30: Electromagnetic Induction

We really don't like the order in which your author presents the material in this chapter, so we're going put in a slightly different order. Also, we're only going to cover sections 1-7 of the chapter.

Introduction:

Recall what the Biot-Savart Law and, **more generally, Ampere's Law** say:

"Electric Currents Create Magnetic Fields"

Also, remember this was discovered accidentally by Oersted in 1819.

For some years after, people wondered about the inverse of this process:

"Can Magnetic Fields Create Currents?"

In 1831, Michael Faraday (English) and Joseph Henry (American) made another accidental discovery.

Video Demo (active link) **of Faraday's Experiments.**

The size of the induced current depends on the strength of the magnetic field, the relative speed of the magnet or coil, the area of the coil, and the relative orientation of the field and the coil.

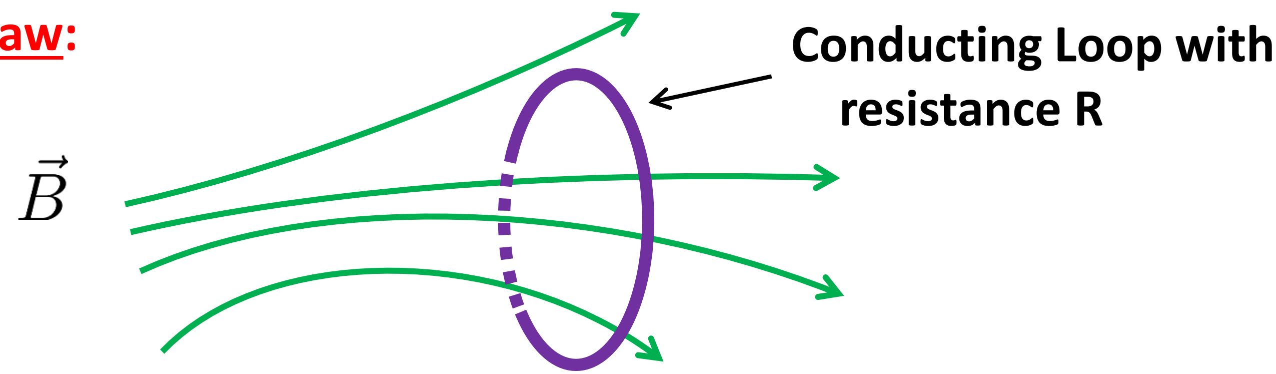
Faraday's Law

Faraday realized that all of these experiments have one feature in common:

"There is an Induced Current in a conducting coil when the Magnetic Flux through the coil is Changing."

Faraday was also able to deduce a precise quantitative description of this:

Faraday's Law:



The Induced EMF* in the Loop is:

$$\mathcal{E} = \left| \frac{d}{dt} \Phi_m \right|$$

where: Φ_m = the Magnetic Flux

The Induced EMF then causes an Induced Current: $I_{in} = \frac{\mathcal{E}}{R}$

**The absolute value sign is there because Faraday's Law in this form doesn't tell us anything about the direction of the induced EMF or current; for that we'll need Lenz's Law (coming soon).*

What About Magnetic Flux?

Back in chapter 24, we used the flux of the electric field in our work with Gauss' Law.

Everything that we learned about the electric flux there also applies to the magnetic flux here. All we have to do is:

$$\text{Replace } \vec{E} \text{ with } \vec{B}$$

$$\text{Replace } \Phi_e \text{ with } \Phi_m$$

Then all of the equations and the ways we calculated the flux of the electric field in chapter 24 will work for flux of the magnetic field as well.

The following two slides are the exact same slides that we used in chapter 24 but changed to the magnetic flux using the above replacements:

Flux of the Magnetic Field

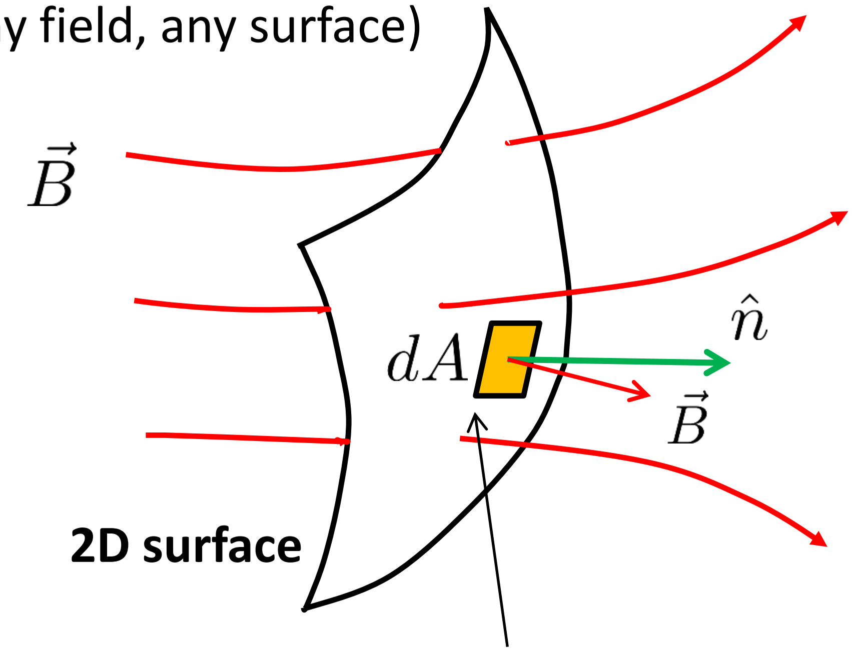
Flux is a measure of how much a magnetic field passes through a 2D surface.

Flux depends on the size and direction of the field and the size, shape, and orientation of the surface.

Your author starts with flux for special cases and builds to a general definition; read his account, we'll do the opposite:

Flux for the General Case

(any field, any surface)



Element of area

$\hat{n}$ = unit vector normal to the surface

$$d\vec{A} = \hat{n}\, dA$$

Magnetic Flux:

$$\Phi_m = \int_{\text{surface}} \vec{B} \cdot d\vec{A}$$

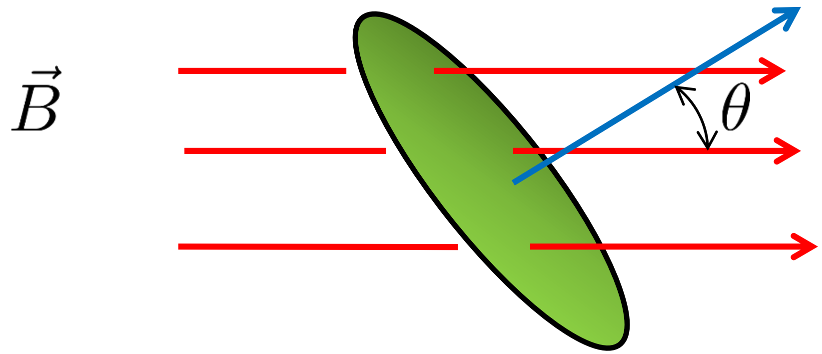
units: $1\ Tm^2 \equiv 1$ Weber (Wb)

(New, different for magnetic!)

