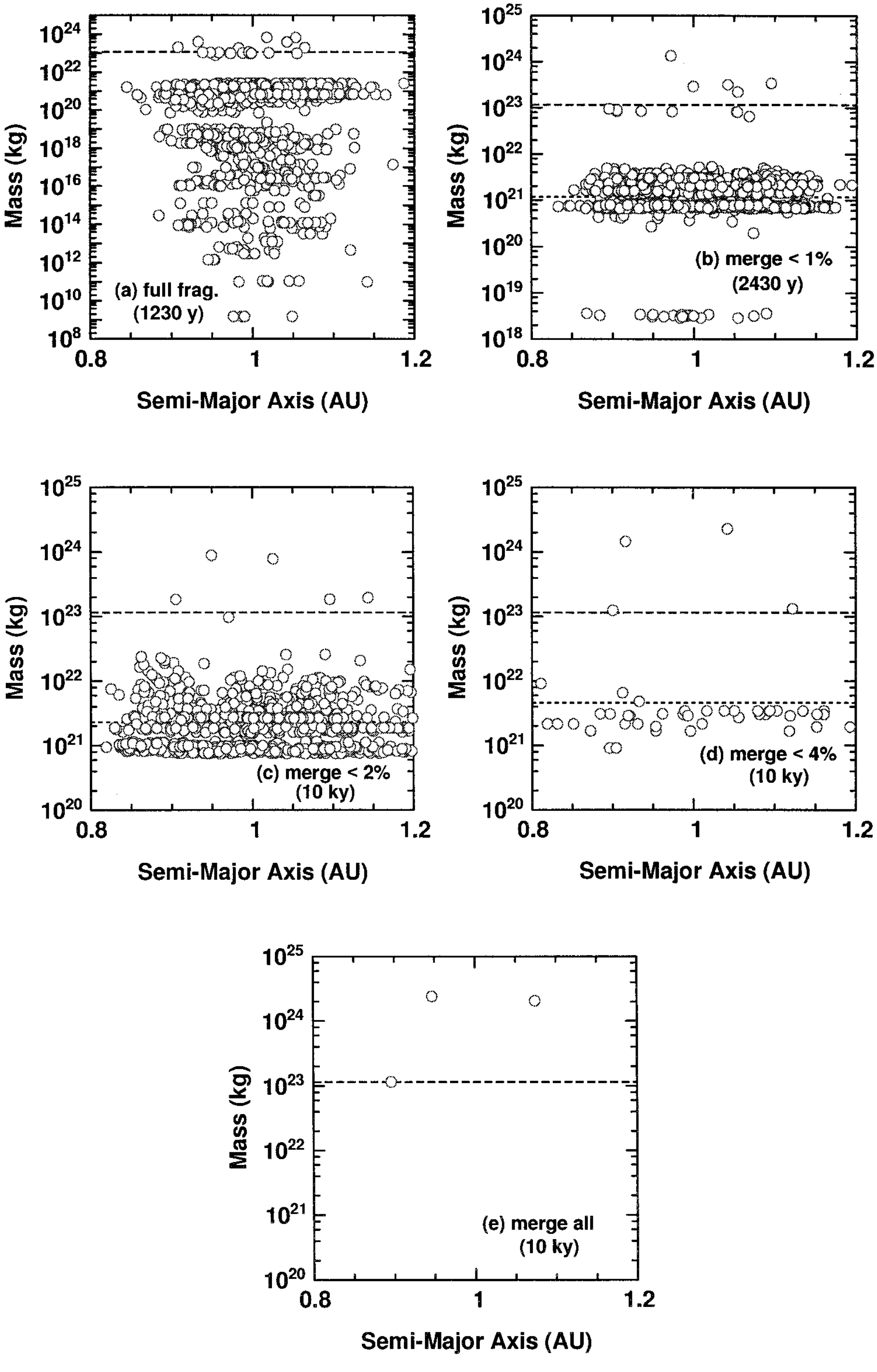


**FIG. 8.** Mass distribution in semimajor axis at the end of the simulation for the five fragmentation cases: (a) full fragmentation, terminated at 1,230 yr; (b) merge masses less than 1% of the initial mass on their next collision, terminated at 2,430 yr; (c) merge masses less than 2% of the initial mass on their next collision, ran to 10,000 yr; (d) merge masses less than 4% of the initial mass on their next collision, ran to 10,000 yr; and (e) all collisions are treated as inelastic mergers, ran to 10,000 