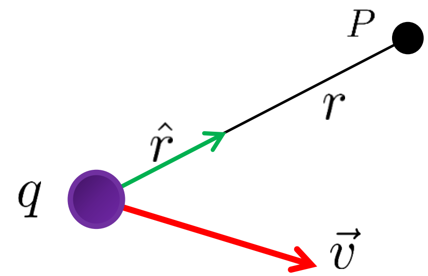
29-2: Sources of Magnetic Field: Summary

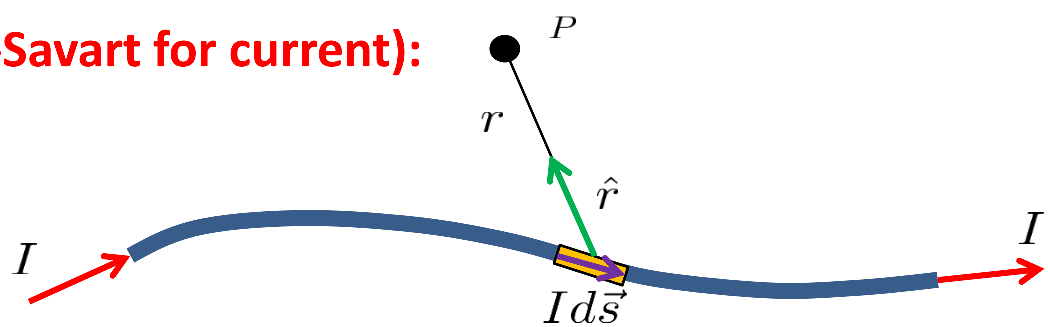
Single Moving Charge (Biot-Savart for a charge):

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

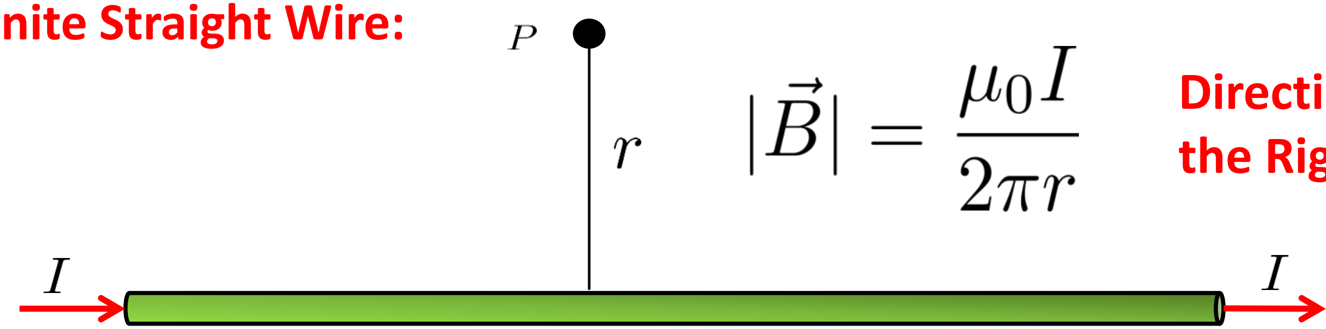


Steady Current in a Wire (Biot-Savart for current):

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$



Infinite Straight Wire:



$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

Direction is from the Right Hand Rule

Ampere's Law

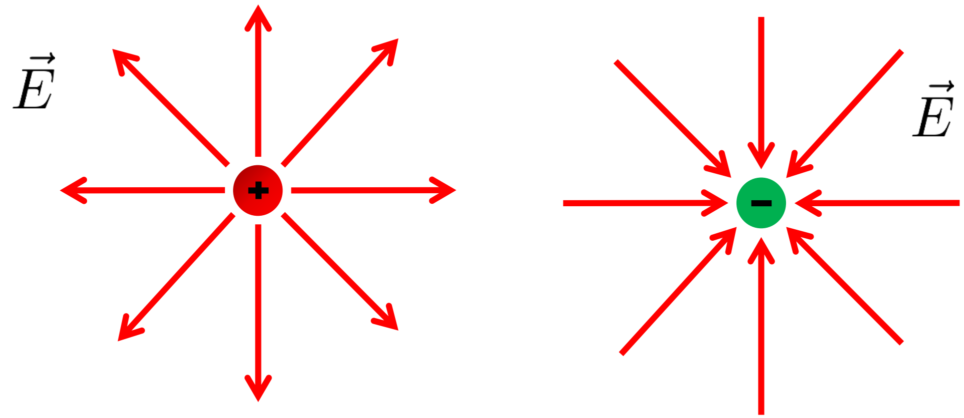
Introduction:

We're at the same place in our study of the magnetic field as we were with the electric field at the end of Chapter 23:

We can calculate $\vec{B}$ for any current, but it's messy.

Recall what Gauss said for the Electric Field:

"$\vec{E}$ always comes out of positive charge and goes into negative charge."



Of course, as we saw in Chapter 24, **Gauss' Law** is a concise mathematical statement of this. **For any closed 2D surface S, the electric field will be such that:**

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

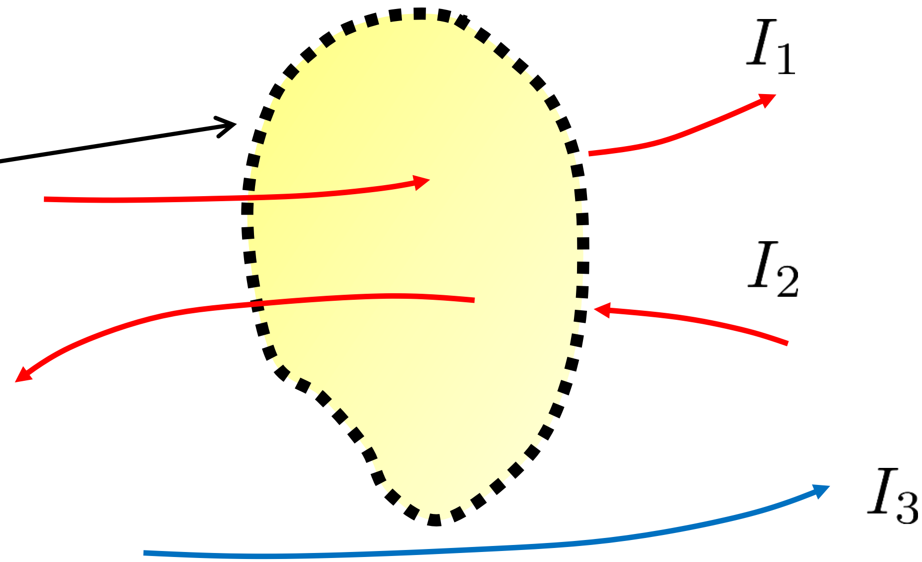
Ampere's Law

Ampere noticed something similar for the magnetic field and currents:

"$\vec{B}$ always goes around current in closed loops; one way for current in one direction, the other way for the opposite direction."

Ampere's Law: General Statement

For any currents and any closed 1D path in space:



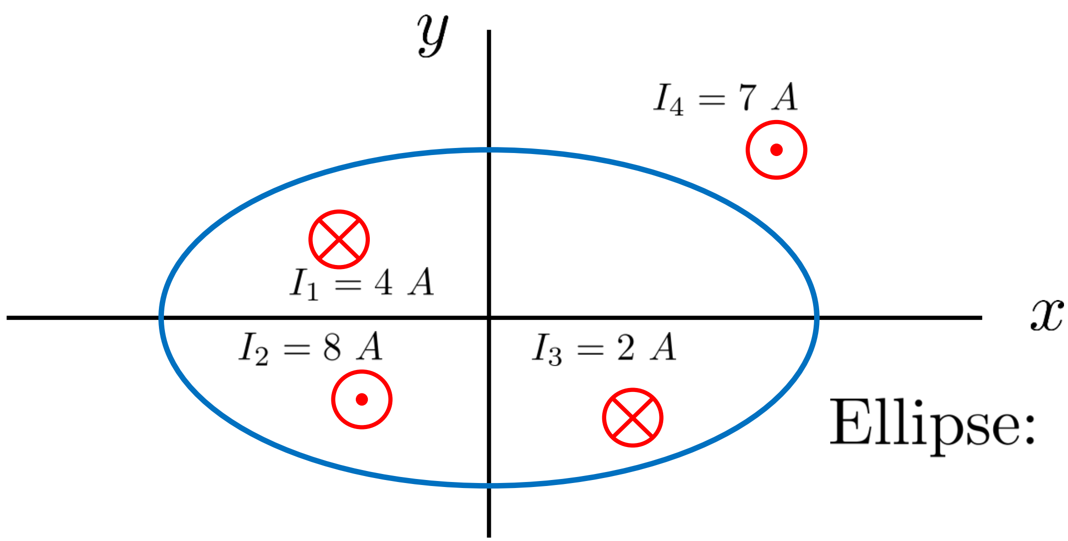
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}}$$

$\oint \Rightarrow$ line integral is around the closed path

$I_{\text{thru}} =$ net current through the path (for above: I_1 and I_2, but not I_3)

Note that we have to choose a positive direction for the current: a current one way will produce a field in one direction on the path, and an opposite current will produce a field the other way.