Note: this is called a **surface integral**

Special Cases of Flux (*we'll use these a lot*)

Uniform Magnetic Field & Planar Surface:

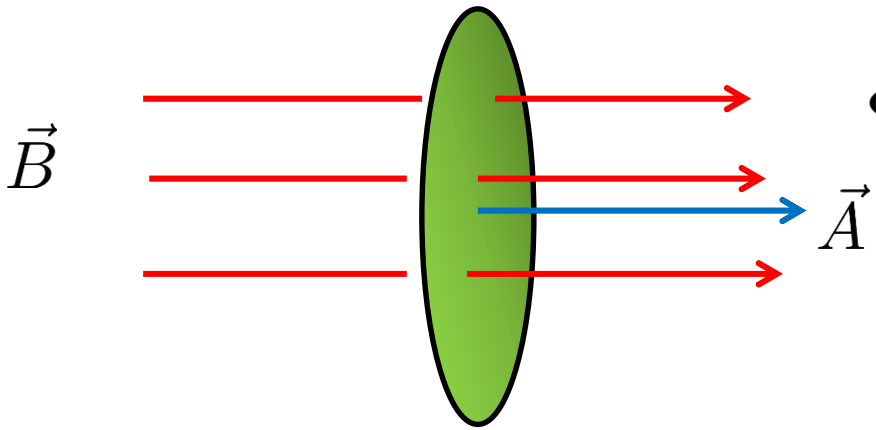


$$\vec{A} = \{A, \perp \text{ to surface}\}$$

$$\text{or, } \vec{A} = A\hat{n}$$

$$\Phi_m = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \int d\vec{A} = \vec{B} \cdot \vec{A} = BA\cos\theta$$

Uniform Magnetic Field & Planar Surface perpendicular to the field:



$$\Phi_m = \vec{B} \cdot \vec{A} = BA\cos 0° = BA$$

For either of these cases, the relation to remember for a planar surface in a uniform field is:

$$\Phi_m = \vec{B} \cdot \vec{A}$$

Back to Faraday's Law*

The induced EMF: $$\mathcal{E} = \left| \frac{d}{dt} \Phi_m \right|$$

And the magnetic flux is: $$\Phi_m = \int_{\mathrm{S}} \vec{B} \cdot d\vec{A}$$

Where S is the surface of the conducting loop.

Faraday's Law says: **"The induced EMF is given by the rate at which the magnetic flux is changing."**

What can produce a changing flux?

$$\text{The magnitude of } B \text{ can change, } i.e. \ B(t)$$

$$\text{The size of the area, } A\text{, can change, } i.e. \ A(t)$$

$$\text{The orientation of } \vec{B} \text{ and } \vec{A} \text{ can change}$$

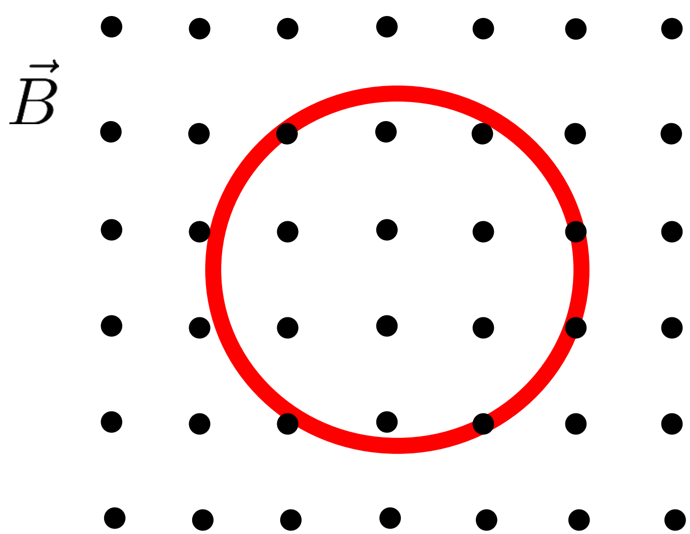
Any of these can occur simultaneously – we'll see problems like that!

**By the way: Faraday's Law is our 4th Maxwell Equation, but we'll want to write it in a different form later in the class, i.e. using just the fields.*

Whiteboard Problem: 30-1

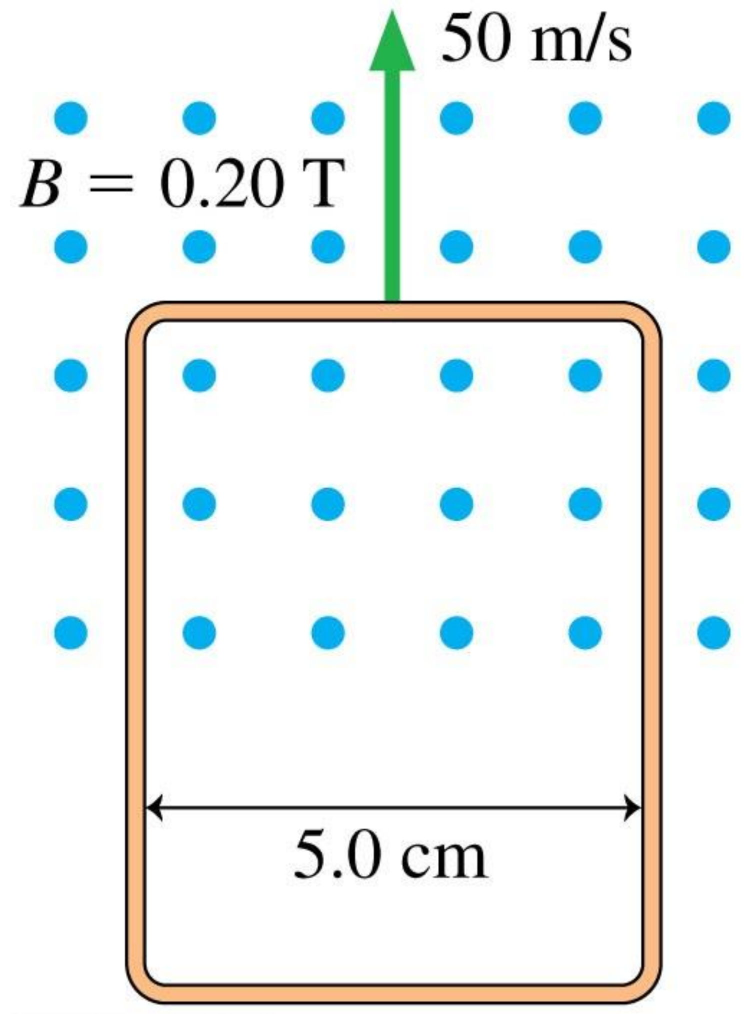
A circular coil of wire has a diameter of 1.0 cm and is in a uniform magnetic field that increases in strength from 100 T to 300 T in 10 ms. The axis of the coil is parallel to the field. **What is the induced EMF in the coil? (LC)**

