29-1: The Magnetic Field

In the first two sections of this chapter, your author briefly recounts some of the history of how we became aware of magnetism.

Certain naturally occurring rocks were known to attract and repel each other and attract certain metals. Also these rocks could be fashioned into a crude compass.

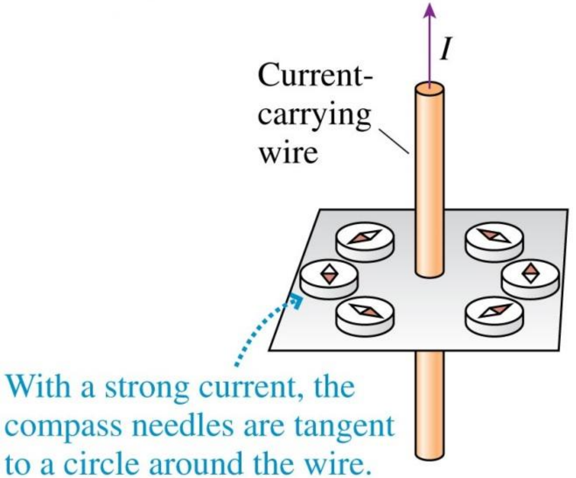
(dogs & compass)

Notice that magnets exert and feel forces when there is no contact. These are long range forces (like the electric force), **and hence we'll use the language of fields to describe them.**

Each magnet has two poles, north and south, that behave somewhat like positive and negative charges, i.e. like poles repel and unlike poles attract, **but every magnet always has two poles** – **magnetic monopoles don't exist**

Perhaps the most important discovery about magnetism was in 1819 when Hans Oersted accidentally found that **electric currents can exert a force on compasses.** *(As we'll see, it's actually a torque.)*

This is our starting point for our study of Magnetism: Moving charges create magnetic fields. **First, we'll look at the source of fields, then how fields affect charges.**



(We'll solve this problem today.)

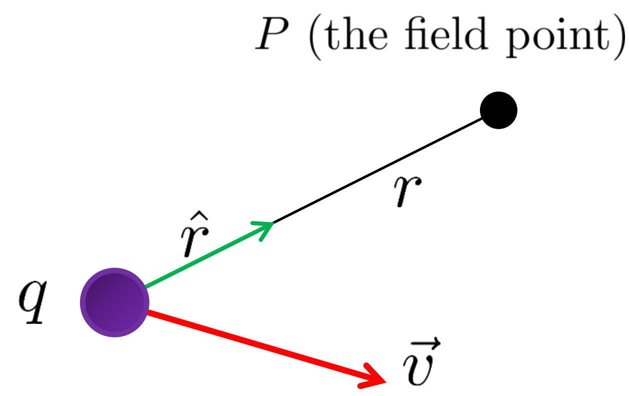
Biot-Savart Law for a Point Charge

Most of the time, we'll be concerned with magnetic fields created by a whole bunch of moving charges – **i.e. currents, but you can calculate the field produced by a single moving charge:**

The magnetic field created by q at P is:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

MKS Units: Tesla (T)



Store in your calculator

Where: $\mu_0 = \text{permeability constant} = 1.257 \times 10^{-6}\ \frac{Tm}{A}$

$r = \text{distance from } q \text{ to } P$

$\hat{r} = \text{unit vector from } q \text{ to } P$

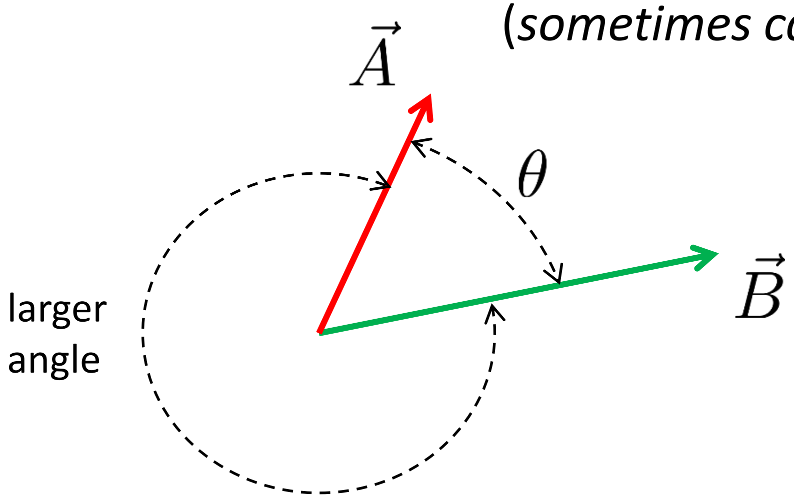
$\vec{v} = \text{velocity of } q$

Note: there are some similarities between this and the electric field of a point charge: both are inverse square fields, and both have opposite directions for different charges. **The big difference is the velocity dependence and the cross product.** **(Every important equation in magnetism has a cross product in it!)**

These slides are from PHY181 – we're going to skim through them, but they're here for your reference.