Whiteboard Problem 29-7



Ellipse: $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

Evaluate the line integral, $\oint \vec{B} \cdot d\vec{s}$, around the ellipse. **(LC)**

Solution: Use Ampere's Law with current coming out as positive:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}} = \mu_0 \sum_i I_i = \mu_0 \left(-4\ A + 8\ A - 2\ A\right)$$

$$= \mu_0 \left(2\ A\right) = 2.513 \times 10^{-6}\ T\ m$$

(Is this a mean problem or what?)

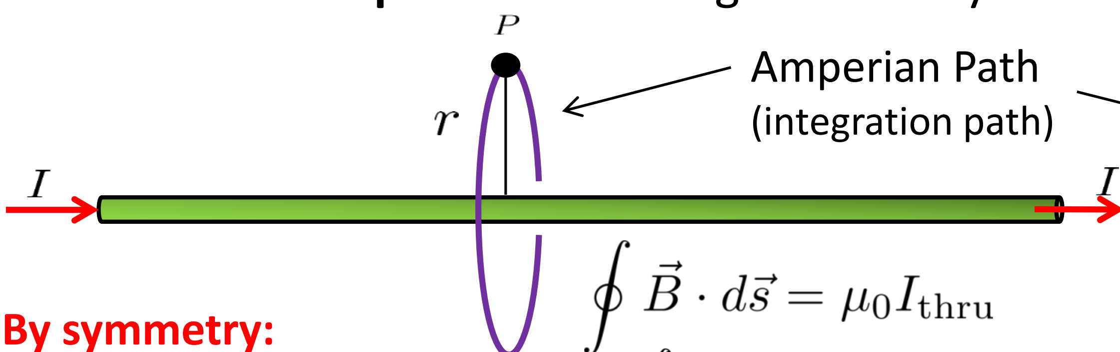
<u>How Do We Use Ampere's Law To Find the Magnetic Field?</u>

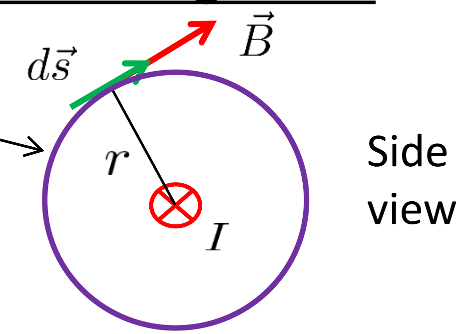
Recall that Gauss' Law contains the same physics as Coulomb's Law, but it uses a different language – The Language of Fields.

Ampere's Law is similar; it contains the same physics as the Biot-Savart Law, but it too is in the language of fields. **How can we use Ampere's Law to find the magnetic field?**

Ampere's Law is always true for any distribution of currents and any path; **however, it is only useful in finding the field if there is enough symmetry to easily determine the line integral – i.e. do it in your head!**

Here's an **example** of something we already know: **<u>an infinite straight wire</u>**:





$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}}$$

By symmetry:

$\vec{B}$ is everywhere $\parallel d\vec{s}$

$|\vec{B}| = \text{constant for any } r$

$$\oint B ds = \mu_0 I$$ (all of the current goes thru the path)

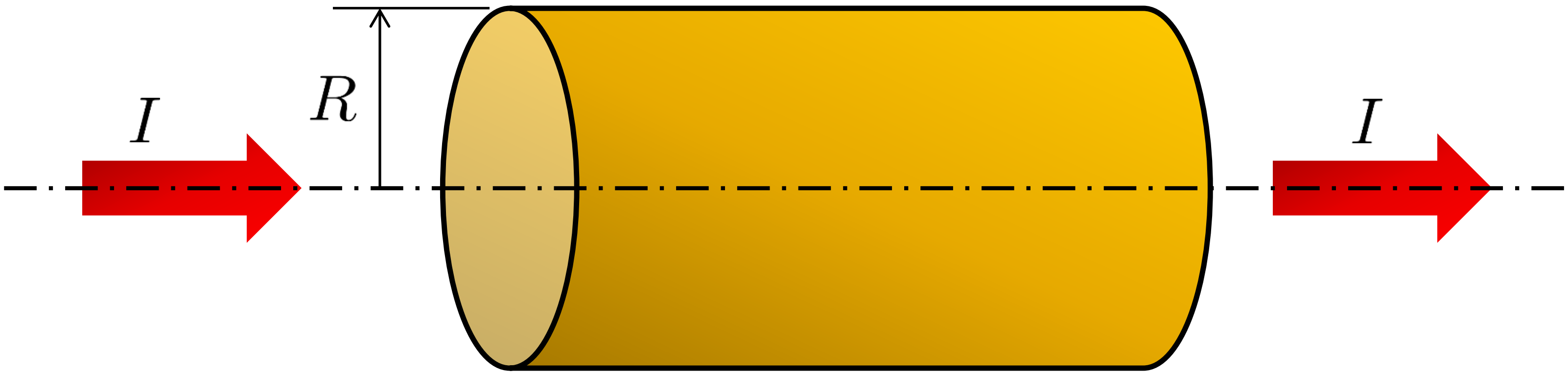
$$B \oint ds = \mu_0 I$$

$$B 2\pi r = \mu_0 I \quad \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Same thing we got using the Biot-Savart Law.

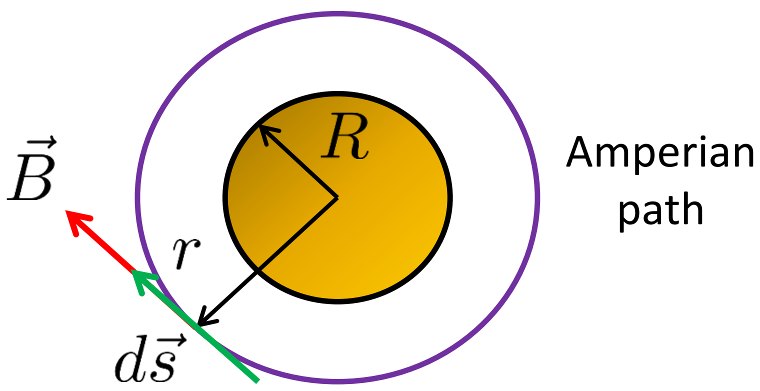
Whiteboard Problem 29-8: An Infinitely Long Wire of Radius R

An infinitely long wire of radius R carries current I assumed to be uniform over the cross section of the wire.



Part 1: Find the magnitude of the magnetic field outside the wire for r > R: (LC)

Cross Section Diagram:



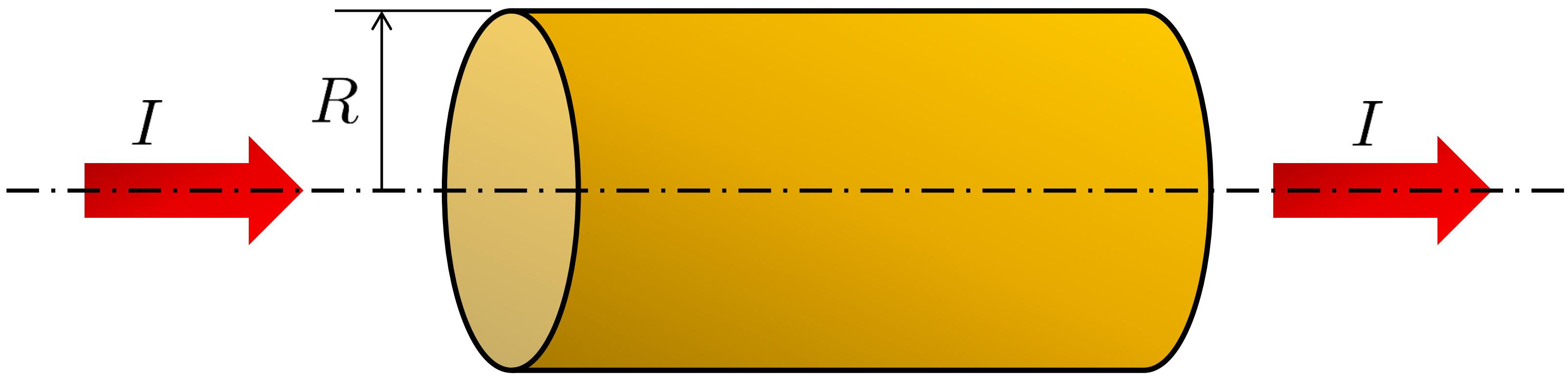
Since all of the current goes through the path, the steps are the same as for a thin wire.