Your sketch should look like this:



Whiteboard Problem: 30-2

The loop in the figure is being pushed into the 0.2 T magnetic field at 50 m/s. The resistance of the loop is 0.1 Ohms.
What are the magnitude and direction of the current in the loop?
(LC, magnitude)

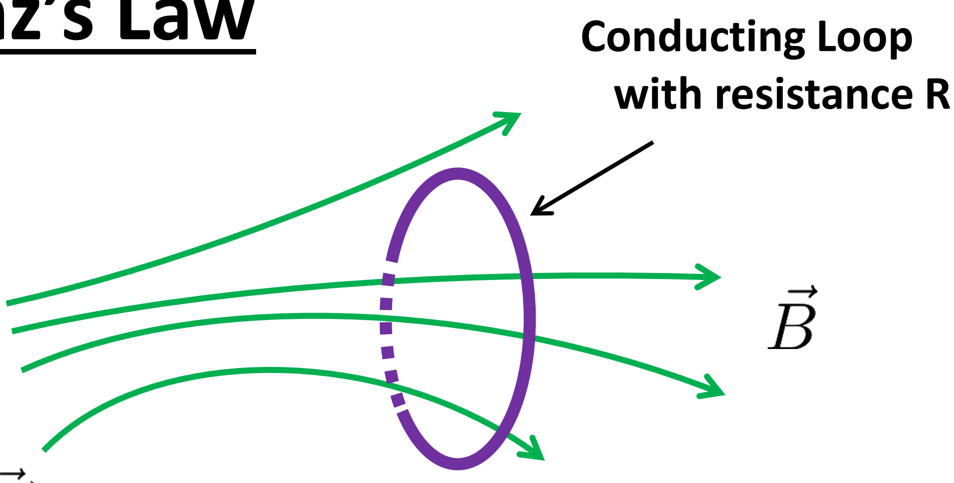


How can you figure out which way the induced current goes, clockwise or counterclockwise?

Lenz's Law

Faraday's Law:

$$\mathcal{E} = \left| \frac{d}{dt} \Phi_m \right| \qquad I_{in} = \frac{\mathcal{E}}{R}$$



I_{in} creates an <u>induced</u> magnetic field, $\vec{B}_{in}$ (this is different than $\vec{B}$)

<u>Lenz's Law</u> can be used to find the direction of the induced current:

> "The direction of the induced current, I_{in}, is such that the induced magnetic field, $\vec{B}_{in}$, creates a flux that opposes the original change in flux."

i.e. $\vec{B}_{in}$ will try to keep the flux the same

This is a very odd physical law – no equations, just words.

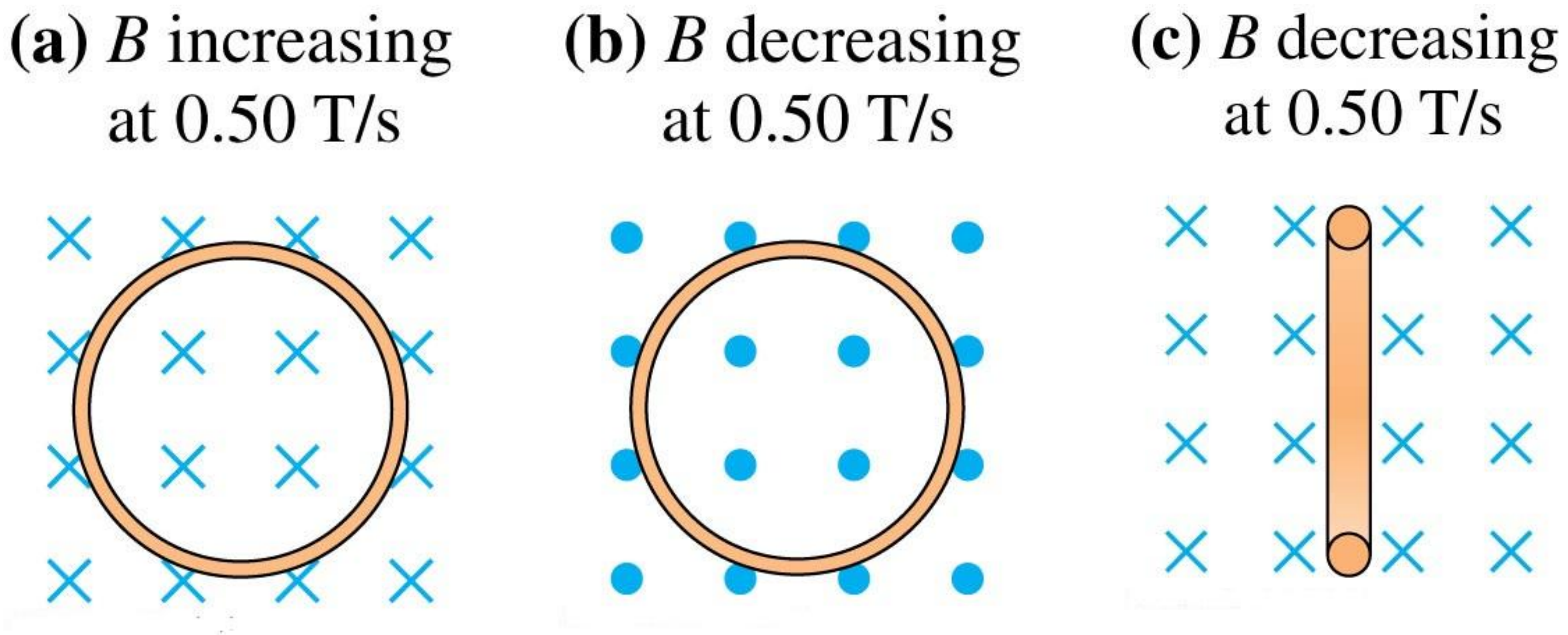
The most important step in applying Lenz's Law is to ask:

"What is the Flux doing, increasing or decreasing?"

