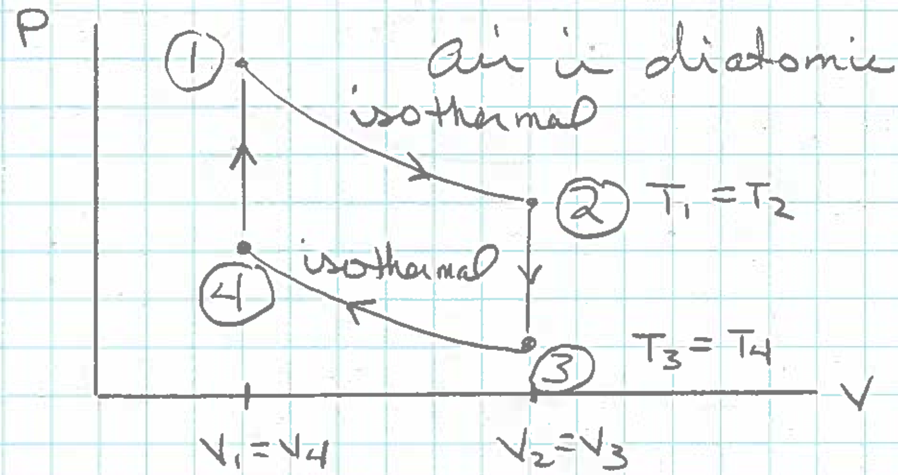
Stirling Engine



a.)

Use $\Delta E_{th} = Q - W_s$

$1 \to 2$: $\Delta E_{th} = 0$; $W_{s\,1\to2} = +\int P\,dV = nRT_1 \ln\left(\frac{V_2}{V_1}\right)$

$Q_{1\to2} = nRT_1 \ln(V_2/V_1) > 0$

$2 \to 3$: $W_s = 0$; $\Delta E_{th} = nC_V \Delta T = nC_V (T_3 - T_1)$

$Q_{2\to3} = \Delta E_{th} = nC_V (T_3 - T_1) < 0$

$3 \to 4$: $\Delta E_{th} = 0$; $W_{s\,3\to4} = nRT_4 \ln\left(\frac{V_4}{V_3}\right)$

$Q_{3\to4} = nRT_4 \ln\left(\frac{V_4}{V_3}\right) < 0$

$4 \to 1$: $W_s = 0$; $\Delta E_{th} = nC_V \Delta T = nC_V (T_1 - T_4)$

$Q_{4\to1} = nC_V (T_1 - T_4) > 0$

$W_{out} = W_{s\,1\to2} + W_{s\,3\to4}$

$= nRT_1 \ln\left(\frac{V_2}{V_1}\right) + nRT_4 \underbrace{\ln\left(\frac{V_4}{V_3}\right)}_{= \ln\left(\frac{V_1}{V_2}\right) = -\ln\left(\frac{V_2}{V_1}\right)}$

So: $W_{out} = nR \ln\left(\frac{V_2}{V_1}\right)(T_1 - T_4)$

$$Q_H = \Sigma \text{ (positive } Q\text{'s)}$$
$$= Q_{1\to 2} + Q_{4\to 1}$$
$$= nRT_1 \ln\left(\frac{V_2}{V_1}\right) + nC_V(T_1 - T_4)$$

Now: $$\eta = \frac{W_{OUT}}{Q_H}$$
$$= \frac{nR\ln\left(\frac{V_2}{V_1}\right)(T_1 - T_4)}{nRT_1\ln\left(\frac{V_2}{V_1}\right) + nC_V(T_1 - T_4)}$$

$$\eta = \frac{(T_1 - T_4)}{T_1 + \frac{C_V(T_1 - T_4)}{R\ln(V_2/V_1)}}$$

Now: $C_V = \frac{5}{2}R$ for diatomic

So: $$\boxed{\eta = \frac{(T_1 - T_4)}{T_1 + \frac{5(T_1 - T_4)}{2\ln(V_2/V_1)}}}$$

Also, for a Carnot cycle between T_1 & T_4:

$$\eta_{carnot} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H} = \frac{T_1 - T_4}{T_1}$$

and, $T_1 > T_4$ & $V_2 > V_1 \Rightarrow \eta = \dfrac{T_1 - T_4}{T_1 + (\text{something} > 0)}$

$\therefore$ $\underline{\eta < \eta_{carnot}}$

b.) for coffee cup engine:

$T_1 \approx 100°C = 373K$

$T_4 \approx 20°C = 293K$

and $\frac{V_2}{V_1} \approx 4$

So: $$\eta = \frac{(T_1 - T_4)}{T_1 + \frac{5(T_1 - T_4)}{2\ln(V_2/V_1)}} = 0.155 = \underline{15.5\%}$$

c.) for $T_1 = 373K = T_H$

and, $T_4 = 293K = T_C$

a <u>Carnot engine</u> between T_H & T_C:

$$\eta_{carnot} = 1 - \frac{T_C}{T_H} = 0.214 = \underline{21.4\%}$$