



Potential at P : $dv = \frac{k dq}{r}$

where $dq = \lambda dx$ and $r = \sqrt{x^2 + z^2}$

So:

$$dv = \frac{k \lambda dx}{\sqrt{x^2 + z^2}}$$

and

$$V = \int_{@} dv = \int_{-L/2}^{L/2} \frac{k \lambda dx}{\sqrt{x^2 + z^2}} = k \lambda \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2 + z^2}}$$

Table: $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$

So

$$V = k \lambda \ln(x + \sqrt{x^2 + z^2}) \Big|_{-L/2}^{L/2}$$

$$= k \lambda \left\{ \ln\left(\frac{L}{2} + \sqrt{\frac{L^2}{4} + z^2}\right) - \ln\left(-\frac{L}{2} + \sqrt{\frac{L^2}{4} + z^2}\right) \right\}$$

So

$$V = \frac{kQ}{L} \ln \left\{ \frac{\sqrt{\frac{L^2}{4} + z^2} + \frac{L}{2}}{\sqrt{\frac{L^2}{4} + z^2} - \frac{L}{2}} \right\}$$