Let's go back to Problem WB30-2 and figure out which way the induced current goes.

Whiteboard Problem 30-3

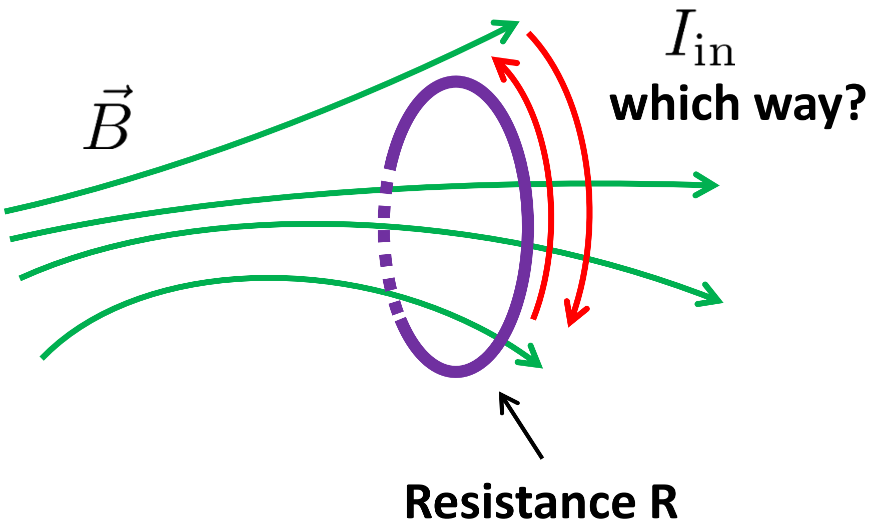
The figures below show a circular conducting loop in a magnetic field. **Determine the direction of the induced current in the loop?**



(LC: CW, CCW, or 0) (LC: CW, CCW, or 0) (LC: CW, CCW, or 0)

Summary of Faraday & Lenz's Laws

For a conducting loop in a magnetic field:



Faraday's Law gives the Induced EMF and Current:

$$\mathcal{E} = \left| \frac{d}{dt}\Phi_m \right| \qquad I_{\text{in}} = \frac{\mathcal{E}}{R}$$

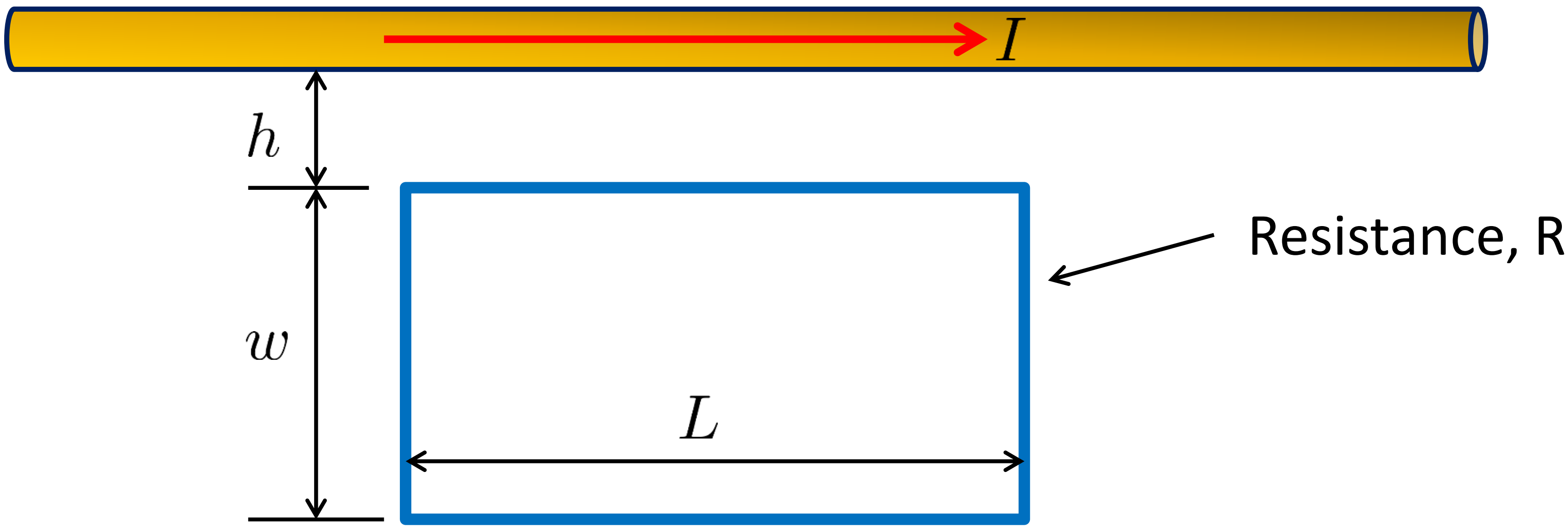
Lenz's Law gives the direction of the induced current:

"The direction of the induced current, I_{in}, is such that the induced magnetic field, $\vec{B}_{\text{in}}$, creates a flux that opposes the original change in flux."

Now that we have some experience with Faraday's and Lenz's Laws, we're going to spend some time looking at some more interesting (& fun?) problems.

Whiteboard Problem: 30-4

A rectangular conducting loop of resistance R is a distance h away from a long current carrying wire. The width of the loop is w and the length is L.