Vector Cross Product

(*sometimes called a Vector Product*)



Vector, $\vec{C} = \vec{A} \times \vec{B}$

Where: $|\vec{C}| = AB\sin\theta$

Direction of $\vec{C}$ is from the Right Hand Rule (RHR)

Where: A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$

θ is the smaller of the two angles between $\vec{A}$ and $\vec{B}$

$$\text{So, } \vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta, \text{direction by RHR})$$

What does the cross product mean in words?

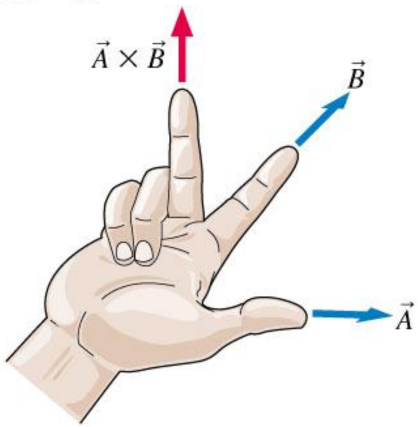
$\vec{A} \times \vec{B}$ is a measure of how much of $\vec{A}$ is perpendicular to $\vec{B}$ and in what direction, and vice versa.

What is the Right Hand Rule (RHR)?

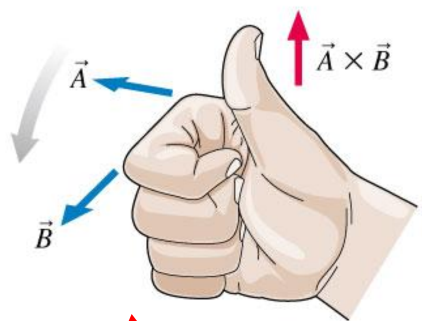
The Right Hand Rule (From your text)

Using the right-hand rule

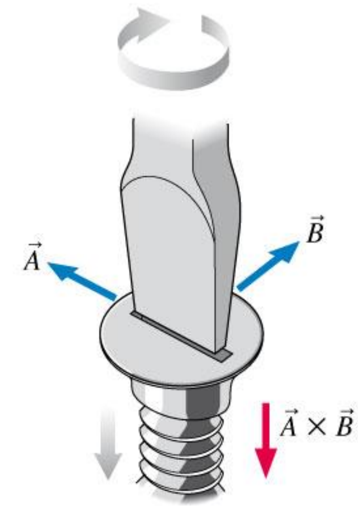
Spread your *right* thumb and index finger apart by angle α. Bend your middle finger so that it is *perpendicular* to your thumb and index finger. Orient your hand so that your thumb points in the direction of $\vec{A}$ and your index finger in the direction of $\vec{B}$. Your middle finger now points in the direction of $\vec{A} \times \vec{B}$.



Make a loose fist with your *right* hand with your thumb extended outward. Orient your hand so that your thumb is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and your fingers are curling *from* the line of vector $\vec{A}$ *toward* the line of vector $\vec{B}$. Your thumb now points in the direction of $\vec{A} \times \vec{B}$.

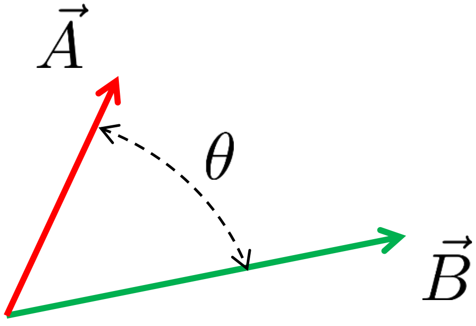


Imagine using a screwdriver to turn the slot in the head of a screw from the direction of $\vec{A}$ to the direction of $\vec{B}$. The screw will move either "in" or "out." The direction in which the screw moves is the direction of $\vec{A} \times \vec{B}$.



There are many ways to do the RHR. The way that I do the Right Hand Rule is a variation of this way.

The Right Hand Rule (My Way)



1. **Use your right hand!**

2. With your four fingers extended and together, point them in the direction of the first vector of the cross product.

3. Rotate your hand about an axis through your forearm until you can close your fingers through the angle θ towards the second vector in the product.

4. Your thumb points in the direction of the cross product.

So, $\vec{C} = \vec{A} \times \vec{B}$ points into the screen.

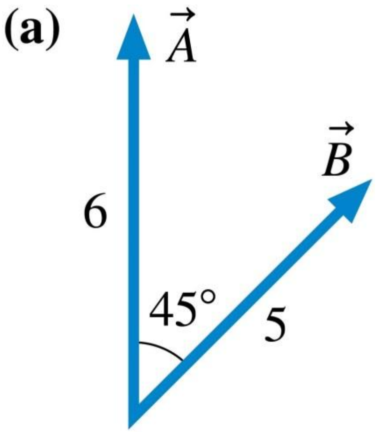
And, $\vec{D} = \vec{B} \times \vec{A}$ points out of the screen.

