

Hydrogen \rightarrow Helium Fusion

4 proton \rightarrow 1 Helium Nucleus + energy.
(${}^4\text{He} = 2p \ \& \ 2n$)

a.) mass of ${}^4\text{He} = 6.64 \times 10^{-27} \text{ kg}$

mass of proton = $1.67 \times 10^{-27} \text{ kg}$

Note: $4m_p > m_{\text{He}}$, the difference is converted to energy.

$$\begin{aligned} \text{mass difference, } \Delta &= 4m_p - m_{\text{He}} \\ &= 4.0 \times 10^{-29} \text{ kg.} \end{aligned}$$

$$\begin{aligned} \text{So Energy, } E &= mc^2 = \Delta c^2 \\ &= \underline{\underline{3.6 \times 10^{-12} \text{ J.}}} \end{aligned}$$

So, the fusion of 4 proton \rightarrow ${}^4\text{He}$

releases $3.6 \times 10^{-12} \text{ J}$. Note, not all of the mass is converted to energy.

b.) $M_{\text{H}_2\text{O}} = 1 \text{ kg}$, molecular mass, $M_{\text{H}_2\text{O}} \sim 18u$
mass of Hydrogen atom $m_{\text{H}} \sim 1u$

So, water is $\sim \frac{1}{9}$ hydrogen by mass

So, amount of hydrogen in jug, $M_{\text{H}} \sim \frac{1}{9} \text{ kg}$

Number of hydrogen atoms, $N = \frac{M_{\text{H}}}{m_p} = 6.6 \times 10^{25}$

For $4H \rightarrow {}^4He$, the number
of fusion, $N_f = \frac{N}{4}$

and each fusion releases, $\bar{E} = 3.6 \times 10^{-12} \text{ J}$

So, the total released
energy is

$$E_{\text{released}} = \left(\frac{N}{4}\right) \bar{E} = \underline{6 \times 10^{13} \text{ J}}$$

Energy used in US in 2010 $\sim 1 \times 10^{20} \text{ J}$

So, we would need $\frac{E_{\text{us}}}{E_{\text{rel. per jug}}}$

~ 1.6 million jugs of water
to power the US for one year!