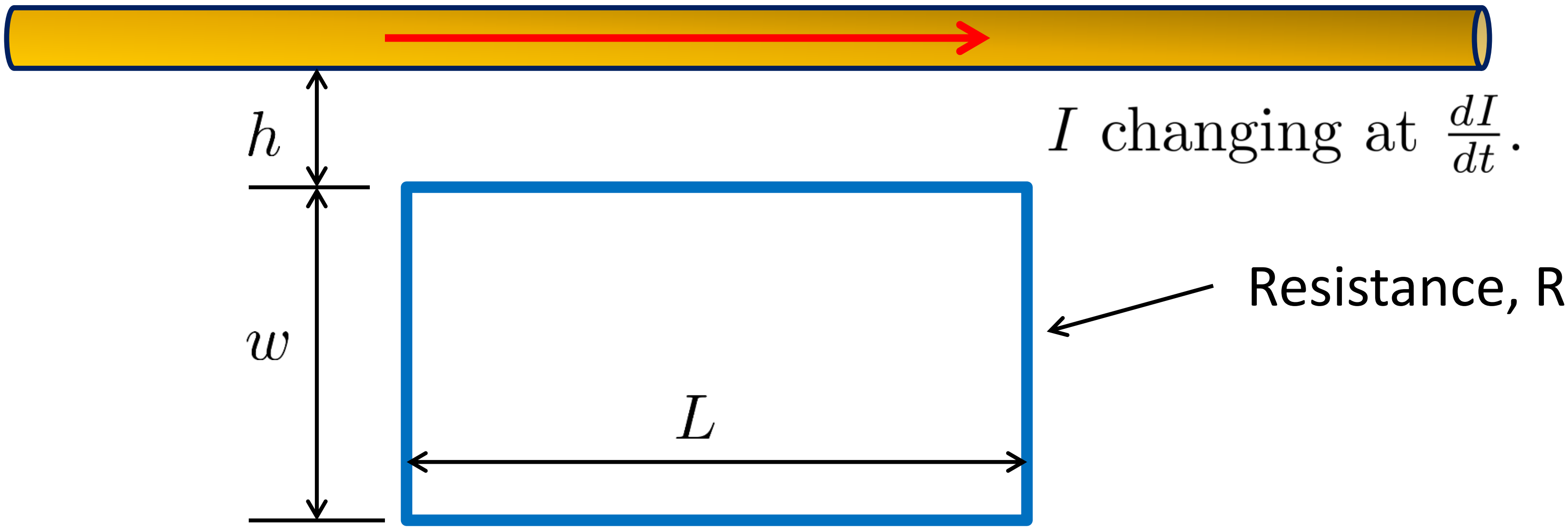


Part a. Find an expression for the magnetic flux through the loop in terms of the given quantities (I, L, w, & h). (LC)

Hint: you have to integrate this one.

Answer: $$\Phi_m = \frac{\mu_0 I L}{2\pi} \ln\left(\frac{w+h}{h}\right)$$ ***copy this on to your whiteboard***

Whiteboard Problem: 30-4



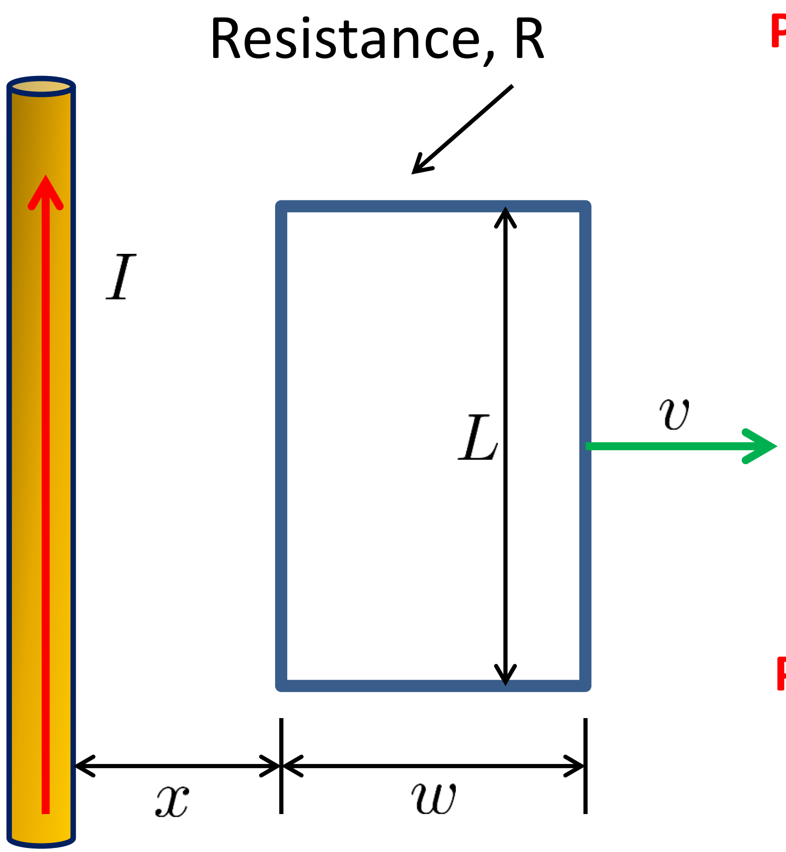
Part b. If the current in the wire is changing at a rate of dI/dt, find an expression for the induced current in the loop. (LC)

$$\text{Answer:} \qquad I_{\text{in}} = \frac{\mu_0 L}{2\pi R} \ln\left(\frac{w+h}{h}\right)\left|\frac{dI}{dt}\right|$$

Part c. If the current in the wire is increasing, determine the direction of the induced current in the loop. (LC)

Whiteboard Problem: 30-5

A rectangular conducting loop of resistance R is a distance x away from a long wire that carries **constant current I**. At this instant, the loop is moving away from the wire at speed v.
The width of the loop is w and the length is L.



Part a. Find an expression for the induced current in the loop in terms of given quantities. (LC)

Hint: *use the flux from the previous WB problem.*

$$\Phi_m = \frac{\mu_0 I L}{2\pi} \ln\left(\frac{w+h}{h}\right)$$

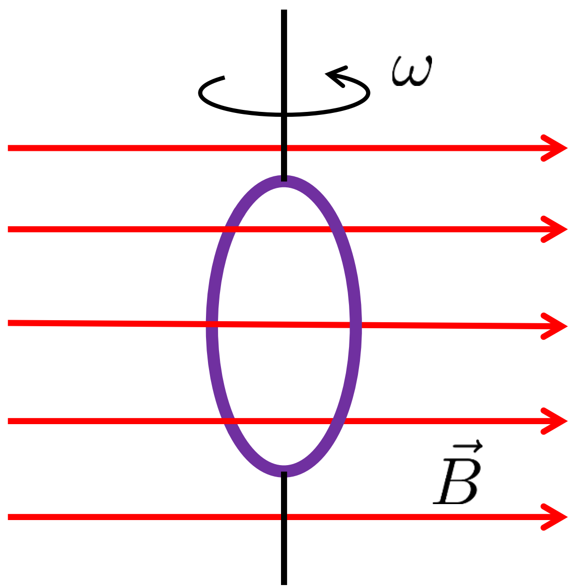
with x instead of h.