Note: $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

And, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

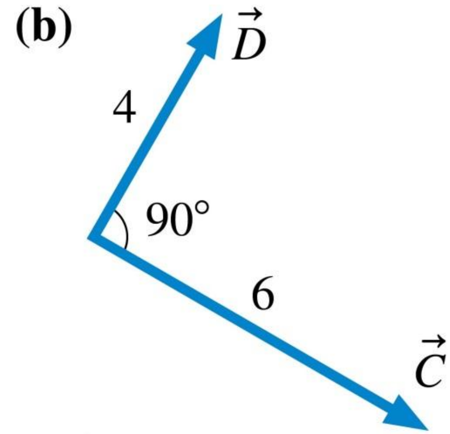
WB Problem: 29-1   Some Cross Products

Evaluate the following cross products (give direction as in or out):



$\vec{A} \times \vec{B}$

LC (mag first)
LC (direction)
Ans: (21.21, in)



$\vec{C} \times \vec{D}$

LC (mag first)
LC (direction)
Ans: (24.0,out)

(c) Find cross product in Component Form.

$$\vec{A} = 3\hat{\imath} + \hat{\jmath} \qquad \vec{B} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$$

Find: $\vec{A} \times \vec{B}$

Hmmm, maybe we need another way to do a Cross Product

Vector Cross Product using a Determinant

When you know only the components of the vectors in a cross product, it may difficult to find the angle between them, and it can be even more difficult to apply the RHR and find the components of the cross product vector. **Here's how you do it easily:**

A determinant is something that you'll learn about in a Math course, perhaps linear algebra. All we need to know now is how to expand one. **A 2 X 2 determinant is the simplest (a,b,c,d are just numbers):**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Note, the negative sign; when you expand a determinant, the signs on the terms always alternate.

Now a 3 X 3 determinant can be expanded into three 2 X 2 determinants:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

(Now, just expand the 2 X 2's) $= a(ei - hf) - b(di - gf) + c(dh - ge)$

Note, there are many ways to expand a determinant. If you know one that you like and it works, use that technique.

OK, how does this work for a vector cross product?

Vector Cross Product using a Determinant

For the vectors:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{\jmath}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \underbrace{\hat{\imath}(A_yB_z - B_yA_z)}_{C_x} - \underbrace{\hat{\jmath}(A_xB_z - B_xA_z)}_{C_y = (B_xA_z - A_xB_z)} + \underbrace{\hat{k}(A_xB_y - B_xA_y)}_{C_z}$$

So, this method directly gives the components of the cross product vector. <u>You don't need to know any angles, and you don't need the Right Hand Rule!</u>

See the Equation Sheet

Now, Let's go back to Problem WB 29-1c:

(c) Find cross product in Component Form.

$$\vec{A} = 3\hat{\imath} + \hat{\jmath} \qquad \vec{B} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$$

Find: $\vec{A} \times \vec{B}$ **(LC)**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{\imath} \begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 3 & 0 \\ 3 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 1 \\ 3 & -2 \end{vmatrix}$$

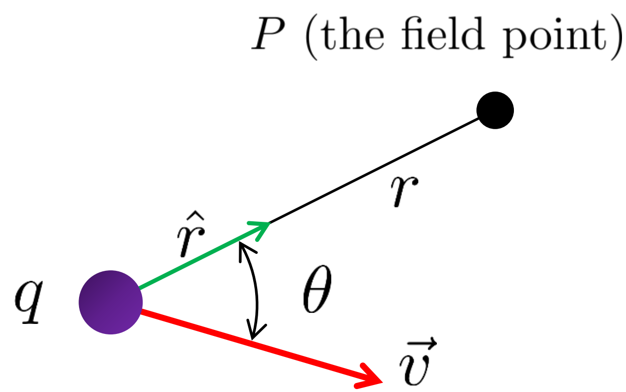
$$= \hat{\imath}[(1)(2) - (-2)(0)] - \hat{\jmath}[(3)(2) - (3)(0)] + \hat{k}[(3)(-2) - (3)(1)]$$

$$\text{So, } \vec{A} \times \vec{B} = 2\hat{\imath} - 6\hat{\jmath} - 9\hat{k} \quad \text{and, } |\vec{A} \times \vec{B}| = 11.0$$

Back to the Biot-Savart Law for Point Charges

The magnetic field created by q at P is:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



So for this geometry: $|\vec{v}\times\hat{r}| = v|\hat{r}|\sin\theta = v\sin\theta$

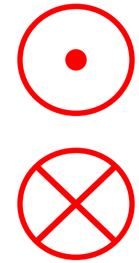
And the field <u>magnitude</u> is: $|\vec{B}| = \frac{\mu_0}{4\pi}\frac{|q|v\sin\theta}{r^2}$