Answer:

$$\text{For } r > R\text{:}\;\; B = \frac{\mu_0 I}{2\pi r}$$

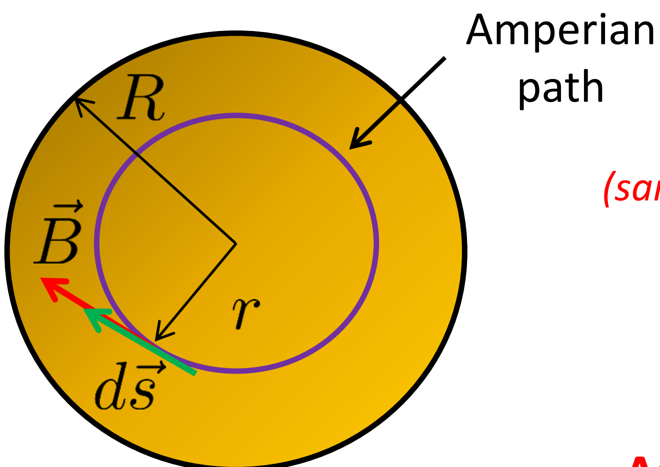
as if all of the current is concentrated on the axis

Whiteboard Problem 29-8 : An Infinitely Long Wire of Radius R



Part 2: Find the magnitude of the magnetic field inside the wire for r < R: (LC)

Cross Section Diagram:



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}}$$

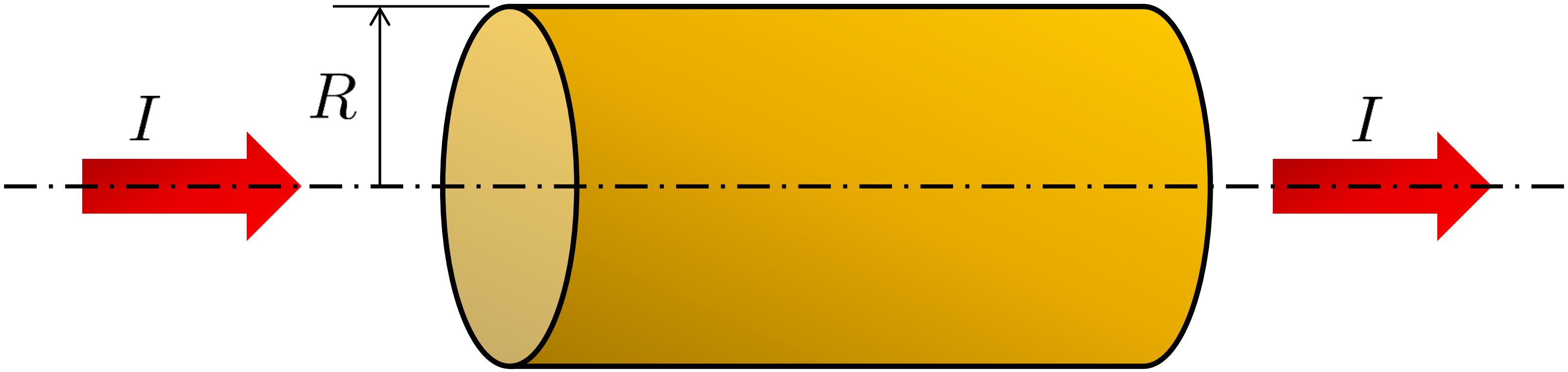
$$B 2\pi r$$

(same symmetry arguments)

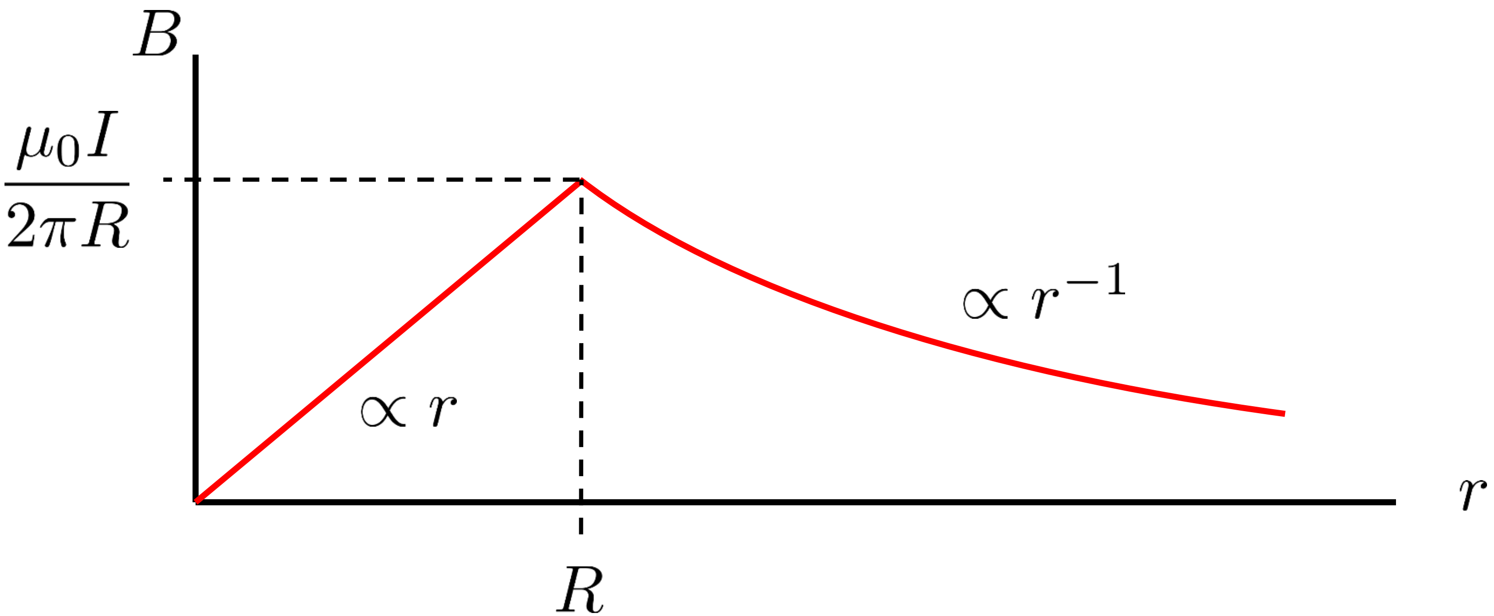
$$\begin{aligned} I_{\text{thru}} &= J(\text{Area})_{\text{thru}} \\ &= J\pi r^2 \\ &= \frac{I}{\pi R^2}\pi r^2 \\ &= I\frac{r^2}{R^2} \end{aligned}$$

Answer: $$B = \frac{\mu_0 I}{2\pi R^2} r$$

Whiteboard Problem 29-8 : An Infinitely Long Wire of Radius R



Part 3: Plot the magnitude of the magnetic field as a function of r: (LC)



A Short Pause: Maxwell's Equations

As I'm sure you remember us saying, one of our goals in our study of electricity and magnetism is to arrive at **Maxwell's Equations, a complete description of all classical Electricity and Magnetism**. So far, we've talked about one, but we have some more:

Maxwell Equation #1: Gauss' Law for $\vec{E}$: $$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

Describes how charge creates $\vec{E}$

Maxwell Equation #2: If charges create the electric field, and we know there are no isolated magnetic charges (poles):

Gauss' Law for $\vec{B}$: $$\Phi_m = \oint \vec{B} \cdot d\vec{A} = 0$$

says there are no isolated magnetic monopoles and $\vec{B}$ forms closed loops

Maxwell Equation #3: Ampere's Law for $\vec{B}$: $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}}$$

Describes how currents create $\vec{B}$

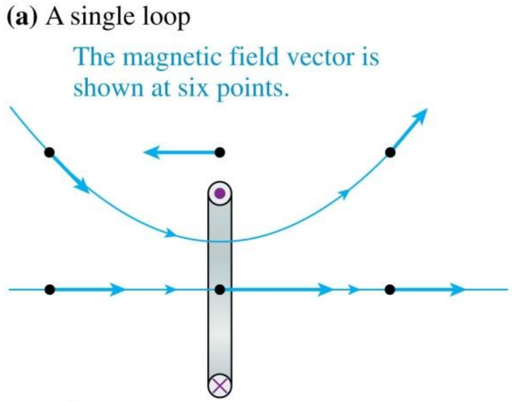
There's only one more Maxwell Equation to go!

The Solenoid Field

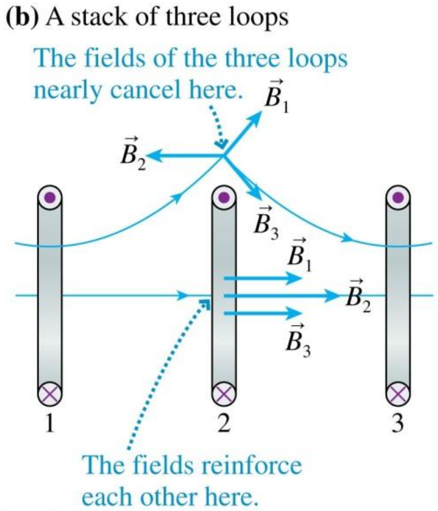
In lab, maybe you wound a wire around a nail and made a strong electromagnet. Or you worked with a large coil of wire with many loops. These are examples of a Solenoid Field. **The Solenoid creates a strong uniform field in a region of space**.