Part b. Determine the direction of the induced current in the loop. (LC)

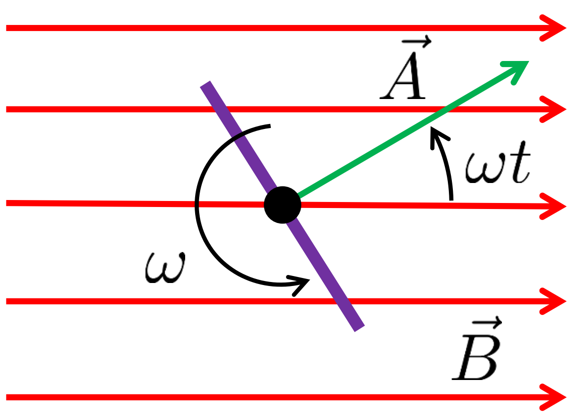
Whiteboard Problem: 30-6

A conducting circular loop of radius r and resistance R rotates at a constant rate about an axis perpendicular to a constant uniform magnetic field.

Find an expression for the magnitude of the induced current in the loop as a function of time. (LC)



Hint: draw it looking down the rotation axis



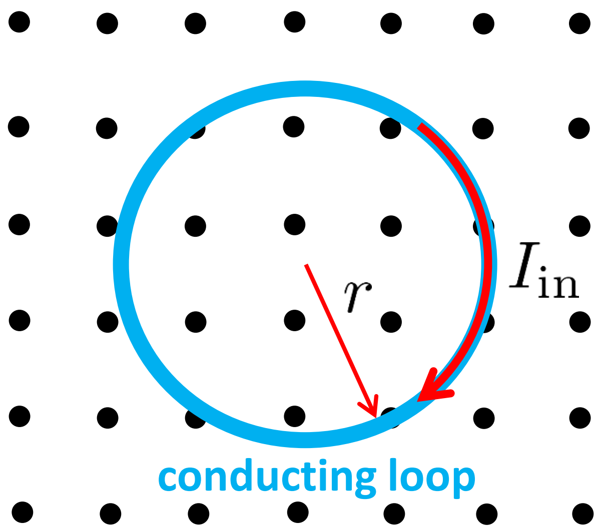
Note: this is a time dependent or Alternating Current (AC);
this is how we generate our electricity, thanks to Nikola Tesla.

PhET gen

Induced Electric Fields

Here's how we've been using Faraday's and Lenz's Laws, suppose for example:

$\vec{B}$ uniform & $\frac{dB}{dt} > 0$



$$\mathcal{E} = \left| \frac{d}{dt}\Phi_m \right| = \frac{d}{dt}(AB) = \pi r^2 \frac{dB}{dt}$$

$$I_{\text{in}} = \frac{\mathcal{E}}{R} = \frac{\pi r^2}{R}\frac{dB}{dt}$$

Flux is increasing $\Rightarrow \vec{B}_{\text{in}}$ is in

So, I_{in} is CW

Now, think back to Chapter 27:

What pushes the induced Current?

The induced EMF, which is a potential difference.

And, if there's a potential difference, there must be an Electric Field.

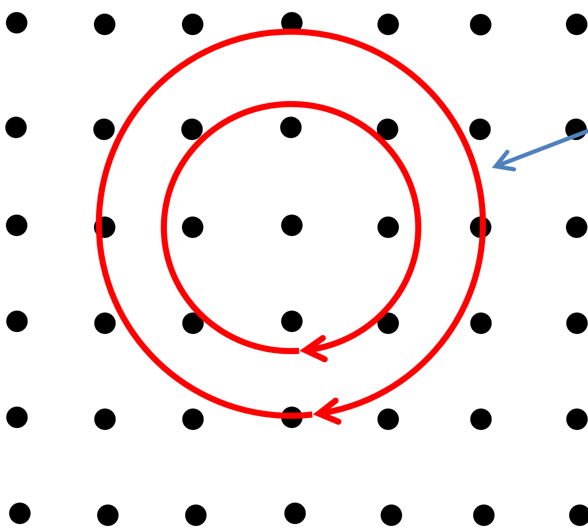
So, there must be an Induced Electric Field in the loop.

Induced Electric Fields

In fact, even if the conducting loop is not present:

$\vec{B}$ uniform & $\frac{dB}{dt} > 0$

The induced Electric Field is still there!



Induced $\vec{E}$ (same direction as I_{in})

What's different about this electric field than any electric field that we've seen before?

This electric field is **NOT** produced by electric charges and it **forms closed loops**.

We call this a "**Non-Coulomb" Electric Field**

This leads to a more fundamental form of Faraday's & Lenz's Laws in terms of the fields:

$$\mathcal{E} = \left| \frac{d}{dt} \Phi_m \right|$$

& Lenz's Law ⟺

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Says: "A Changing $\vec{B}$ creates $\vec{E}$"

This form of Faraday's Law is our 4th Maxwell Equation.