Direction By RHR:

$\vec{B}$ is out of the screen for $q > 0$

$\vec{B}$ is into the screen for $q < 0$

Draw As:

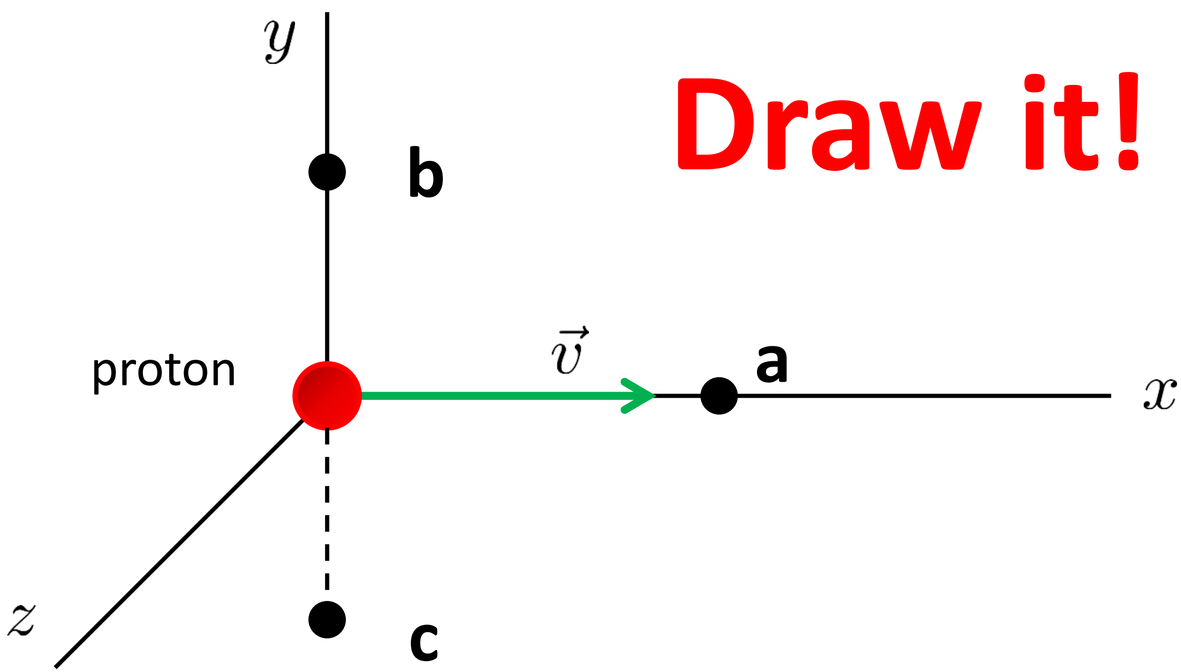


Whiteboard Problem: 29-2

A proton moves along the x-axis with $v_x = 1.0 \times 10^7$ m/s.

As it passes the origin, what are the strength and direction of the magnetic field at the following (x, y, z) points:

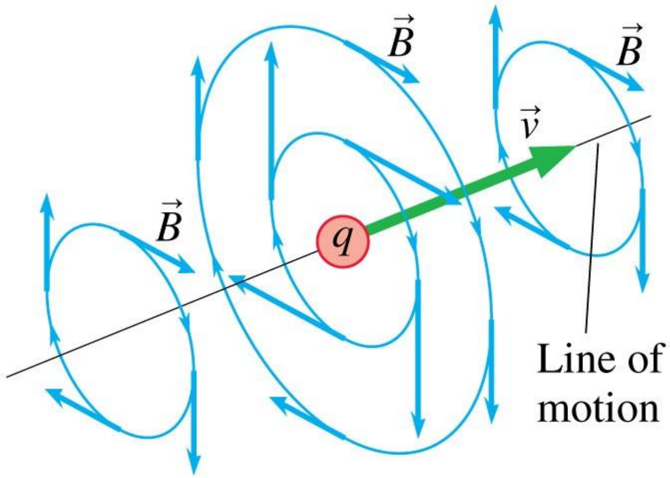
a) **(1 cm, 0 cm, 0 cm) (Enter the field magnitude in LC)**
b) **(0 cm, 1 cm, 0 cm) (Enter the field magnitude in LC)**
c) **(0 cm, -2 cm, 0 cm) (Enter the field magnitude in LC)**



Draw it!