yr. Note the difference in the mass scale. The dashed lines indicate the initial mass and the minimum fragment mass where appropriate.

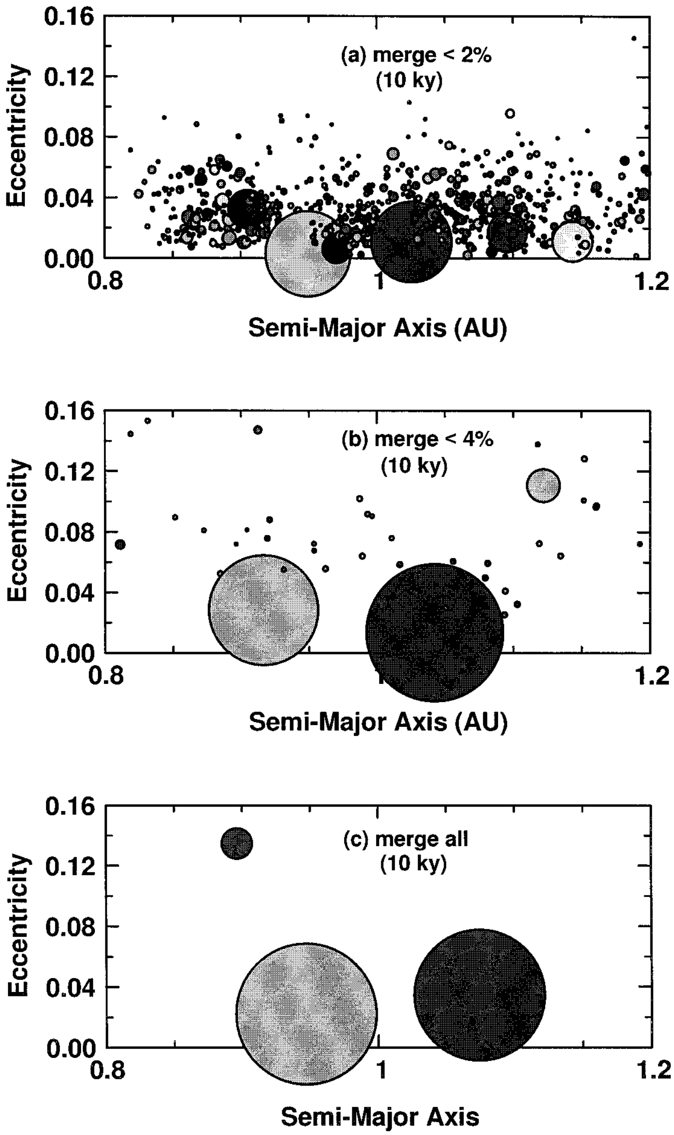


**FIG. 9.** The eccentricity vs semimajor axis of all bodies for the simulations that ran to 10,000 yr, where: (a) merge masses less than 2% of the initial mass on their next collision; (b) merge masses less than 4% of the initial mass on their next collision; and (c) all collisions are treated as inelastic mergers. The area of each circle is proportional to the mass of that body.