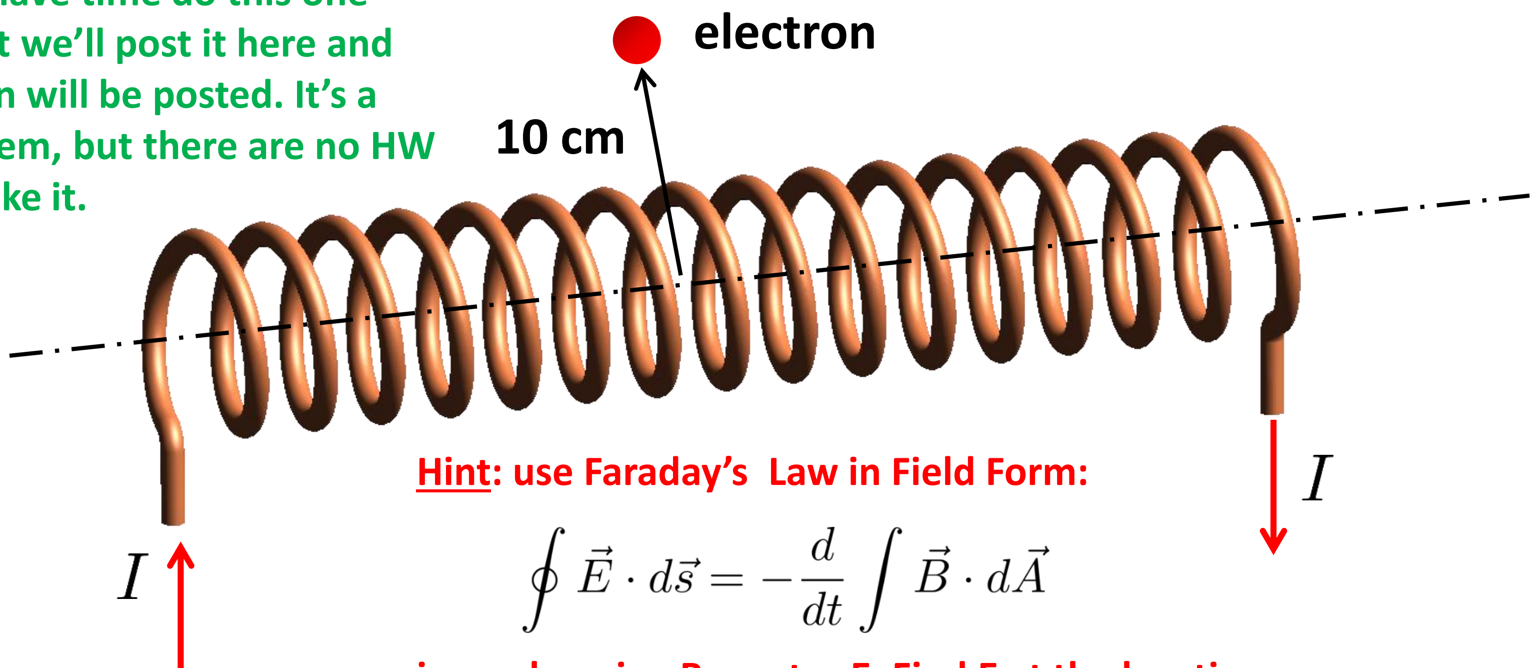
However, our 1st form with Lenz's Law works better in solving problems for induced currents.

Whiteboard Problem: 30-7

The figure shows an **ideal solenoid** of length 50 cm and radius 5 cm with 200 turns of wire. The current in the solenoid is increasing at a rate of 0.5 A/s. An electron is released from rest outside the solenoid at the position shown.

What is the magnitude of the acceleration of the electron at the instant it is released? (LC)

We didn't have time do this one in class, but we'll post it here and the solution will be posted. It's a good problem, but there are no HW problems like it.



Hint: use Faraday's Law in Field Form:

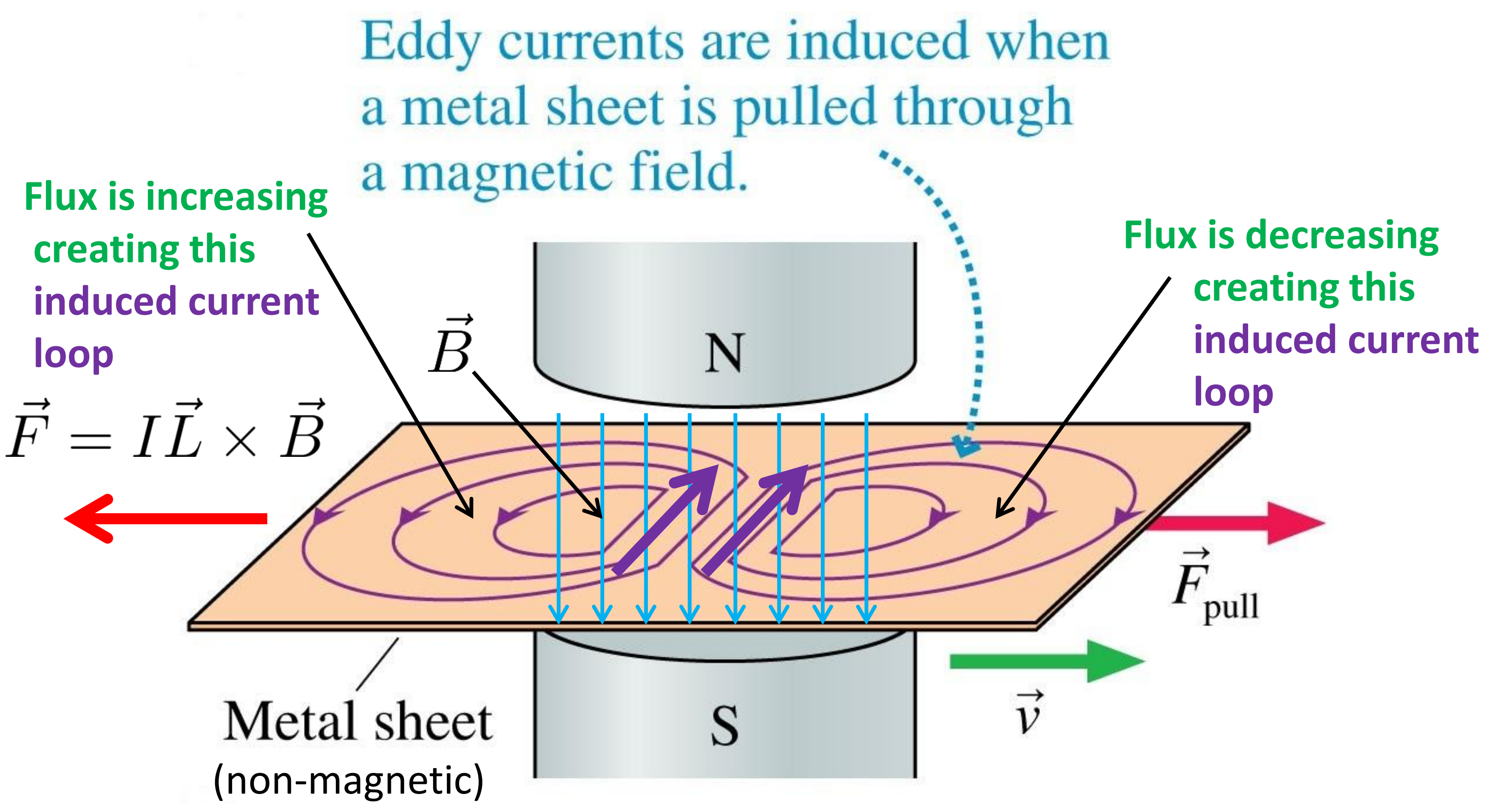
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

i.e. a changing B creates E. Find E at the location of the electron, then a = E/m

