36-2: Special Relativity - 2

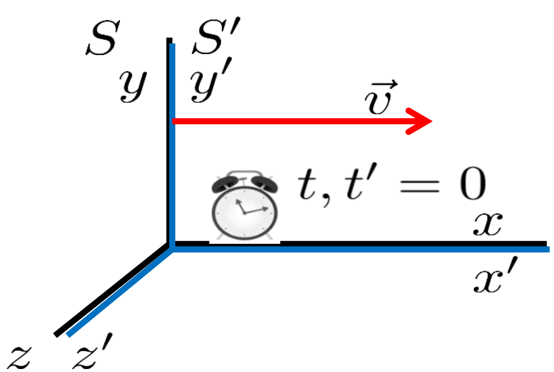
Last time we reviewed classical Galilean relativity and looked at some of the issues presented when you try to include Maxwell's Electromagnetism **– here's a quick review.**

Cass – 2

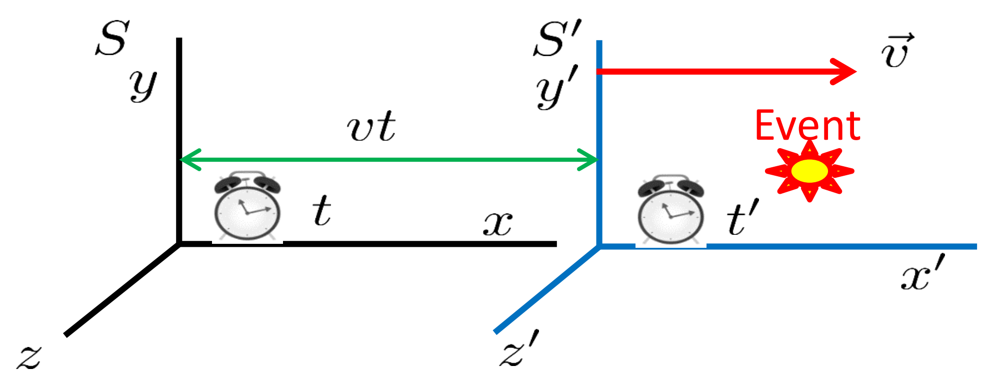
(~4m10s active link)

We also reviewed the **Galilean Transformations** , the postulates of Special Relativity, and the **Lorentz Coordinate Transformations** which are consistent with the postulates.

$\text{At } t = t' = 0$



$\text{At } t, t' > 0$



Galilean Coordinate Transformations

Transformation	Inverse Transformation
$x = x' + vt$	$x' = x - vt$
$y = y'$	$y' = y$
$z = z'$	$z' = z$
$t = t'$	$t' = t$

Lorentz Coordinate Transformations

Transformation	Inverse Transformation
$x = \gamma(x' + vt')$	$x' = \gamma(x - vt)$
$y = y'$	$y' = y$
$z = z'$	$z' = z$
$t = \gamma(t' + \frac{vx'}{c^2})$	$t' = \gamma(t - \frac{vx}{c^2})$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

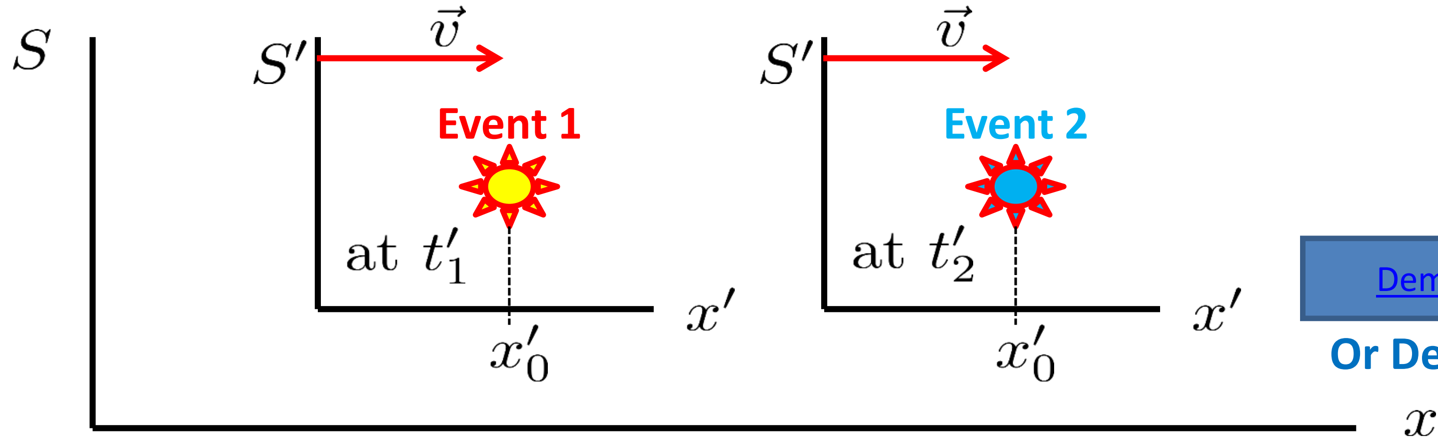
Time Dilation

Time Dilation refers to the result from Special Relativity that moving clocks run more slowly than clocks at rest. Your author does what most textbooks do: derive the time dilation formula using an idealized light clock – here's a quick demonstration of this.