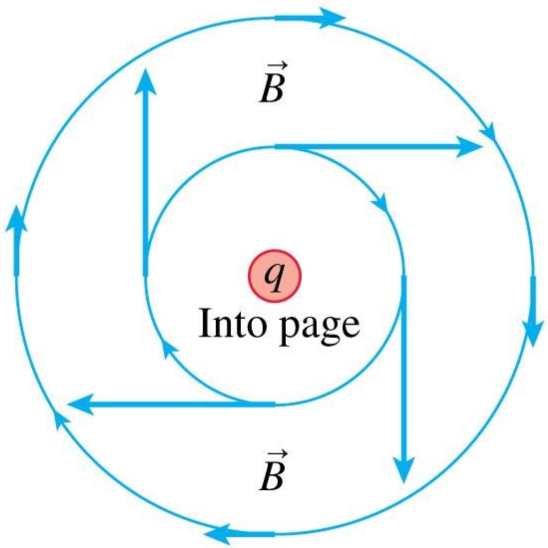
A Snapshot of the Field of a Moving Charge

In problem WB29-2, if you include many more points, you could map out the field in space at a given instant of time:



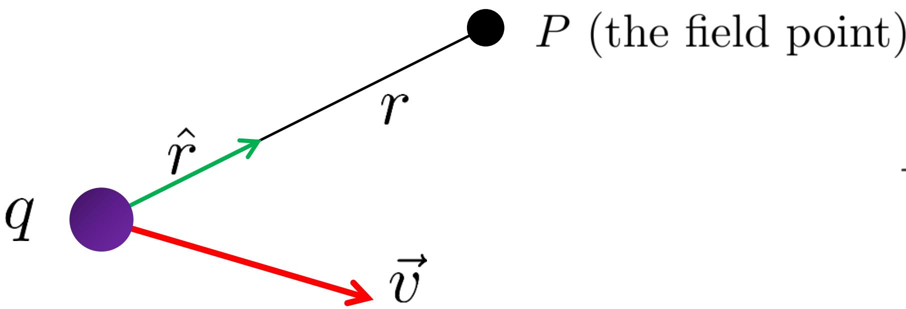
Notice that the field goes around the charge, not away from it like the electric field – magnetic fields always form closed loops. Also, the direction of the field is given by the right hand rule. For a negative charge, the field would go the other way.

This is looking parallel to the velocity vector going into the page, and only shows the field in the plane of the charge.



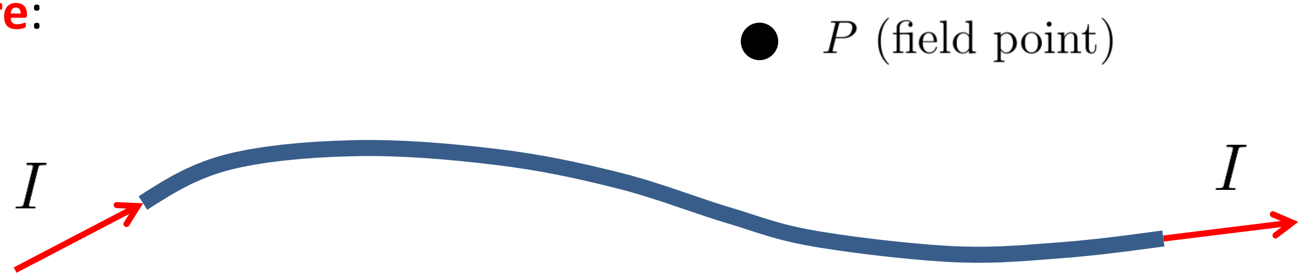
The Biot-Savart Law for Currents

So far, we have introduced the **Biot-Savart Law for a single moving charge:**



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

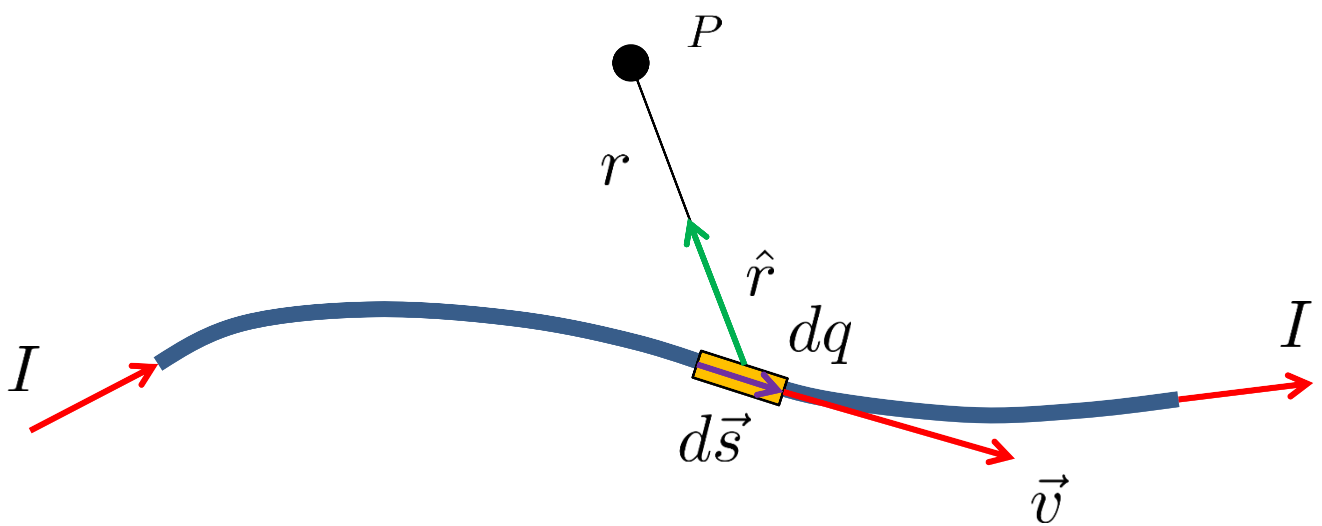
As you will see in the lab activity this week, we usually are interested in magnetic fields created by a large group of moving charges – **e.g. steady current in a section of wire**:



How would we do this?

We treat a small element of the current as a moving point charge, determine the field for it, and then add up all of the charges in the current (i.e. we integrate over the wire).

The Biot-Savart Law for Currents



So, at P, dq creates the field: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{dq\ \vec{v} \times \hat{r}}{r^2}$

Now: $\vec{v}dq = \frac{d\vec{s}}{dt}dq$ where: $d\vec{s} =$ element of length tangent to wire at dq

$$= d\vec{s}\ \frac{dq}{dt} = d\vec{s}\ I$$

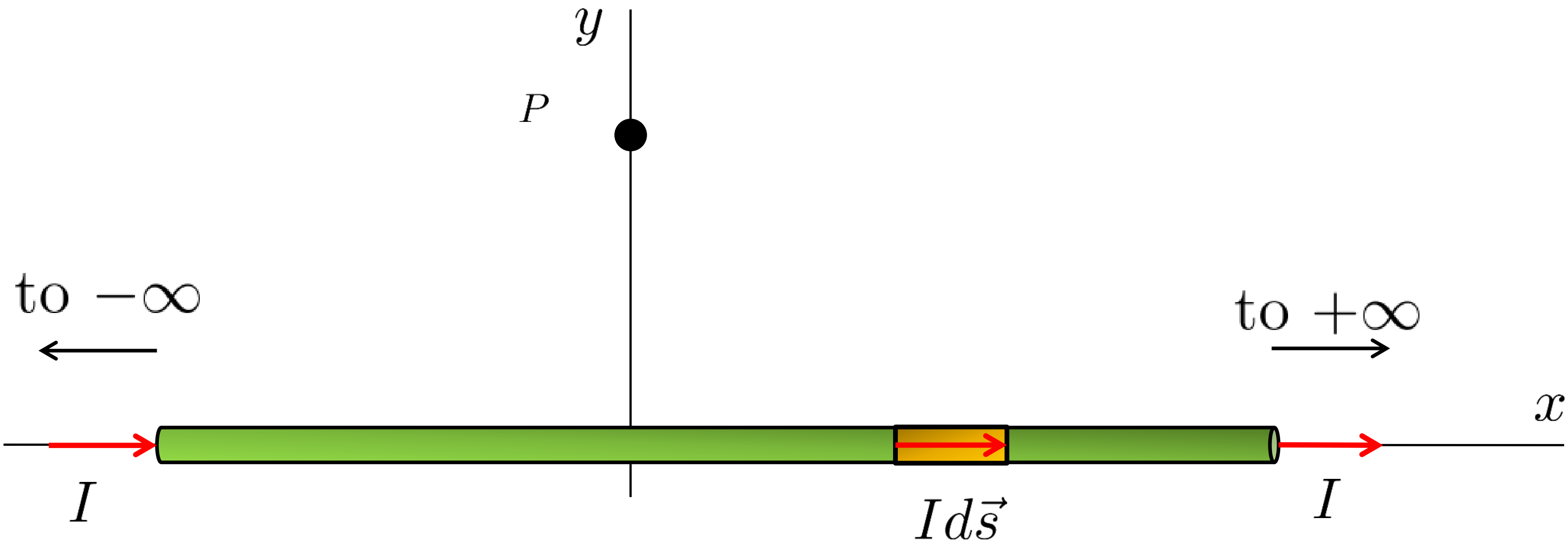
So, the Biot-Savart Law for Currents is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$

Where we have to integrate over the wire to find the total field at P.