When you stack loops of current, they reinforce inside and cancel on the outside:

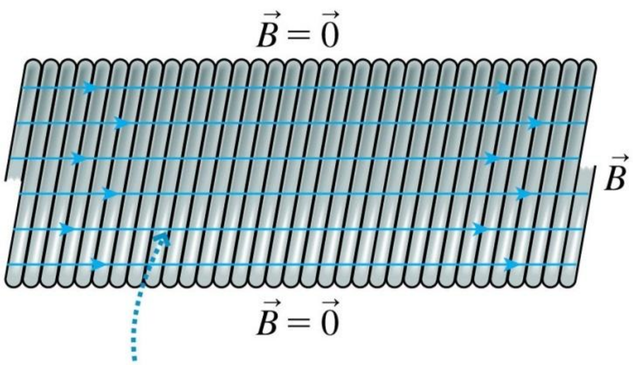
One Loop



Three Loops



Many Loops, A Solenoid



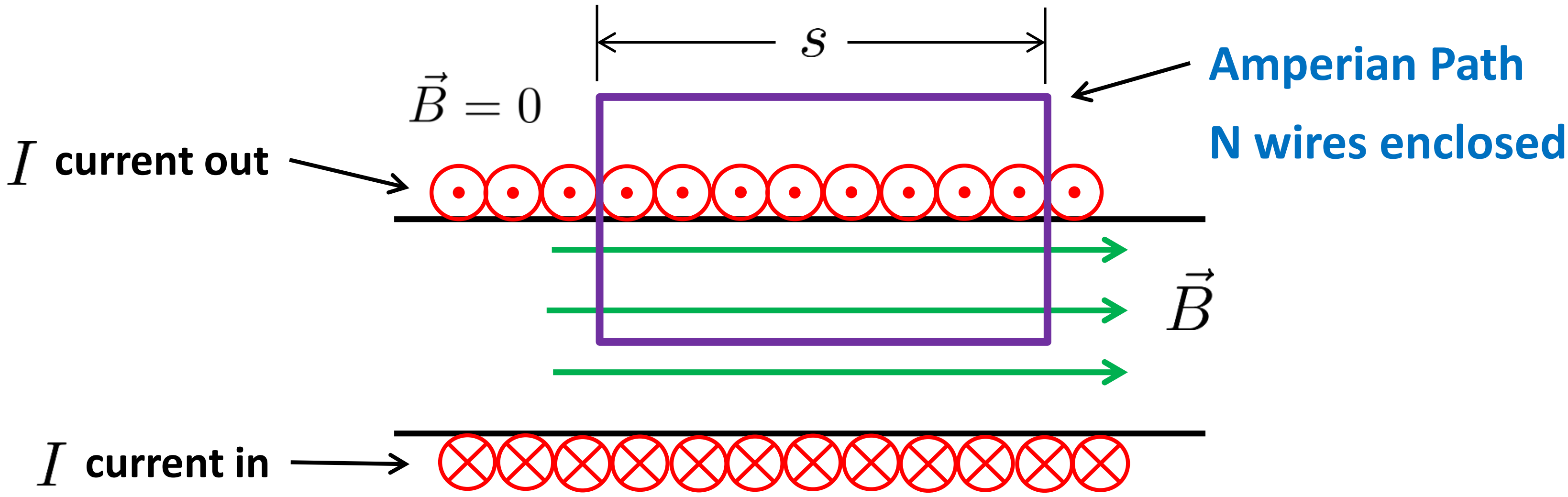
The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

For many loops where the length of the solenoid >> diameter, we have an Ideal Solenoid where there is a uniform field inside and no field outside.

Can we determine the magnitude and direction of the field inside the solenoid?

The Ideal Solenoid Field

Cross Section of Solenoid:



Apply Ampere's Law to the path:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru}}$$

Starting at the lower left corner (CCW): $Bs + 0 + 0 + 0 = \mu_0 NI$

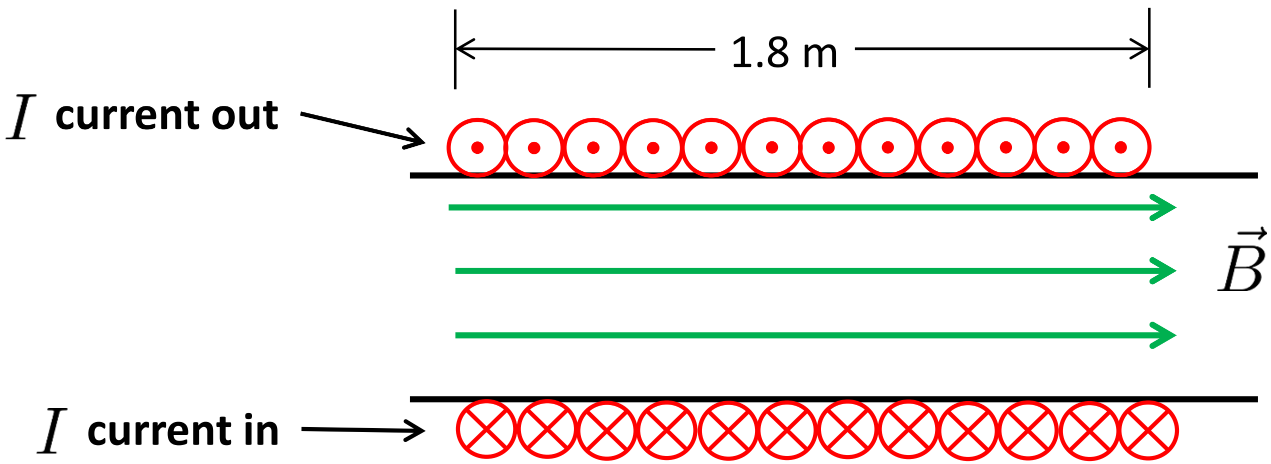
So: $$B = \frac{\mu_0 NI}{s}$$ Or $$B = \mu_0 nI$$

where: $n = \dfrac{N}{s}$ is the number of turns per length

Whiteboard Problem: 29-9

Magnetic Resonance Imaging (MRI) requires a magnetic field strength of 1.5 T produced by a solenoid. The MRI is 1.8 m long and 75 cm in diameter so that a person can fit inside. The solenoid is made of a tightly wound single layer of 2.0 mm diameter superconducting wire.

Cross Section of Solenoid:



How much current is needed to produce the 1.5 T magnetic field? (LC)

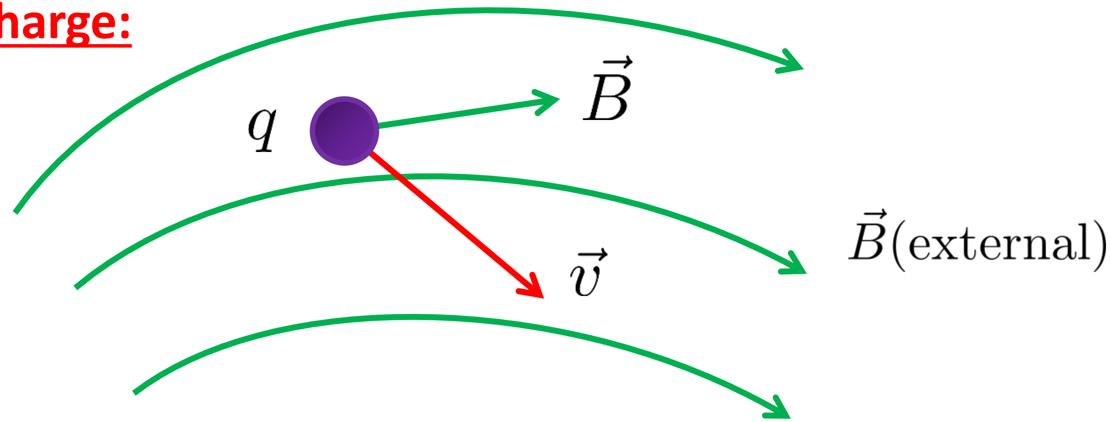
Magnetic Forces

Now that we know how to determine the magnetic fields produced by charges and currents, we're ready to study **how externally created fields exert forces on charges**.