Cass – TD

(~2m35s active link)

We can easily derive the time dilation formula directly from the **Lorentz Coordinate Transformations**. **Consider two events that occur at the same place in the moving frame S':**



Demo-1

Or Demo

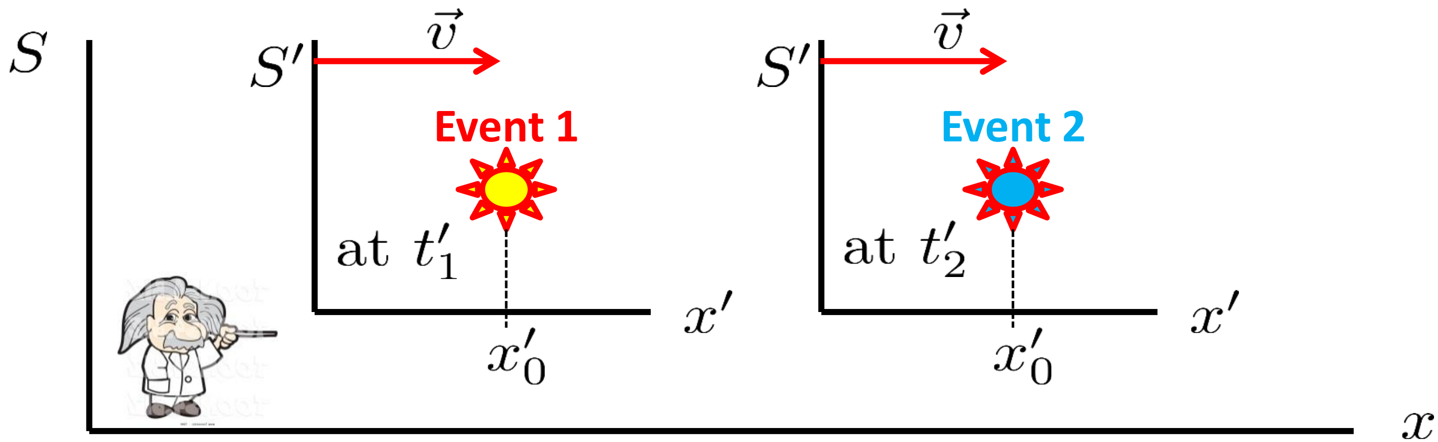
In frame S': the coordinates of **Event 1** are: (x'_0, t'_1)

the coordinates of **Event 2** are: (x'_0, t'_2)

The time between the events as seen in frame S' is: $t'_2 - t'_1 = \Delta t_p$

Where: $\Delta t_p =$ Proper Time between two events.
Time between two events measured in a frame where the two events have the same space coordinate (*i.e.* occur at same place).

Time Dilation



Now, the really important question: What does an observer in frame S measure for the time between the two events?

Use the LCT to transform the times in S' to S:

$$t_1 = \gamma(t'_1 + \frac{vx'_0}{c^2})$$

$$t_2 = \gamma(t'_2 + \frac{vx'_0}{c^2})$$

So, observers in S measure the time between the events as:

$$\Delta t = \underbrace{t_2 - t_1}_{\Delta t} = \gamma\underbrace{(t'_2 - t'_1)}_{\Delta t_p} + \gamma(\frac{vx'_0}{c^2} - \frac{vx'_0}{c^2})$$

= 0

Time Dilation Formula:

$$\text{So: } \boxed{\Delta t = \gamma \Delta t_p}$$

$$\text{Note: } \gamma \overset{\text{always}}{\geq} 1 \Rightarrow \Delta t \overset{\text{always}}{\geq} \Delta t_p$$

i.e. the proper time is always the shortest time between two events.

Whiteboard Problem: 36-6

The diameter of the solar system is 10 light hours (*1 light hour is the distance that light travels in one hour*). A spaceship crosses the solar system in a time of 15 hours, as measured on the Earth. **How long, in hours, does the passage take according to passengers on the spaceship? (LC)**

Suggestion: This problem is set up to use a convenient set of units: distance in light hours (lh) and time in hours (h); **the speed of light, c = 1.0 lh/h.** The problem can be solved in MKS units, **but it's a lot more difficult!**

Here's another hint: what are the two events that we are measuring the time between, and **who measures the proper time between these two events?**

Demo-2

Or Demo

Travelling to the Stars: Whiteboard Problem: 36-7

In July 2015, NASA announced that the Kepler Spacecraft had discovered an Earth-sized planet in an Earth-sized orbit about a Sun-like star. The planet bears the name Kepler-452b and is 1400 lightyears away. You have been selected by NASA to command a mission to explore Kepler-452b. Your starship can travel at 0.9999c, and you are to spend one year exploring the planet and then return.

You leave Earth on Jan 1, 2020, your 20th birthday. When you get back to Earth:

a) **What year is it (i.e. calendar year)? (LC)**

b) **How old are you? (LC)**

(Assume that the time needed to accelerate and decelerate is negligible.
***Strong Suggestion**: use the units: length in ly and time in y; then c = 1 ly/y)*

As you calculated in this problem, the people on the ship age only a few years, while many years pass on Earth – **this is time dilation**. However, as seen from the frame of the ship, the Earth travels off in the other direction and returns. So the people on Earth should have aged less than the people on the ship!

This discrepancy is known as the **Twin Paradox**: one twin stays on Earth, the other voyages to the stars and returns to Earth. Which one is younger?