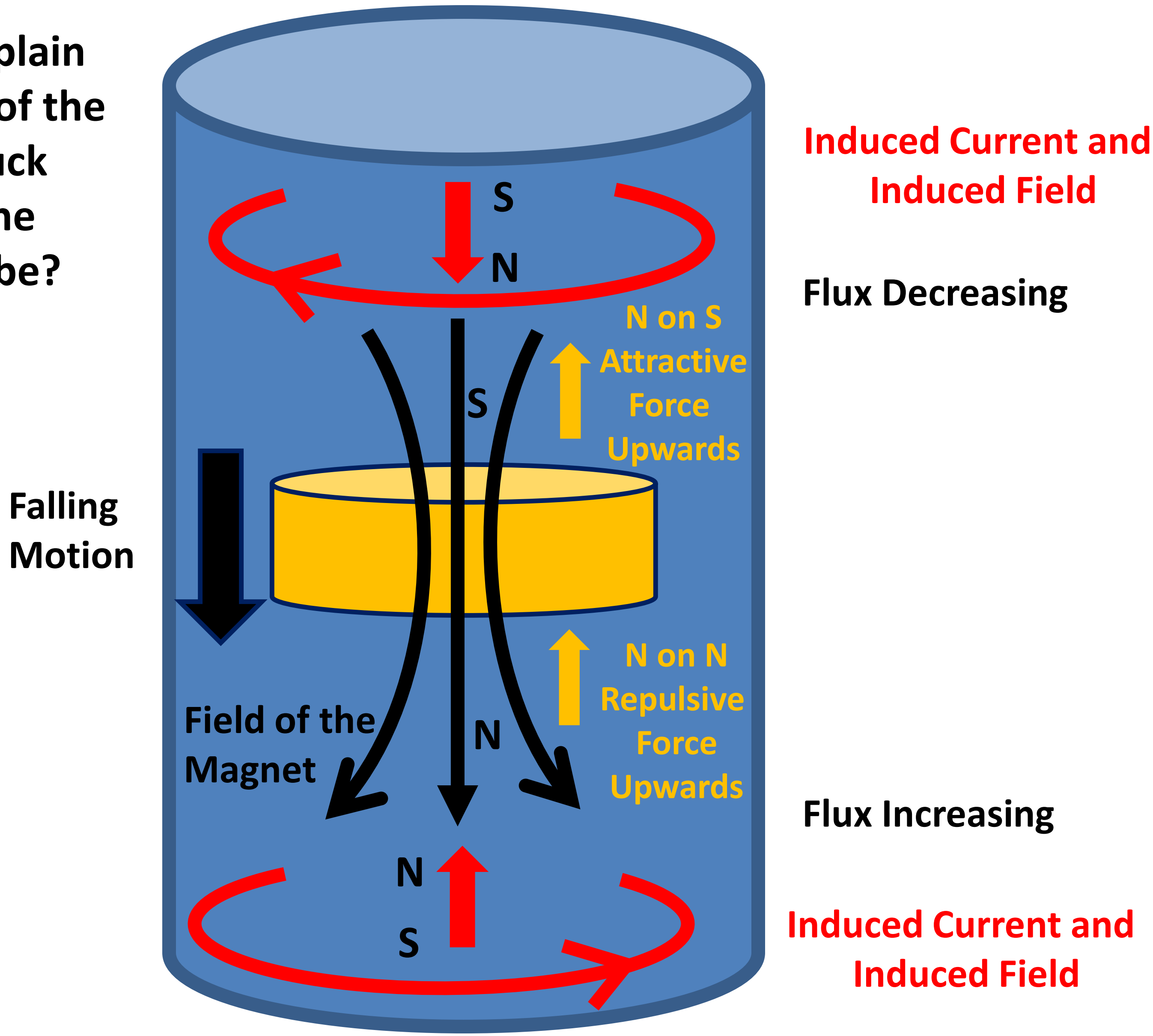
Eddy Currents

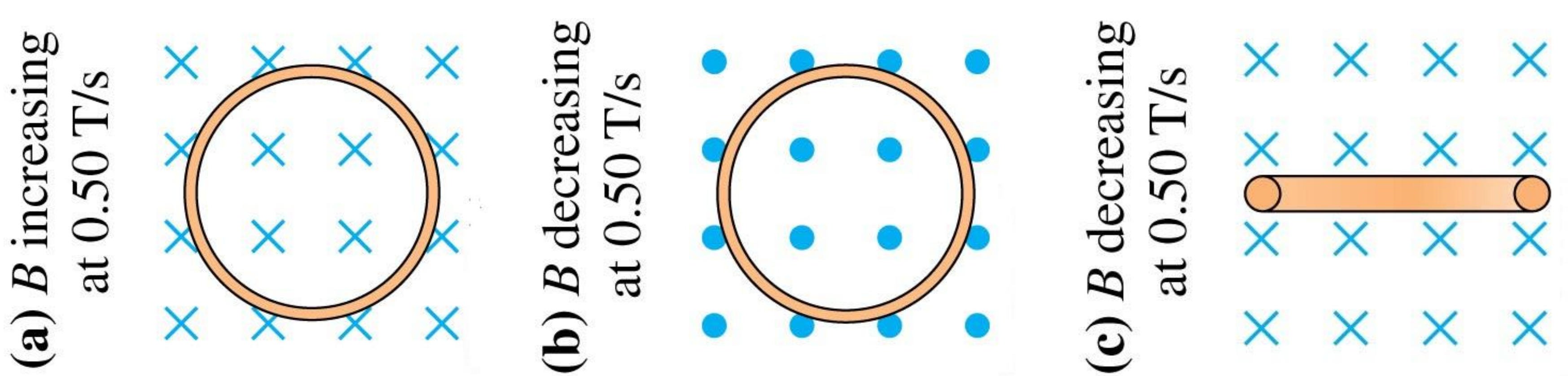
What is an Eddy Current?

- Here's a neat **demo video** (*an active link*) of what we're talking about.
- Below, is your author's example of how eddy currents are induced and how a drag-like retarding force is generated.



Can we explain the demo of the Magnet Puck falling in the Copper Tube?




(a) B increasing
at 0.50 T/s
(b) B decreasing
at 0.50 T/s
(c) B decreasing
at 0.50 T/s