The Magnetic Force on a Charge:

Force on q:

$$\vec{F} = q\vec{v} \times \vec{B}$$



Note:

-$\vec{F}$ is always perpendicular to both $\vec{v}$ and $\vec{B}$ (because of $\vec{v} \times \vec{B}$)

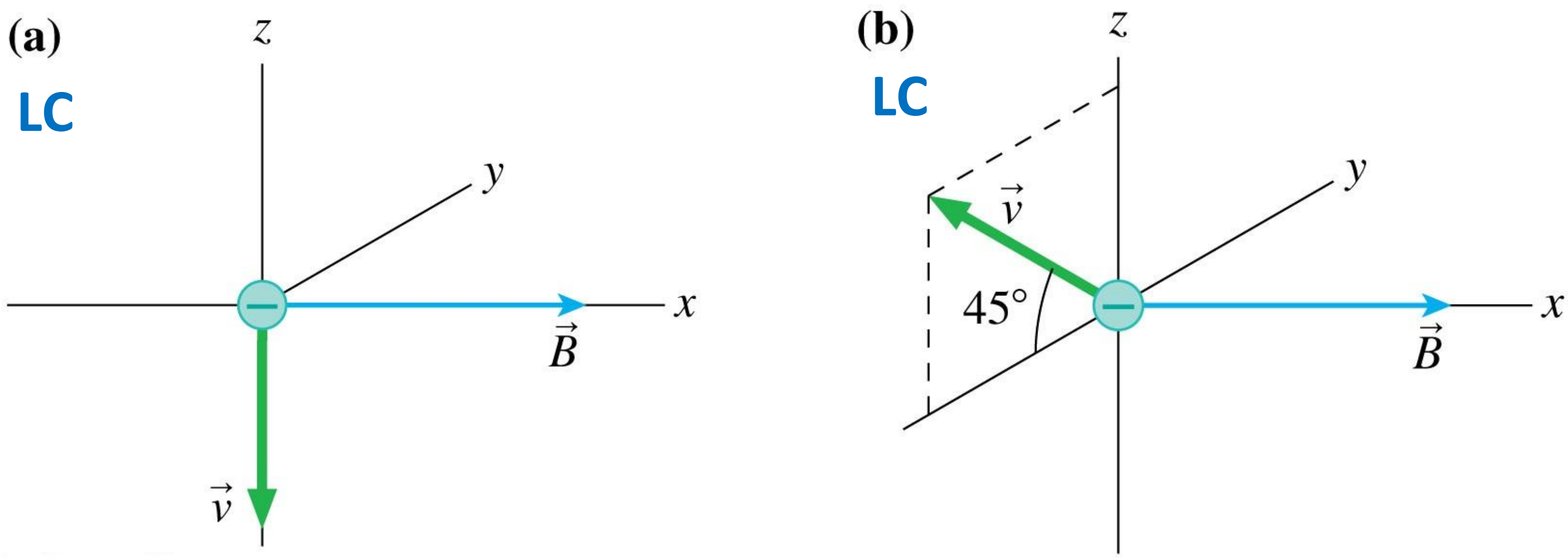
-Equal magnitude of $\vec{F}$, but opposite directions for $q > 0$ and $q < 0$

-The magnetic force does not change the speed of the particle just the direction of the velocity. Why?

$\vec{F}$ is always $\perp$ $\vec{v}$, so it does no work.

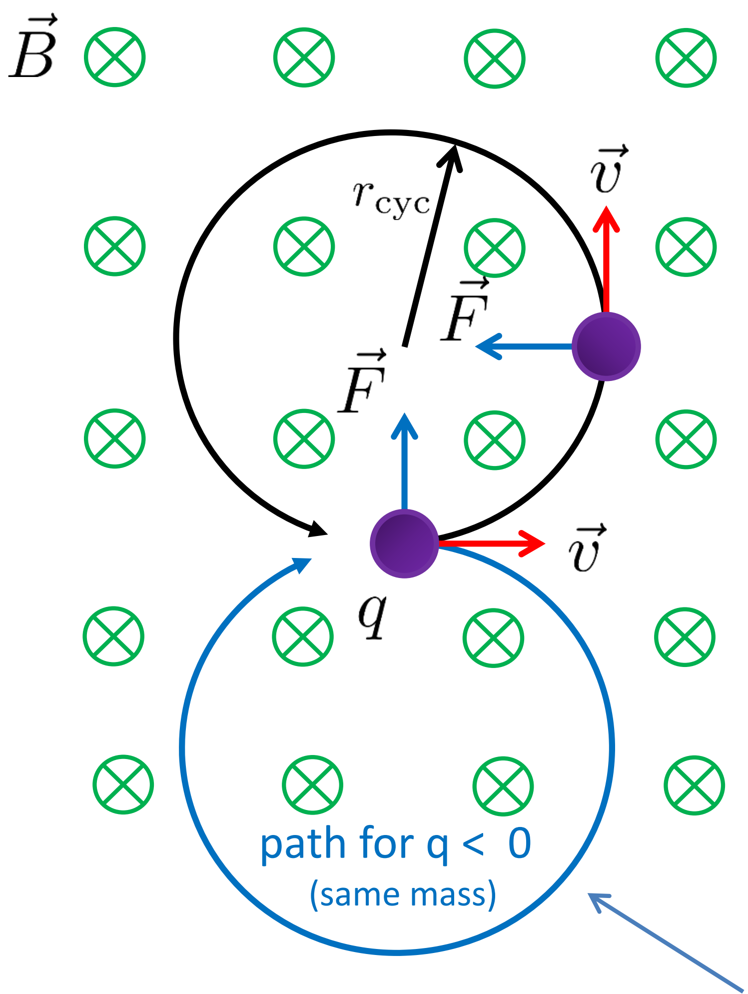
Whiteboard Problem: 29-10

An electron moves in the magnetic field $\vec{B} = 0.5\hat{\imath}\ T$ with a speed of $1.0 \times 10^7\ m/s$ in the direction shown. What is the magnetic force $\vec{F}$ on the electron?
Give your answer in component form.



Cyclotron Motion

When a charged particle moves in a uniform magnetic field, it's speed remains constant, but it's direction changes. The resulting motion is called **cyclotron motion**.



$$\text{For } q > 0: \quad \vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = (qvB, \text{ towards center})$$

So, we have **Uniform Circular Motion** from PHY181 (chapter 8):

$$\sum F_r = qvB = ma_r = \frac{mv^2}{r}$$

So, the **Radius** is: $$r_{cyc} = \frac{mv}{qB}$$

The Period of the orbit is:

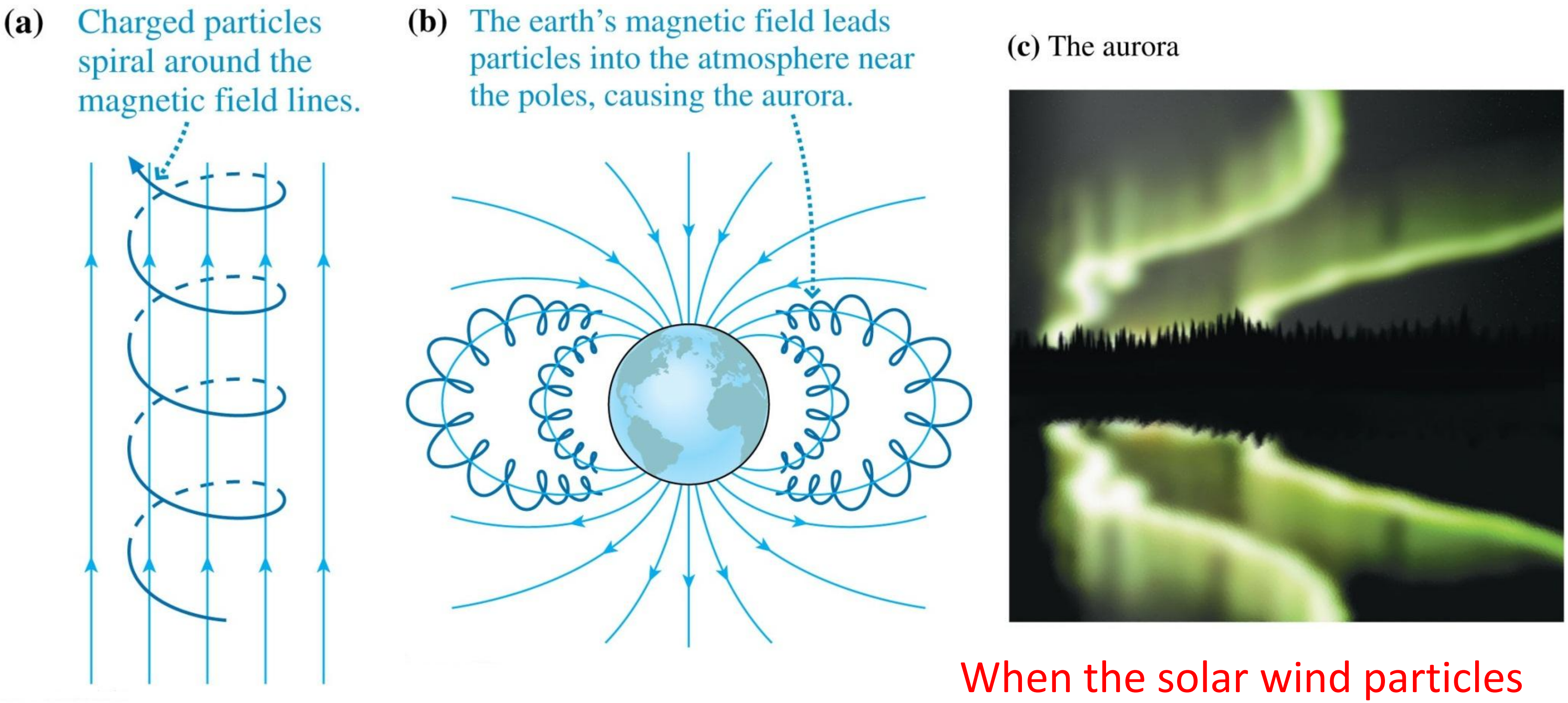
$$P = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Or, the Cyclotron Frequency (= 1/P): $$f_{cyc} = \frac{qB}{2\pi m}$$

What if the charge is negative?