PG_Twin

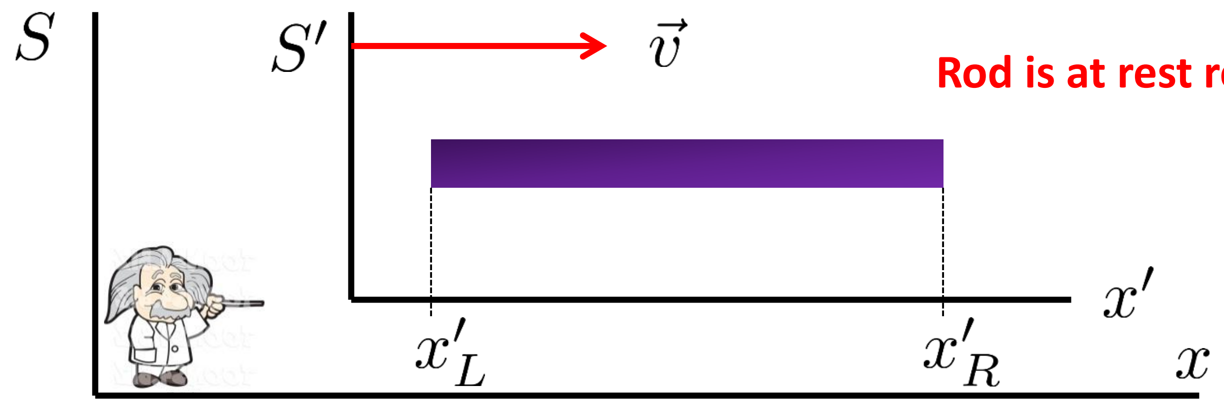
(~5m45s active link)

(this is also an excellent review of a lot of what we've done.)

As Physics Girl says, the time dilation in the movie **Interstellar** is a little different. It's caused by time slowing down in a strong gravitational field – **a consequence of General Relativity**.

Length Contraction

Another oddity of Special Relativity is that moving objects are measured to be shorter along their direction of motion. **This is known as Length Contraction, and we can derive it also from the LCT. Consider a moving rod:**



Demo-3

Or Demo

How do you measure the length of a rod? You determine the coordinates of the ends **at the same time**, and subtract.

So, observers in S' measure: $\text{Length} = x'_R - x'_L \equiv L_p$, the Proper Length

Note: the **Proper Length** of an object is its length in its own **Rest Frame**.

What does an observer in S measure for the length? Start with the ILCT:

$$x'_L = \gamma(x_L - vt_L)$$
$$x'_R = \gamma(x_R - vt_R)$$

Subtract: $x'_R - x'_L = L_p = \gamma(x_R - x_L) - \gamma(vt_R - vt_L)$

Length in S = L **= 0**

Length Contraction Formula

$$L = \frac{L_p}{\gamma}$$

Note: $\gamma \overset{\text{always}}{\geq} 1 \Rightarrow L \overset{\text{always}}{\leq} L_p$

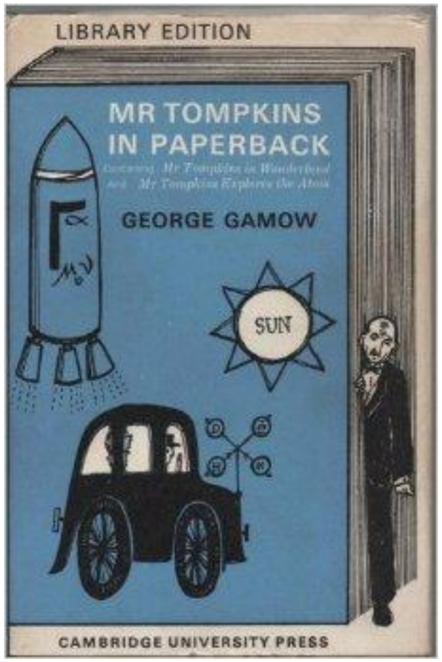
Whiteboard Problem: 36-8

Jill claims that her new rocket is 100 m long. As she flies past your house, you measure the rocket's length and find that it is only 80 m.

What is the speed of Jill's rocket as a fraction of the speed of light, c? (LC)

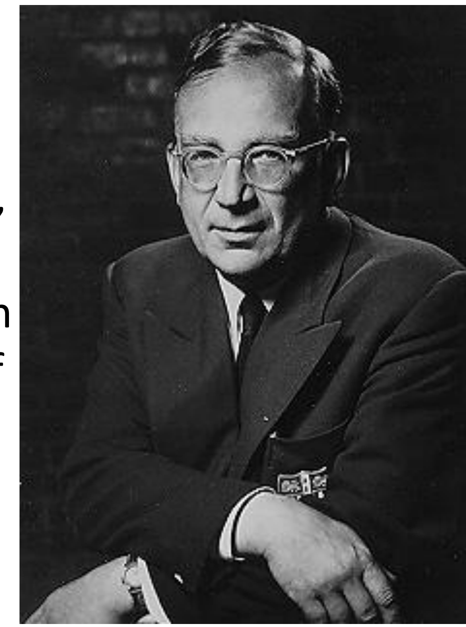
Mr. Tompkins in Wonderland

Of course, we don't experience things like **time dilation** and **length contraction** in our everyday lives because the speed of light is so fast, and for the effects to be noticeable, you have to travel near the speed of light.



What if the speed of light was slower?

In 1940, the Russian born American physicist, George Gamow wrote a delightful story called, Mr. Tompkins in Wonderland (*click on the link to see a website describing the book*) where, in a dream, a banker, Mr. Tompkins, finds himself in a universe where the speed of light is 10 km/hr, but all of the rules of Special Relativity are still valid.



