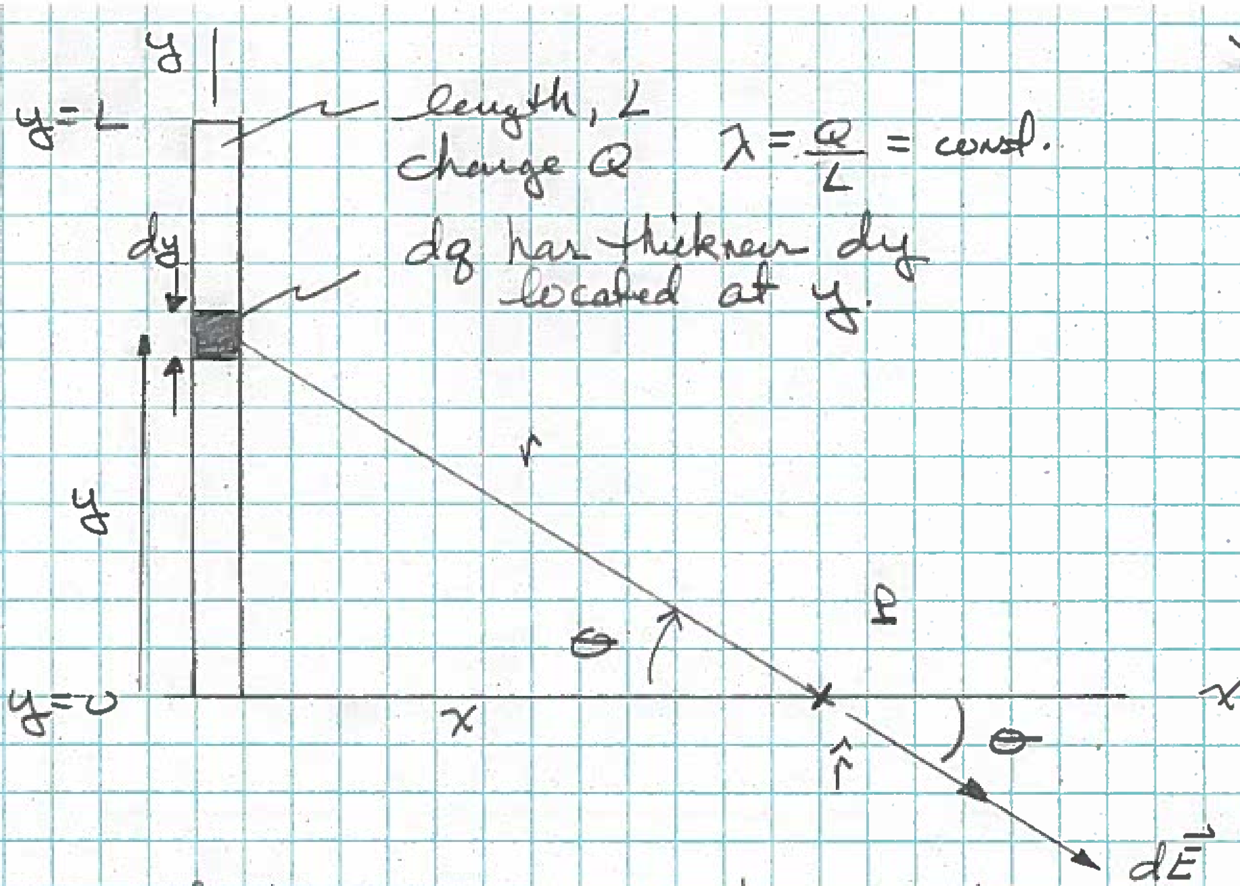

Treat dq as a point charge which creates field $d\vec{E}$ at P:

$$d\vec{E} = \frac{K\,dq}{r^2}\hat{r} \quad \text{and} \quad |d\vec{E}| = \frac{K\,dq}{r^2}$$

Now, to integrate this, we have to express dq, r, and $\hat{r}$ in terms of constants, the coordinates of P, and the coordinates of dq.

So: $\quad dq = \lambda\,dy$

$$r^2 = x^2 + y^2$$

$$d\vec{E} = |d\vec{E}|\cos\theta\,\hat{\imath} - |d\vec{E}|\sin\theta\,\hat{\jmath}$$

where: $\quad \cos\theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2+y^2}}$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

Or, $\hat{r} = \cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$

So:

$$d\vec{E} = \frac{K\lambda\, dy}{(x^2+y^2)} \left\{ \frac{x}{\sqrt{x^2+y^2}}\,\hat{i} - \frac{y}{\sqrt{x^2+y^2}}\,\hat{j} \right\}$$

or,

$$d\vec{E} = \underbrace{\frac{K\lambda x\, dy}{(x^2+y^2)^{3/2}}}_{dE_x}\,\hat{i} - \underbrace{\frac{K\lambda y\, dy}{(x^2+y^2)^{3/2}}}_{dE_y}\,\hat{j}$$

Integrate components separately:

$$dE_x = \frac{K\lambda x\, dy}{(x^2+y^2)^{3/2}}$$

$$E_x = \int_{\text{charge}} dE_x = \int_0^L \frac{K\lambda x\, dy}{(x^2+y^2)^{3/2}}$$

$$= K\lambda x \int_0^L \frac{dy}{(x^2+y^2)^{3/2}}$$

How do we do this integral?

Use the table in appendix A-3:

find: $\displaystyle\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$

$a = \text{const.}$

for our integral:

Table	Us
$x \leftrightarrow$	y
$a \leftrightarrow$	x

So:

$$E_x = K\lambda x \int_0^L \frac{dy}{(x^2+y^2)^{3/2}}$$

$$= K\lambda x \left\{ \frac{y}{x^2\sqrt{x^2+y^2}} \right\} \Bigg|_0^L$$

$$= \frac{K\lambda}{x} \left\{ \frac{L}{\sqrt{x^2+L^2}} - 0 \right\}$$

$$\therefore \quad E_x = \frac{K\lambda L}{x\sqrt{x^2+L^2}}$$

Now:

$$E_y = \int_{\text{charge}} dE_y = -\int_0^L \frac{K\lambda y\,dy}{(x^2+y^2)^{3/2}}$$

$$= -K\lambda \int_0^L \frac{y\,dy}{(x^2+y^2)^{3/2}}$$

<u>Table</u>: $\displaystyle \int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$

<u>Table</u>		<u>Us</u>
x	$\leftrightarrow$	y
a	$\leftrightarrow$	x

So:

$$E_y = -K\lambda \left\{ -\frac{1}{\sqrt{x^2+y^2}} \right\} \Bigg|_0^L$$

$$= K\lambda \left\{ \frac{1}{\sqrt{x^2+L^2}} - \frac{1}{x} \right\}$$

So, finally:

$$\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath}$$

$$= \frac{K\lambda L}{x\sqrt{x^2+L^2}}\hat{\imath} + K\lambda\left[\frac{1}{\sqrt{x^2+L^2}} - \frac{1}{x}\right]\hat{\jmath}$$

or, $\lambda = \frac{Q}{L}$

$$\boxed{\vec{E} = \frac{KQ}{x\sqrt{x^2+L^2}}\hat{\imath} + \frac{KQ}{L}\left[\frac{1}{\sqrt{x^2+L^2}} - \frac{1}{x}\right]\hat{\jmath}}$$

Also, for $x >> L$, ignore L^2 in $\sqrt{x^2+L^2}$

$$\vec{E}(x>>L) \rightarrow \frac{KQ}{x^2}\hat{\imath}$$

i.e. a point charge when viewed from far away.