Ans: the circle goes the other way

More on Cyclotron Motion: The Aurora



General motion of a charged particle in a magnetic field, a helix

Solar wind particles get trapped in the Earth's magnetic field and spiral toward the poles

When the solar wind particles collide with the nitrogen & oxygen in the upper atmosphere, it causes the aurora.

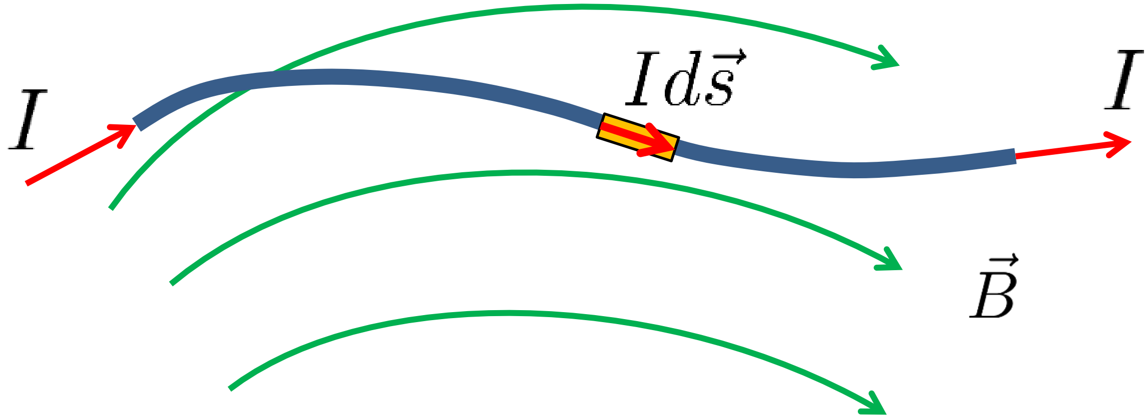
Video: Aurora from 2014

(link works; time is sped up a bit)

Magnetic Force on a Current

A current is just a bunch of moving charges, so when a current carrying wire is in a magnetic field, it will feel a force – the sum of forces on all the individual charges.

Most General Case: Arbitrary Wire in an Arbitrary Magnetic Field:



Just as we did in going from the Biot-Savart Law for a charge to that for a current, we treat an element of current as a point charge. So the force on this element of charge is:

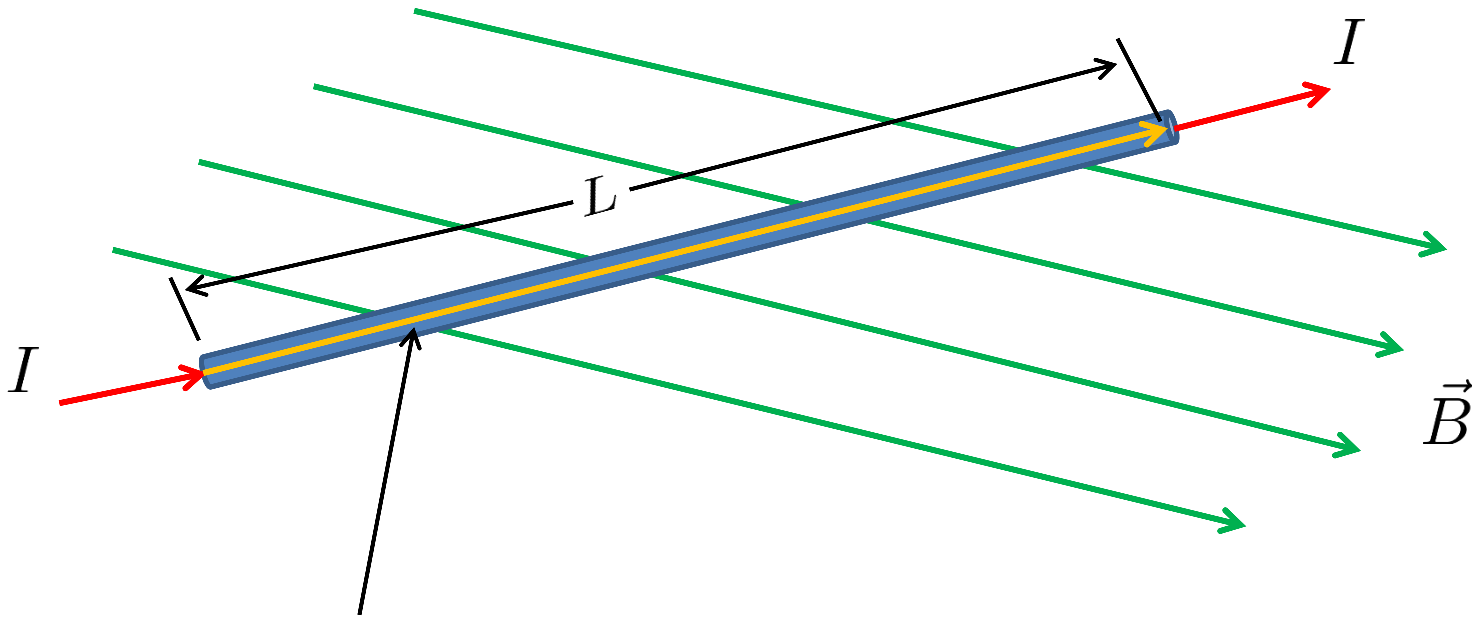
$$d\vec{F} = I d\vec{s} \times \vec{B}$$

Then to get the total force on the wire, we just add up (*i.e. integrate*) the forces on all of the elements:

$$\vec{F} = \int_{\text{wire}} d\vec{F}$$

Magnetic Force on a Current

A Useful Special Case: A Straight Wire in a Uniform Magnetic Field:



Define: $\vec{L} = (L, \text{direction of I})$

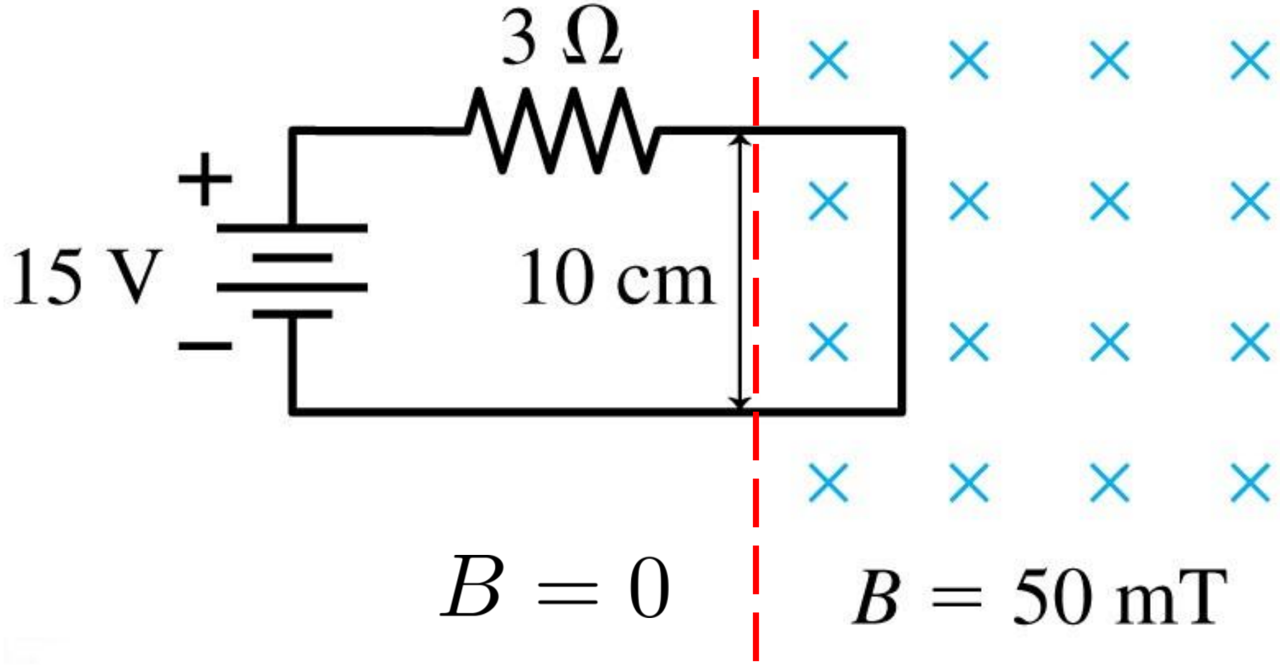
Force on the Wire: $$\vec{F} = I\vec{L} \times \vec{B}$$

We'll use this equation a lot.