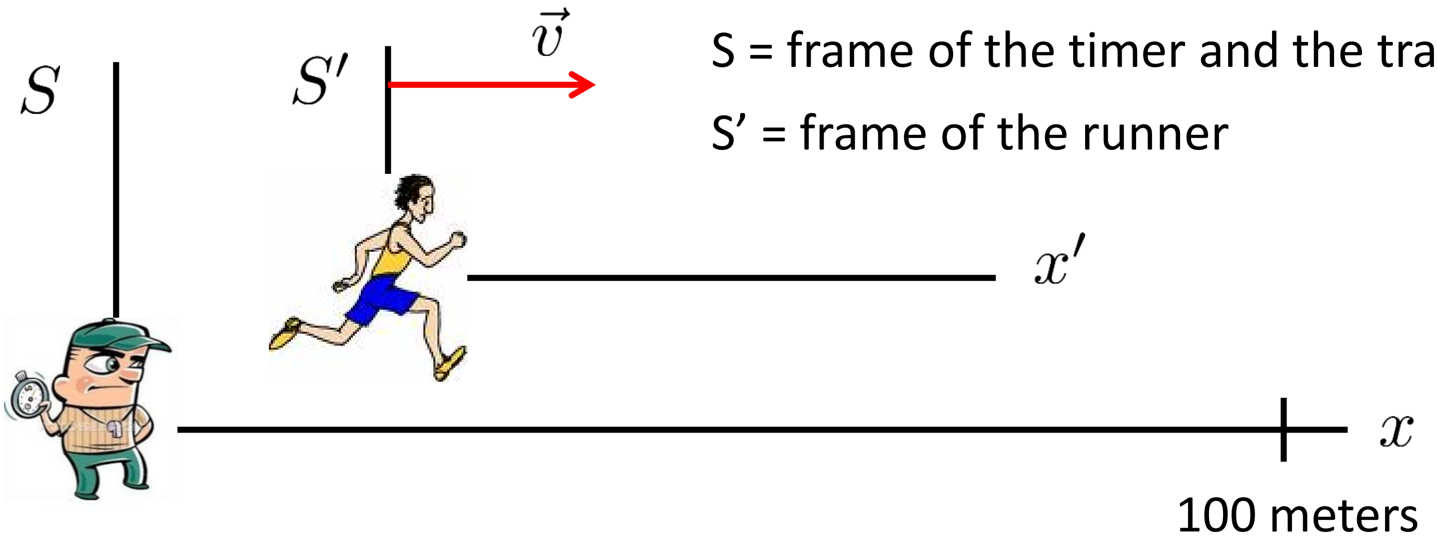
Here's Carl Sagan with his version of Mr. Tompkins: Sagan_Tompkins

(~4m24s active link)

Whiteboard Problem 36-9: Mr. Tompkins Runs a Race

In a dream, you are transported to a universe where the speed of light is not 3 X 10^8 m/s; however, the rules of Special Relativity are still valid. While attending a track meet in this dream universe, you participate in the 100 meter dash. After the race, the official timer says that you won with a time of 10.0 seconds. However, you carried a watch with you, and you claim that, according to your watch, your time was 8.0 seconds, a new world record!

Part a: Draw a diagram to define the frames of reference. What is the relative speed between the frames? (LC)



S = frame of the timer and the track

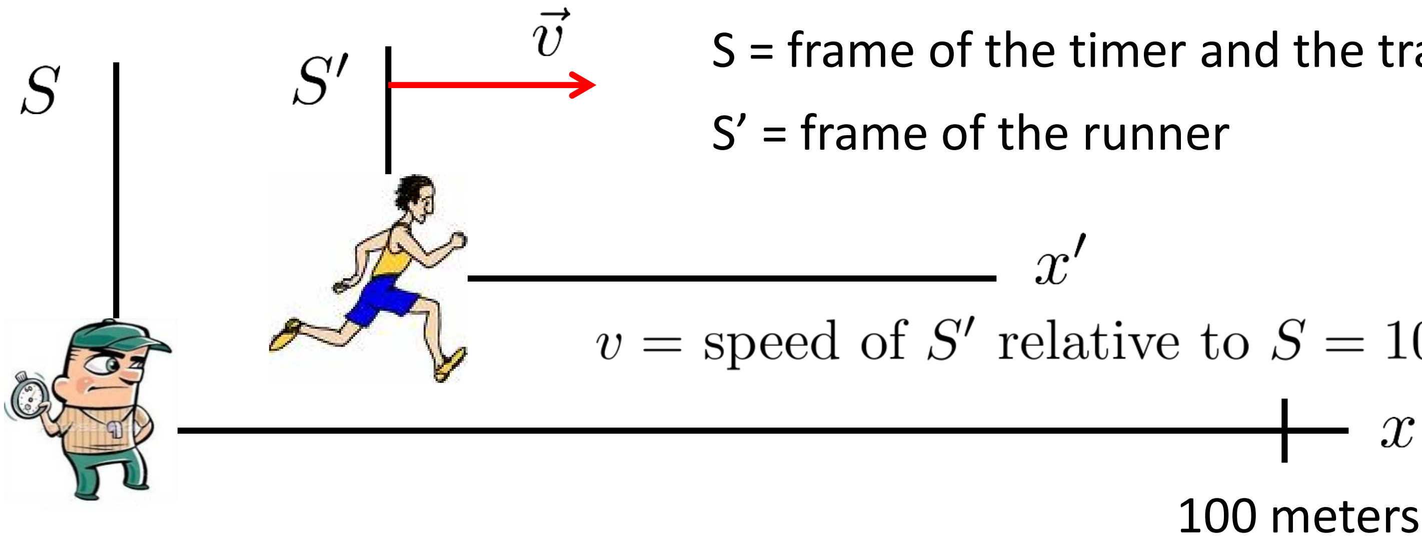
S' = frame of the runner

As measured in frame S, the speed of S' is: $v = \dfrac{100\ m}{10\ s} = 10\ m/s$

Whiteboard Problem 36-9: Mr. Tompkins Runs a Race

S = frame of the timer and the track

S’ = frame of the runner



Part b: Which observer, the runner (you) or the timer, measures the proper length of the track? (LC) Explain why to your partners.