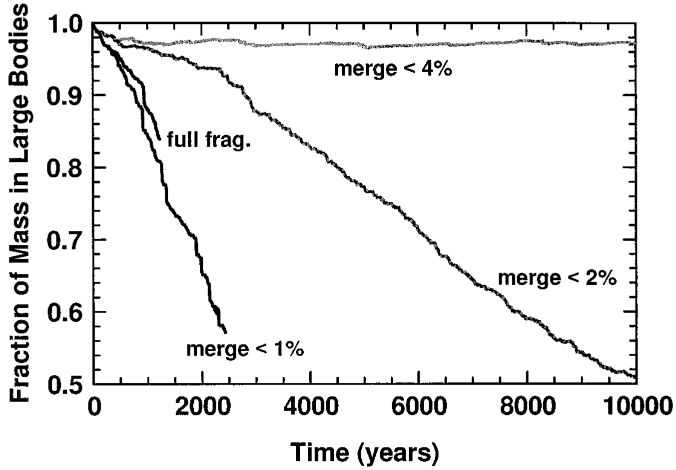


**FIG. 10.** The fraction of the total mass vs time contained in large bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg) for the four fragmentation cases as indicated.

## 4. DISCUSSION AND CONCLUSIONS

We have developed a two-dimensional gravitational *N*-body simulation based on the Hermite Individual Timestep integrator to study the final stage of planetary formation. We have presented results that extend the treatment of Lecar and Aarseth (1986) to include all interactions throughout the simulation. Our results show excellent agreement with theirs in both the collision frequency and the evolution of the orbits.

We have also included and examined the collisional fragmentation model of Beaugé and Aarseth (1990). We have

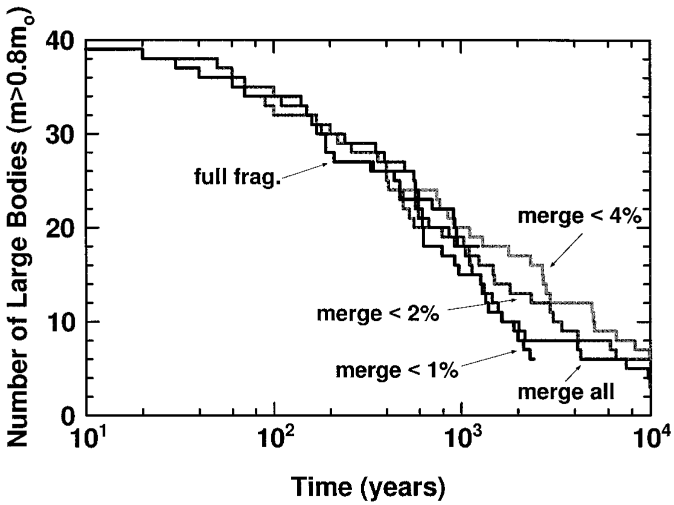


**FIG. 11.** The number of bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg) vs time on a logarithmic time scale for the five different fragmentation cases as indicated.

made. This is shown in Fig. 13. This plot extends out to 2,500 yr, at which time the number of large bodies in each simulation is between 6 and 16. While the average eccentricity of the large bodies fluctuates as they undergo close encounters, it has similar values in each simulation and is seen to evolve in much the same fashion. It is difficult to determine what dynamical effect, if any, that the inclusion of small fragments, according to the model of Beaugé and Aarseth (1990), has on the evolution of the larger bodies in the simulations.

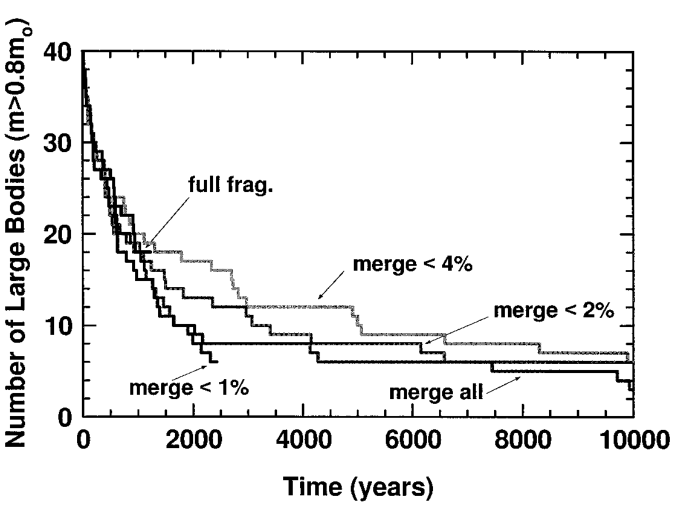


**FIG. 12.** The number of bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg) vs time on a linear time scale for the five different fragmentation cases as indicated.