Whiteboard Problem: 29-11

The right edge of the circuit in the figure extends into a 50 mT uniform magnetic field.

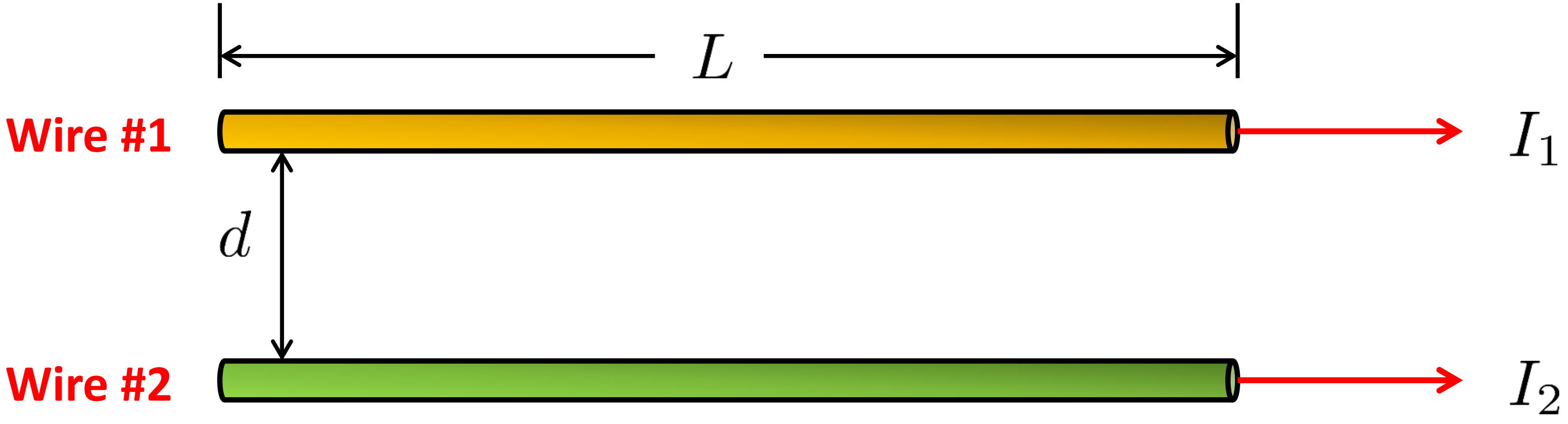
What are the direction (LC) and the magnitude (LC) of the Net Force on the circuit?



Whiteboard Problem 29-12: Force Between Two Parallel Wires

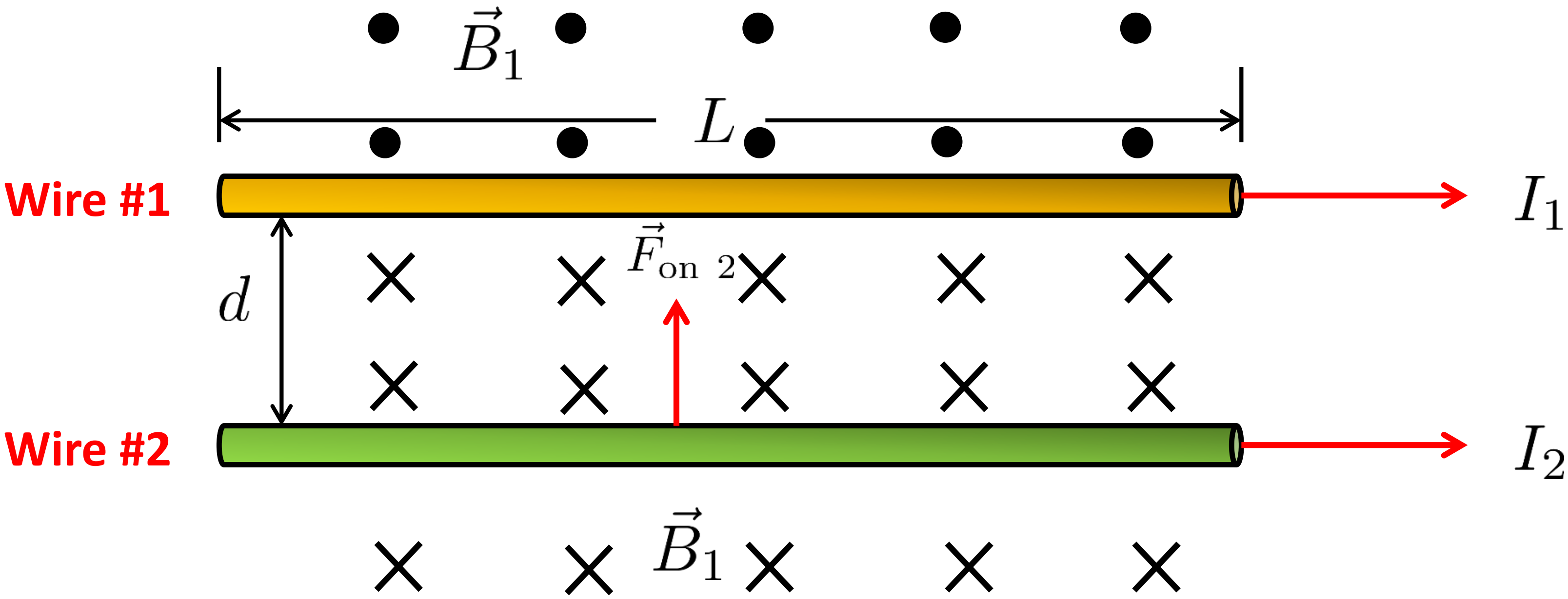
Two straight wires of equal length are parallel and carry currents.

Find an expression, in terms of the length of the wires, the currents, and the distance between the wires, for the force on the lower wire, wire #2. (LC)



Next page for Hints:

Whiteboard Problem 29-12: Parallel Wires – Some Hints



Wire #1 creates a magnetic field B_1.

Wire #2 feels a force in the field of Wire #1: $\vec{F}_{\text{on }2} = I_2\vec{L} \times \vec{B}_1 = (I_2 L B_1, \text{up})$

The magnitude of field of Wire #1 at the location of Wire #2 is: $B_1 = \frac{\mu_0 I_1}{2\pi d}$

So, the force on Wire #2 is:
(LC, magnitude only)

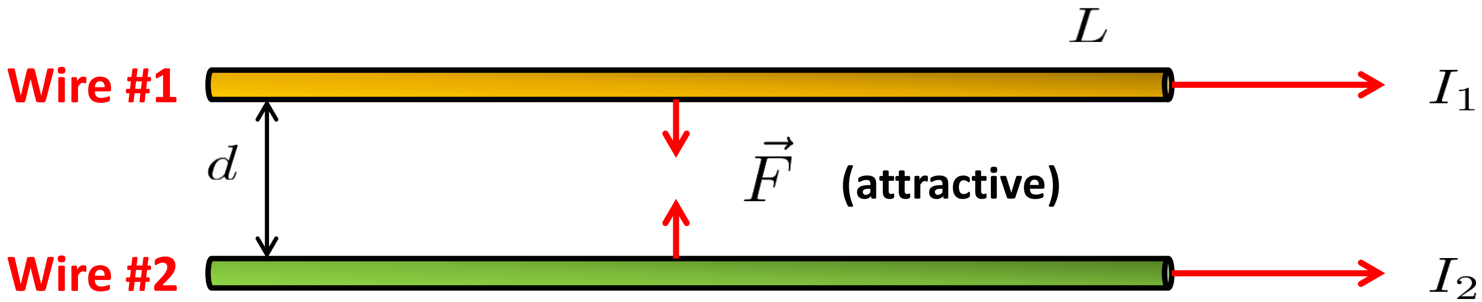
$$\vec{F}_{\text{on }2} = \left(\frac{\mu_0 L I_1 I_2}{2\pi d}, \text{up}\right)$$

What about the force on Wire #1? Equal magnitude, opposite direction (3rd Law).