The timer measures the proper length of the track, L_p = 100m, since he is at rest relative to it.

Part c: Which observer, the runner (you) or the timer, measures the proper time of the race? (LC) Explain why to your partners.

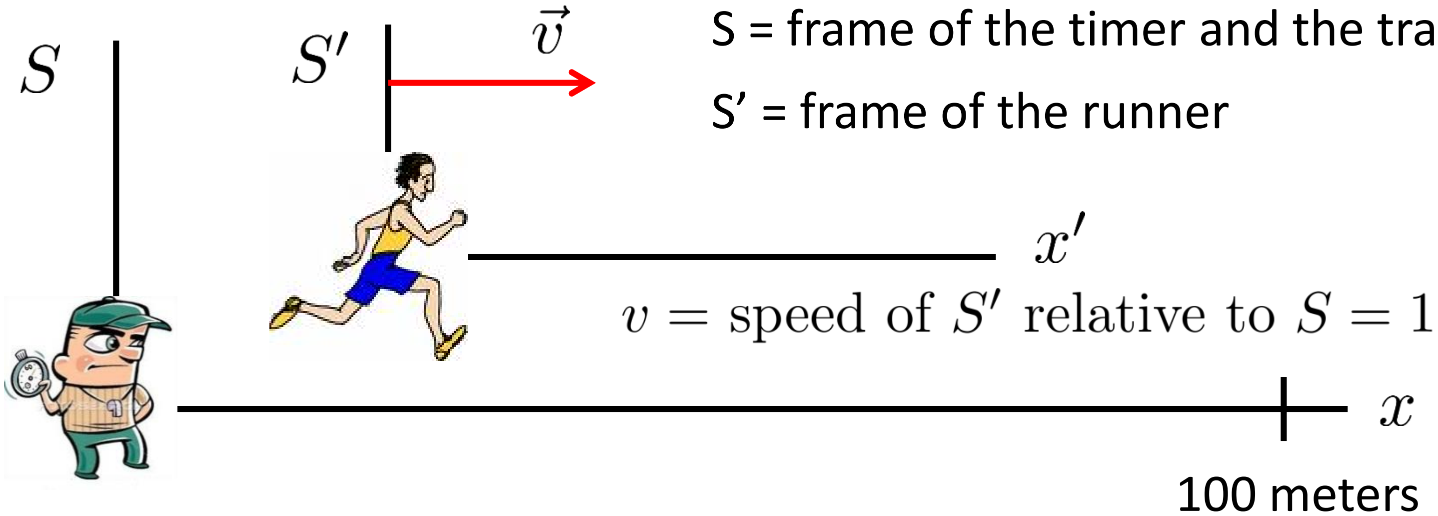
The runner (you) measures the proper time of the race since the two events (start and finish of the race) are at the same place in the runner’s frame (S’).

Or Demo Demo-4

Whiteboard Problem 36-9: Mr. Tompkins Runs a Race

S = frame of the timer and the track

S' = frame of the runner



$$v = \text{speed of } S' \text{ relative to } S = 10\ m/s$$

Part d: What is the speed of light (in m/s) in this dream universe? (LC)

$$\text{In } S\text{: } \Delta t = 10\ s \ \text{ and in } S'\text{: } \Delta t_p = 8\ s$$

Time dilation: $\Delta t = \gamma \Delta t_p \ \Rightarrow \gamma = \dfrac{\Delta t}{\Delta t_p} = 1.25$

So: $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} \ \Rightarrow \dfrac{v}{c} = \sqrt{1 - \dfrac{1}{\gamma^2}} = 0.6 \ \Rightarrow c = \dfrac{v}{0.6} = 16.7\ m/s$

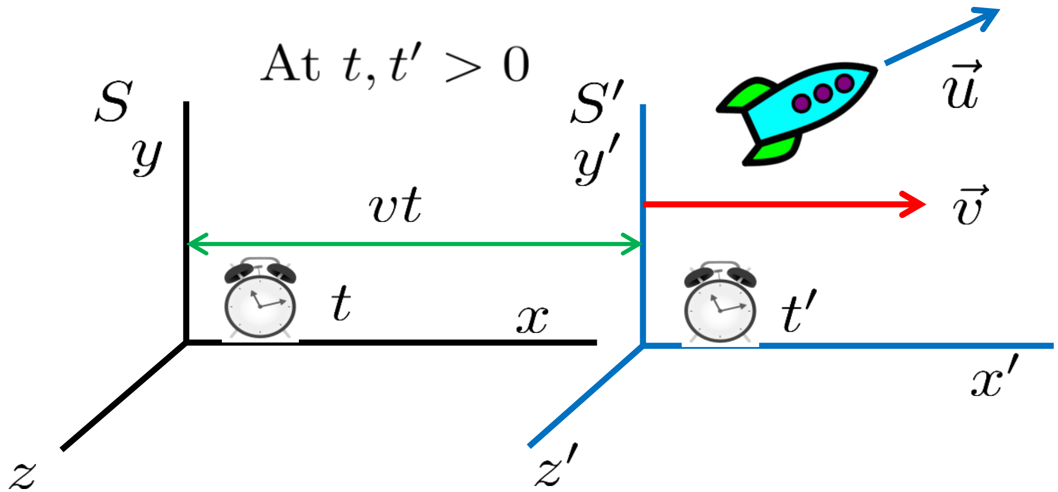
Part e: Which observer, the runner (you) or the timer, measures the <u>*correct* time</u> of the race? (LC) Explain why to your partners.