found that for lunar-sized bodies ($\sim 10^{23}$ kg) that two-body collisions are always sufficiently energetic to fragment both bodies. Thus, according to this model, every collision will result in the production of four small fragments and a massive core that is $\sim$90% of the two-body mass. The question that we have addressed here is: do the small fragments have any significant effect on the subsequent evolution of the larger bodies? Based on the results of five simulations that treated the fragments in different manners, we find no significant effect either on the number of larger bodies in the simulation or their orbital evolution.

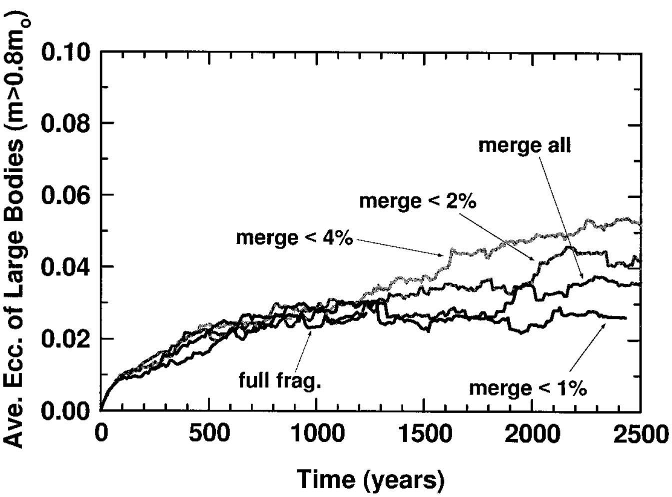


**FIG. 13.** The mean orbital eccentricity of the bodies with mass greater than 80% of the initial mass ($1.15 \times 10^{23}$ kg) vs time for the five different fragmentation cases as indicated.

Therefore, within the constraints of this model, it appears as if late stage planetary formation simulations can ignore collisional fragmentation and treat all collisions as inelastic mergers. This, of course, translates into a significant savings in computation time since the number of bodies in the simulation will not grow and slow the simulation.

It should be pointed out that our results regarding the role of collisional fragmentation in late stage simulations are dependent on the validity of the model as presented in Beaugé and Aarseth (1990). It should be emphasized that the assumptions and parameters employed in this collision model were not investigated in this study. Because the collision and fragmentation of lunar-sized planetesimals is a highly complicated phenomenon, any practical collision model represents a crude approximation of real world collisions. A logical next step in improving our understanding of the role of fragmentation in the late stage of planetary development would be to investigate the dependence of the evolution of the largest planetesimals on the values of the collision parameters and on the assumptions employed in Beaugé and Aarseth's model. While the results presented here are convincing for the collision model and parameters used, these results need to be tested with a wide range of parameter values and colliding masses before it can be concluded that fragmentation plays a minimal role in the late stage growth of the planets.

Finally, we should address the question of two-dimensional versus three-dimensional simulations. While fully three-dimensional simulations are desirable, the additional computation time that is required to produce the same simulation can become prohibitive. We have endeavored in this study to always compare simulation results with other similar simulations of the same dimensionality. Two-dimensional simulation results (e.g., times and final orbits) of planetary formation should not be extended to three dimensions; and thus, we do not claim that our results should have any bearing on the characteristics of the actual planets. Based on theoretical arguments, Lecar and Aarseth (1986) estimate that their simulation beginning with 200 bodies would require 10 Myr of simulation time. At the present time, we are working to extend the simulations presented here to three dimensions.

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