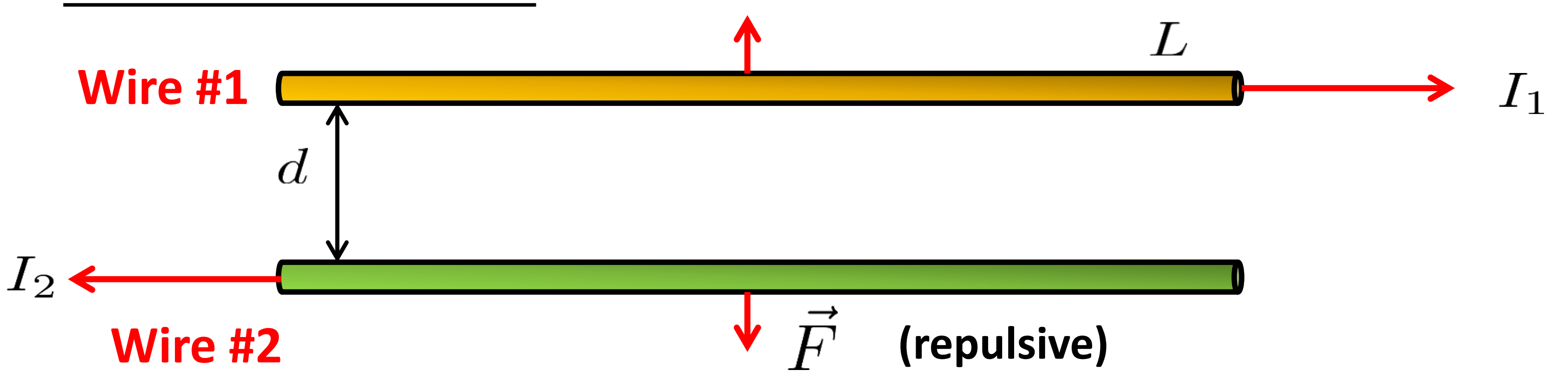
What about anti-parallel currents? Repulsive Forces.

Whiteboard Problem 29-12: Parallel Wires – Summary

Parallel Currents:

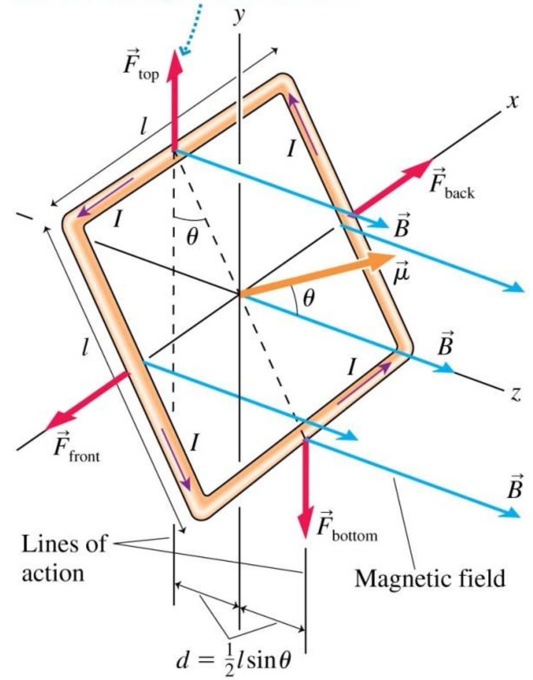


Anti-Parallel Currents:



$$\vec{F} = \left[\frac{\mu_0 L I_1 I_2}{2\pi d}, \begin{pmatrix} \text{attractive for parallel} \\ \text{repulsive for anti} - \text{parallel} \end{pmatrix} \right]$$

Torque on a Current Loop

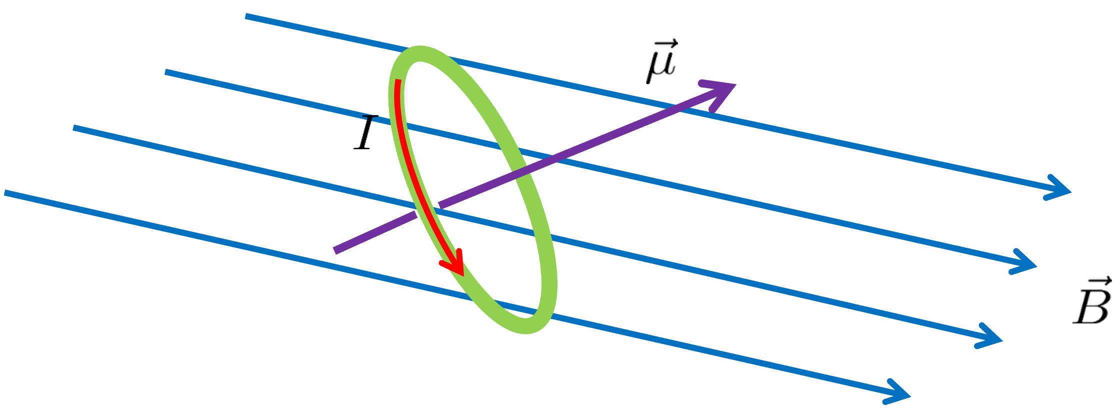


Your author considers a square current loop in an external magnetic field. The net force on the loop is zero – if the field is uniform, but there is a torque on the loop.

This very complicated diagram is used just to establish the expression for the torque on a current loop. We could arrive at the relation using just what we know about dipoles.

A current loop has a magnetic dipole moment, and , as we saw previously:

a magnetic dipole behaves in an external magnetic field just like an electric dipole does in an external electric field, and we know: $\vec{\tau} = \vec{p} \times \vec{E}$ **; So:**



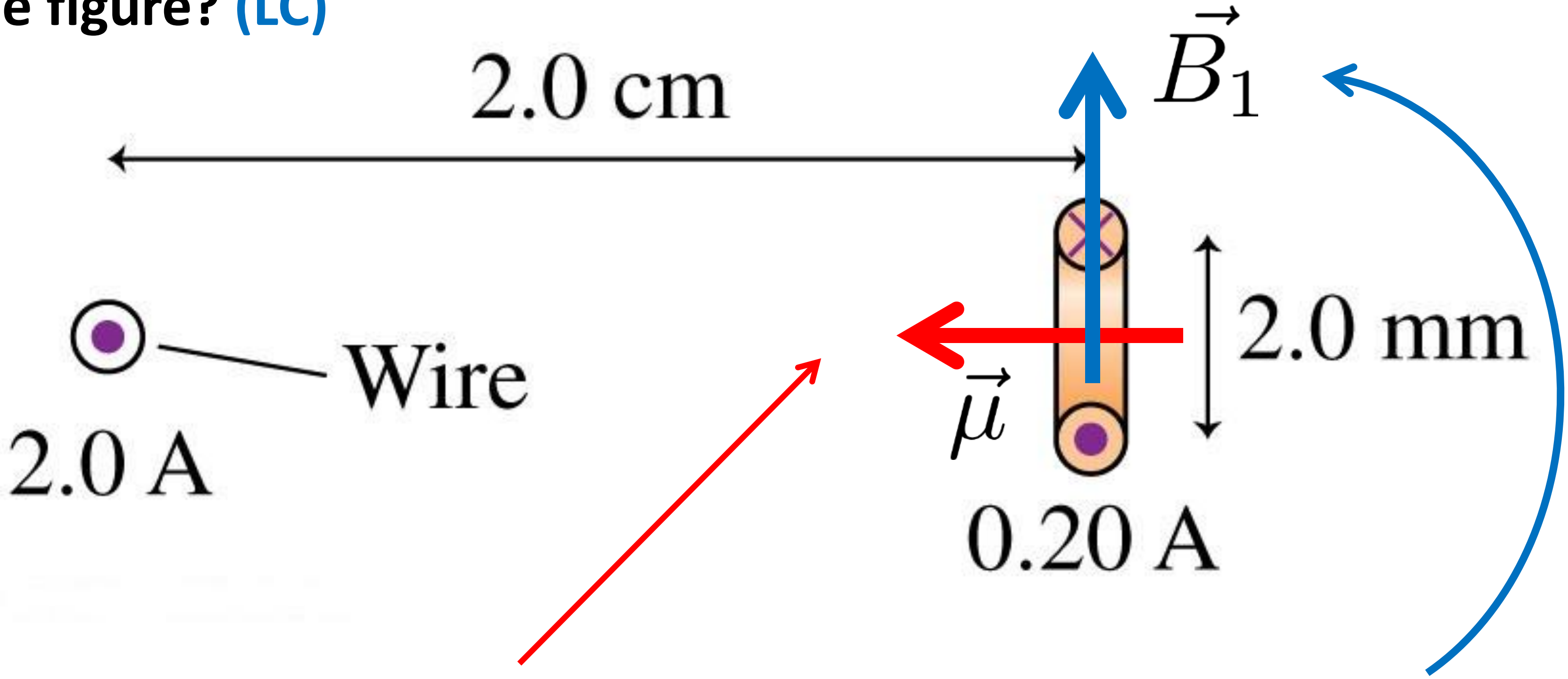
The Torque on the Loop is:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

A Really Wonderful Whiteboard Problem: 29-13

(Or a HW problem – Give hints)

What is the magnitude of the torque on the current loop in the figure? (LC)



Hint: Draw the dipole moment of the loop and the field of the wire at the location of the loop.

What is the loop's equilibrium orientation?