Both measure the ***correct*** time in their own reference frames.

However, only the timer will claim that the race is 100 m. The runner (you) measures a contracted length. **In S':** $L = \frac{L_p}{\gamma} = 80\ m$

Relative Velocities in Special Relativity

In a previous class, we had the **Galilean Velocity Transformations (GVT):**



S measures components: u_x, u_y, u_z

S' measures components: u'_x, u'_y, u'_z

The transformations relate the velocity components as observed in one frame to what is observed in another frame.

Galilean Velocity Transformations (t = t')

Transformation	Inverse Transformation
$u_x = u'_x + v$	$u'_x = u_x - v$
$u_y = u'_y$	$u'_y = u_y$
$u_z = u'_z$	$u'_z = u_z$

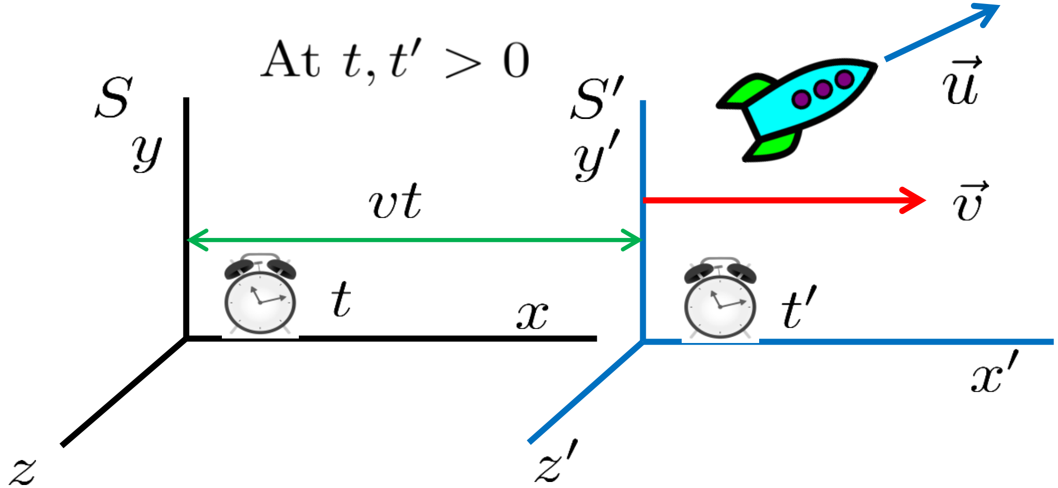
However, we have seen that the GVT violate the 2nd postulate of Special Relativity, i.e. not all observers would measure c for the speed of light.

Can we obtain the relativistically correct velocity transformations?

The Lorentz Velocity Transformations

(a little more than what's in your text)

The Problem:



S measures components: u_x, u_y, u_z

S' measures components: u'_x, u'_y, u'_z

How do these components relate to each other?

We know: $u_x = \frac{dx}{dt}, u_y = \frac{dy}{dt}, u_z = \frac{dz}{dt}$ and $u'_x = \frac{dx'}{dt'}, u'_y = \frac{dy'}{dt'}, u'_z = \frac{dz'}{dt'}$

The LCT give (here t ≠ t') : $x = \gamma(x' + vt')$ and $t = \gamma(t' + \frac{vx'}{c^2})$

Implicitly differentiate: $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{vdx'}{c^2})$

Divide the two equations:

$$\frac{dx}{dt} = \frac{dx' + vdt'}{dt' + \frac{v}{c^2}dx'}\left(\frac{\frac{1}{dt'}}{\frac{1}{dt'}}\right) = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

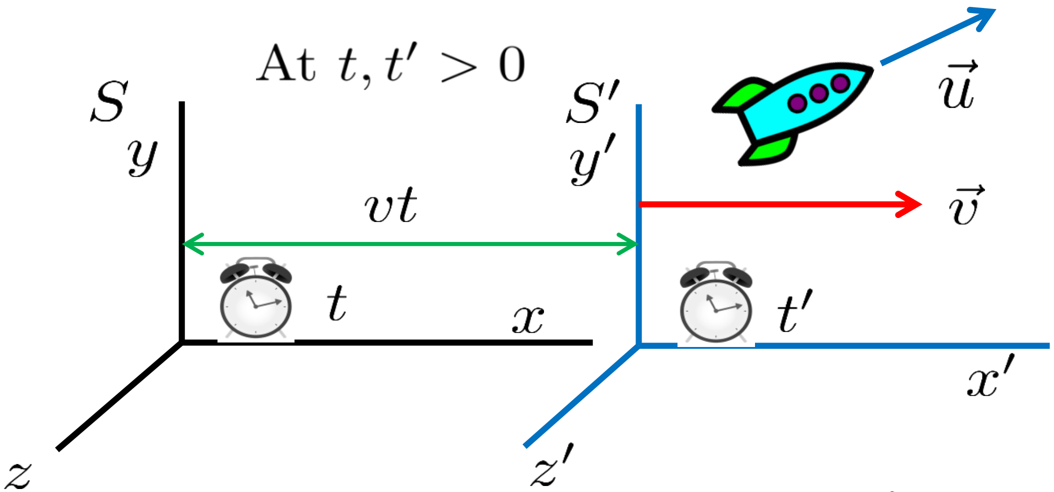
So:

