Where We Stand with Conservation of Energy

For a system going from initial state \( i \) to a final state \( f \):

If there are no dissipative forces (e.g. friction) or applied forces, the Mechanical Energy is Conserved.

\[
\Delta E_{mech} = \Delta K + \Delta U = 0
\]

\[
\Delta K = \frac{1}{2} m v_{f}^2 - \frac{1}{2} m v_{i}^2 = \frac{1}{2} m (v_{f}^2 - v_{i}^2)
\]

Where:

For **Uniform Gravity**:

\[
\Delta U_g = m g y_f - m g y_i = m g (y_f - y_i)
\]

For a **Linear Spring**:

\[
\Delta U_s = \frac{1}{2} k x_{s_f}^2 - \frac{1}{2} k x_{s_i}^2 = \frac{1}{2} k (x_{s_f}^2 - x_{s_i}^2)
\]

*with \( x_s = 0 \) at equilibrium*

What about other forces that can’t be described with a Potential Energy?
Nonconservative* Forces

Definition: A **nonconservative force** is a force for which the work done by the force in going from some initial point \(i\) to some final point \(f\) does depend on the path followed.

For example, Friction:

The work done by the friction force is different for each path from \(i\) to \(f\).

So, we cannot associate a potential energy with a nonconservative force.

But, we can include nonconservative forces in our conservation of energy technique.

*Again, as we’ll see, the name comes from the fact that nonconservative forces do not conserve the mechanical energy.*
The Complete Conservation of Mechanical Energy

We started with the **Work – Kinetic Energy Theorem** for a system going from an initial state, \( i \), to a final state, \( f \):

\[
\Delta K = W_{\text{net}}(i \rightarrow f) = W_c(i \rightarrow f) + W_{nc}(i \rightarrow f)
\]

For Conservative Forces*: \( \Delta U = -W_c(i \rightarrow f) \)

Define: Total Mechanical Energy: \( E_{\text{mech}} = K + U \)

so, \( \Delta E_{\text{mech}} = \Delta K + \Delta U \)

So, the **Work – Kinetic Energy Theorem** can be written as:

\[
\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}
\]

This is the equation we’ve been using with the added feature that we now know how to include forces like friction or applied forces. *They go right here.*

We see that potential energy is energy that can be stored and returned to kinetic. Applied forces can change the kinetic or the potential energies, and hence the mechanical energy. On the other hand, dissipative forces, like friction, will always decrease the mechanical energy and increase the thermal energy.

*If you have more than one conservative force, you’ll need a potential energy for each.
Whiteboard Problem: 9/10-12

A horizontal spring with spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction between the box and the surface is 0.15.

How far does the box slide across the rough surface before it comes to rest?* (LC)

*As before, you could use dynamics and kinematics here, but don’t. Use energy conservation; real soon, we’ll get to problems where using dynamics and kinematics would be very difficult.
Whiteboard Problem: 9/10-13

Kathy’s baby brother sits on a sled. The combined mass of the sled and the baby is \( m \). Kathy pulls on the rope that is at an angle \( \theta \) above the horizontal. The tension \( T \) is constant and the coefficient of kinetic friction between the sled and the snow is \( \mu_k \).

Find an expression for the speed of the sled after Kathy has pulled it a distance \( d \) starting from rest. \( \text{(LC)} \)

There are two traps in this problem -- don’t fall into them!
Whiteboard Problem: 9/10-14

A block of mass $m$ starts from rest at height $h$. It slides down a frictionless slope, across a rough horizontal surface of length $L$, then up another frictionless slope. The coefficient of kinetic friction on the rough surface is $\mu_k$.

Find an expression for the height that the block goes up the second slope. (LC)
Force from Potential Energy

We know how to find the Work done by a Force:

\[ W = \int \vec{F} \cdot d\vec{s} \]

Or, in one dimension:

\[ W = \int F_x dx \]

And for a conservative force:

\[ \Delta U = -W \text{(done by } F_x) = - \int F_x dx \]

i.e. if we know \( F_x \), we can find \( \Delta U \).

Can we go the other way? i.e. if we know the potential energy, can we find the force? As your author shows (for one dimension):

\[ F_x = -\frac{dU}{dx} = -(\text{slope of the } U \text{ vs. } x \text{ curve}) \]

Everyone forgets this negative sign, but it’s important; you just have to remember it. It will even come back to haunt us in PHY192 when we do electric potential!

So, a changing potential energy implies a force is present.
A particle has the potential energy shown below.

a) **What is the x-component of the force on the particle at** \( x = 5 \text{ cm} \)? *(LC)*

b) **What is the x-component of the force on the particle at** \( x = 15 \text{ cm} \)? *(LC)*

c) **What is the x-component of the force on the particle at** \( x = 30 \text{ cm} \)? *(LC)*
Whiteboard Problem 9/10 - 16

A particle moving along the x-axis has the potential energy:

\[ U(x) = \frac{10}{x} \text{ J} \]

where \( x \) is in m.

What is the x-component of the force on the particle at \( x = 5 \text{ m} \)? (LC)
For a system, **Power is the rate that energy is transferred into or out of the system:**

\[
\text{Power, } P = \frac{dE_{\text{sys}}}{dt} \quad \text{Units: } 1 \frac{J}{s} = 1 \text{ Watt (W)}
\]

For a mass for which the energy is changing because a force is doing work on it, your author shows:

\[
\text{Power, } P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}
\]
You push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s.

How much work did you do, and what was your power output (LC) while pushing?

For steel on steel: $\mu_k = 0.6$