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}}$$

(Note: $u_x \rightarrow u'_x + v$ (the GVT) for $v << c$)

The Lorentz Velocity Transformations

(a little more than what's in your text)



S measures components: u_x, u_y, u_z

S' measures components: u'_x, u'_y, u'_z

Doing the same for y (or z): $y = y'$ and $t = \gamma(t' + \frac{vx'}{c^2})$

$dy = dy'$ and $dt = \gamma(dt' + \frac{vdx'}{c^2})$

Divide:

$$\frac{dy}{dt} = \frac{dy'}{\gamma(dt' + \frac{vdx'}{c^2})}\left(\frac{\frac{1}{dt'}}{\frac{1}{dt'}}\right)$$

So:

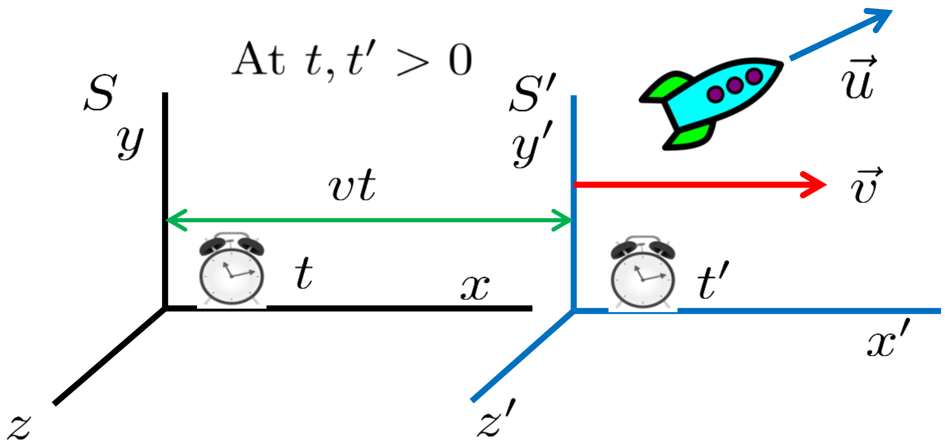
$$u_y = \frac{u'_y}{\gamma(1 + \frac{vu'_x}{c^2})}$$

(Note: $u_y \to u'_y$ (the GVT) for $v << c$)

Should that be u'_y in the denominator instead of u'_x?

No, this comes from the time transformation between S and S'.

The Lorentz Velocity Transformations(LVT)



S measures components: u_x, u_y, u_z

S' measures components: u'_x, u'_y, u'_z

Transformation	**Inverse* Transformation**
$u_x = \dfrac{u'_x + v}{1 + \frac{vu'_x}{c^2}}$	$u'_x = \dfrac{u_x - v}{1 - \frac{vu_x}{c^2}}$
$u_y = \dfrac{u'_y}{\gamma(1 + \frac{vu'_x}{c^2})}$	$u'_y = \dfrac{u_y}{\gamma(1 - \frac{vu_x}{c^2})}$
$u_z = \dfrac{u'_z}{\gamma(1 + \frac{vu'_x}{c^2})}$	$u'_z = \dfrac{u_z}{\gamma(1 - \frac{vu_x}{c^2})}$

Notes:

These are velocity components, so they can be positive or negative.

The Lorentz Factor is still calculated with v, not with u:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

For: $v << c$, $\gamma \sim 1$, and

$$1 \pm \frac{vu_x}{c^2} \sim 1$$

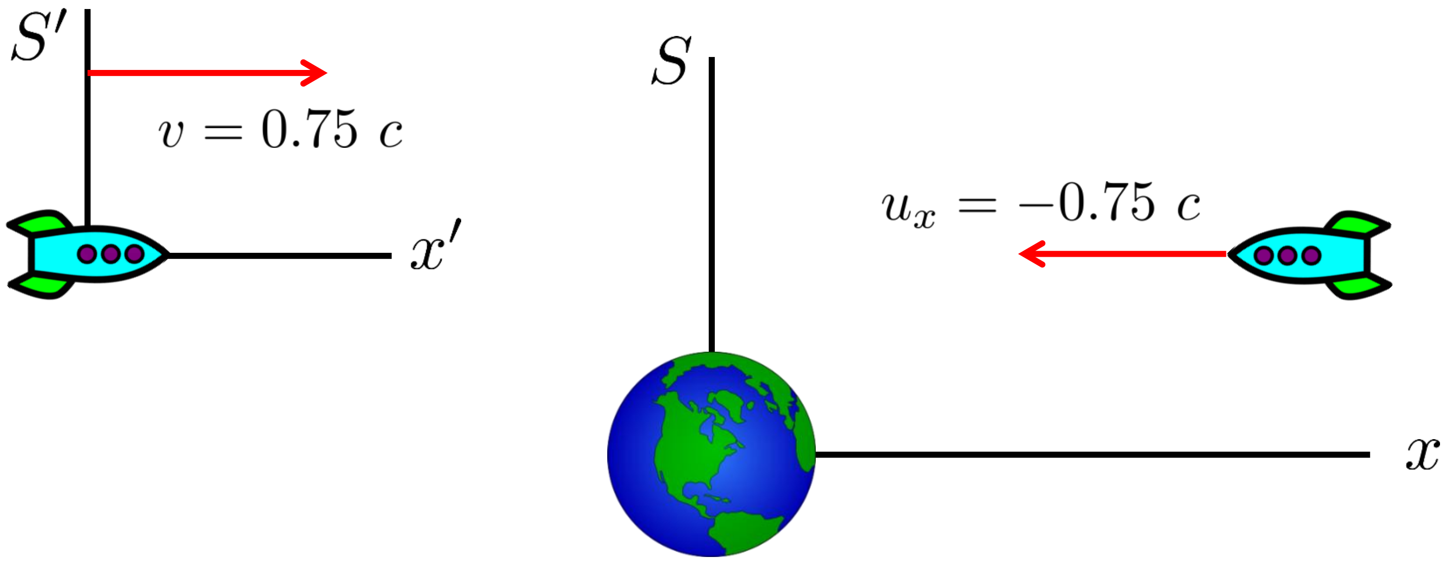
and we recover the GVT.

*Note: to get the inverse transformation, just replace primes with unprimed and replace v with –v.

Whiteboard Problem: 36-10

Two rockets approach the Earth from opposite directions travelling at 0.75c relative to the Earth.

a.) Make a sketch that defines frames of reference and velocities.

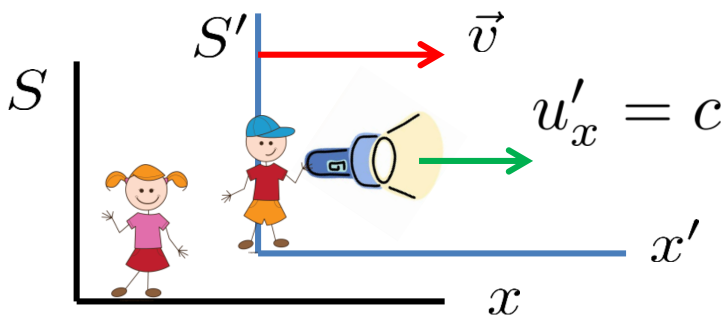


b.) According to the Galilean Velocity Transformations, what is the speed of one rocket relative to the other? (LC)

c.) According to the Lorentz Velocity Transformations, what is the speed of one rocket relative to the other? (LC)

A Little More on the LVT

Remember one of the issues with the Galilean Velocity Transformations was reconciling it with the constancy of the speed of light, i.e. the 2nd postulate.



Frame S' (the boy) measures for the speed of the beam: $u'_x = c$

So, according to the Galilean Velocity Transformations, frame S (the girl) measures:

$$u_x = u'_x + v = c + v$$

What do the Lorentz Velocity Transformation give for this scenario?

$$u_x = \frac{u'_x + v}{1 + \frac{v u'_x}{c^2}} = \frac{c + v}{1 + \frac{vc}{c^2}} = \frac{c + v}{1 + \frac{v}{c}} = \frac{c + v}{\frac{v+c}{c}} = c\left(\frac{c + v}{c + v}\right) = c$$

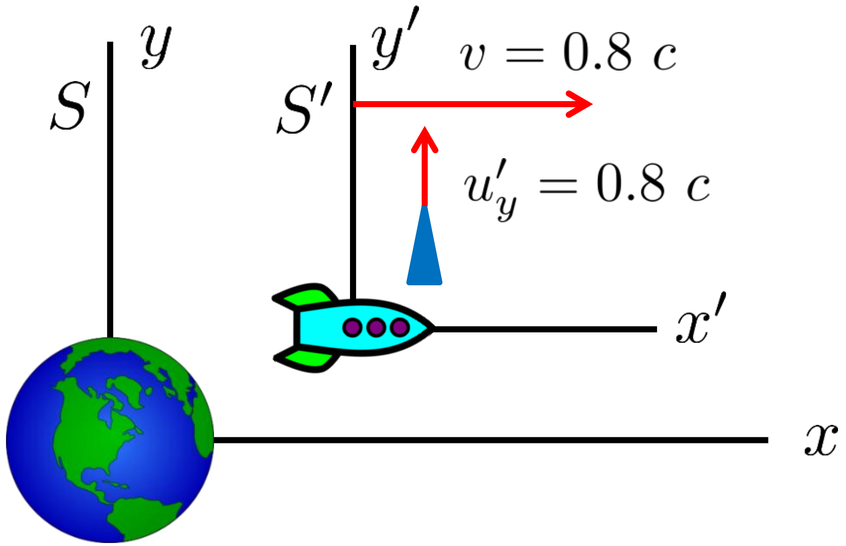
So, the girl will also measure c for the speed of the light beam.

Note, this really shouldn't be a surprise. The derivation of the Lorentz Transformations imposes the constancy of the speed of light in all IRF's

Whiteboard Problem 36-11

A spaceship moving at 0.80c away from the Earth fires a missile that the crew of the spaceship measure to be moving at 0.80c perpendicular to the ship's direction of travel.

a) Make a sketch that defines frames of reference and velocities.



b) Find the velocity components and speed (LC) of the missile as measured by observers on the Earth.

(If you've finished part b, what speed do the GVT give for the speed?)