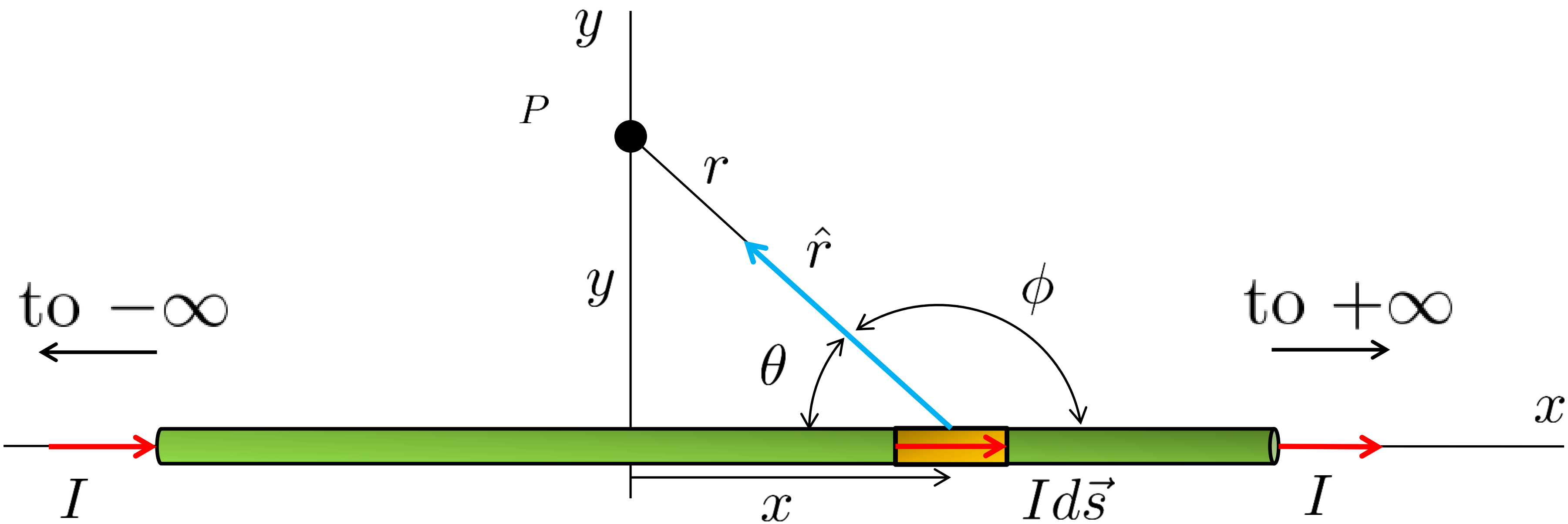
An Important Example: The Field for an Infinite Straight Wire

The most important problem using the Biot-Savart Law is to find **the magnetic field at some perpendicular distance from an infinitely long straight wire carrying a steady current – this is what Oersted discovered:**



Step 1: Choose coordinates and select current element

The Field for an Infinite Straight Wire



Step 2: The current element creates a field at P: $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{s}\times\hat{r}}{r^2}$

Step 3: Express quantities in terms of the coordinates of the current element (x,0) and the field point (0,y). (Similar to what we did for electric field problems.)

$$r^2 = x^2 + y^2 \qquad |d\vec{s}\times\hat{r}| = |d\vec{s}||\hat{r}|\sin\phi \;= dx\sin\phi$$

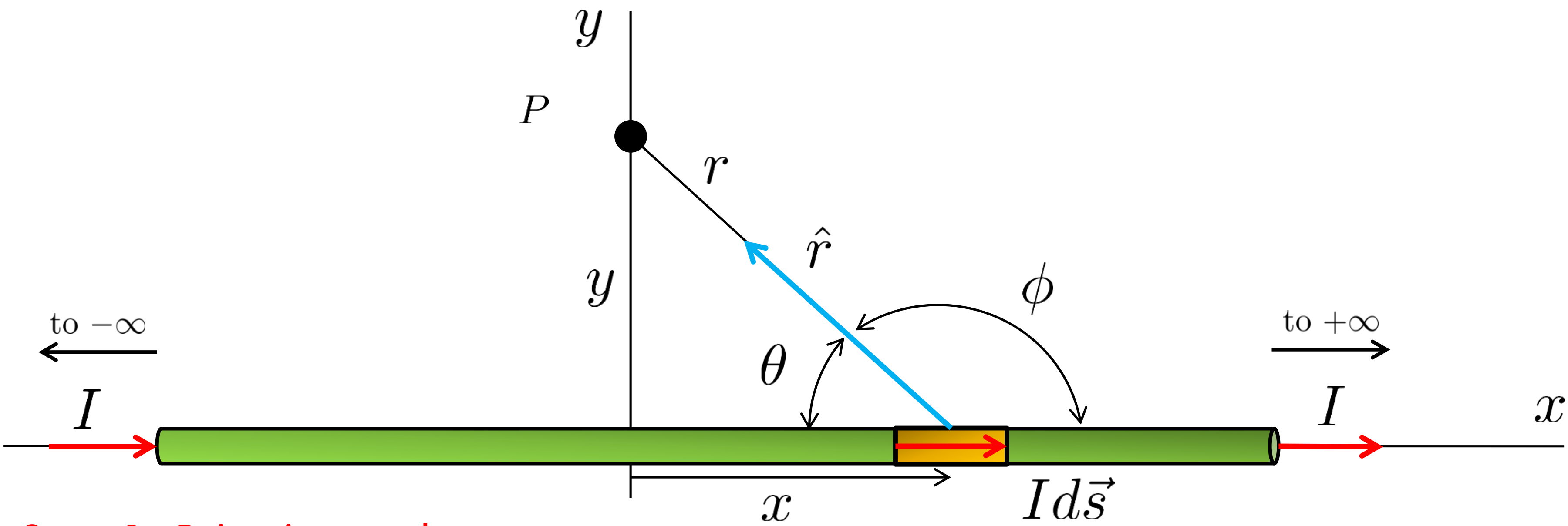
Now:

$$\sin\phi = \sin\theta \;= \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

So:

$$|d\vec{s}\times\hat{r}| = \frac{y\,dx}{\sqrt{x^2+y^2}}$$

The Field for an Infinite Straight Wire



Step 4: Bring it together:

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{|Id\vec{s}\times\hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{1}{x^2+y^2}\frac{y\,dx}{\sqrt{x^2+y^2}} = \frac{\mu_0 I}{4\pi}\frac{y\,dx}{(x^2+y^2)^{3/2}}$$

So, the magnitude of the field of the current element at P is:

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi}\frac{y\,dx}{(x^2+y^2)^{3/2}}$$

Direction: for all current elements, $Id\vec{s}$, in the wire, the RHR gives $d\vec{s}\times\hat{r}$ as out of the page

(This is for the location of P; what if P was below the x-axis? The field would be into the page.)

The Field for an Infinite Straight Wire

Step 5: Integrate over the wire from $x = -\infty \to x = +\infty$

$$|\vec{B}| = \int_{\text{wire}} |d\vec{B}| = \int_{-\infty}^{+\infty} \frac{\mu_0 I}{4\pi} \frac{y dx}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\mu_0 I y}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + y^2)^{3/2}}$$

Using our Table of Integrals:

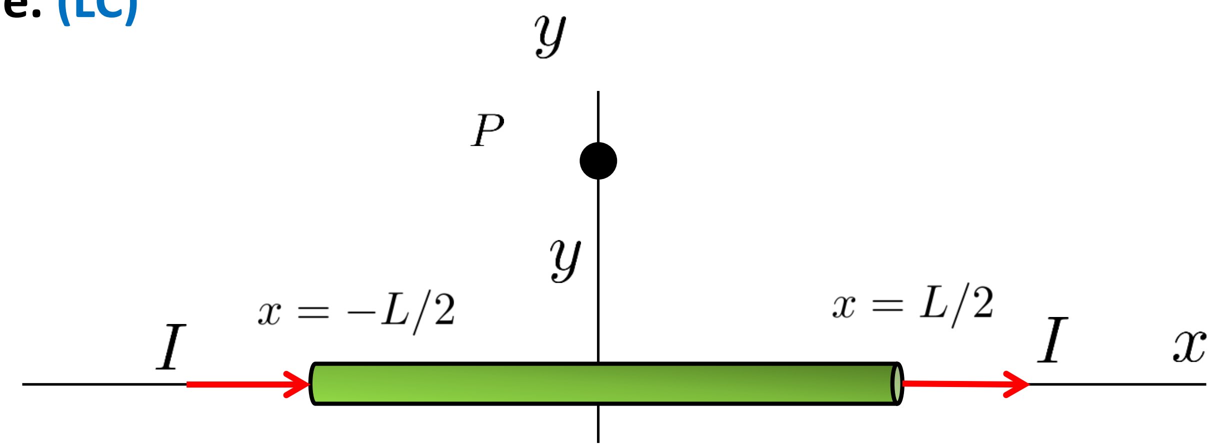
$$|\vec{B}| = \frac{\mu_0 I y}{4\pi} \left[\frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_{-\infty}^{\infty} = \frac{\mu_0 I}{4\pi y} [(1) - (-1)]$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi y}$$

(Direction is out of the page by the RHR)

Whiteboard Problem 29-3: The Finite Wire

Consider, not an infinite wire, but a wire of length L. **Find an expression for the magnitude of the magnetic field at the point P on the bisecting axis of the wire.** **(LC)**



Hint: Step 5 from the example of the infinite wire

Integrate over the wire from $x = -\infty \rightarrow x = +\infty$

$$|\vec{B}| = \int_{\text{wire}} |d\vec{B}| = \int_{-\infty}^{+\infty} \frac{\mu_0 I}{4\pi} \frac{y\,dx}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I y}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{(x^2+y^2)^{3/2}} \rightarrow \frac{\mu_0 I y}{4\pi} \int_{-L/2}^{+L/2} \frac{dx}{(x^2+y^2)^{3/2}}$$

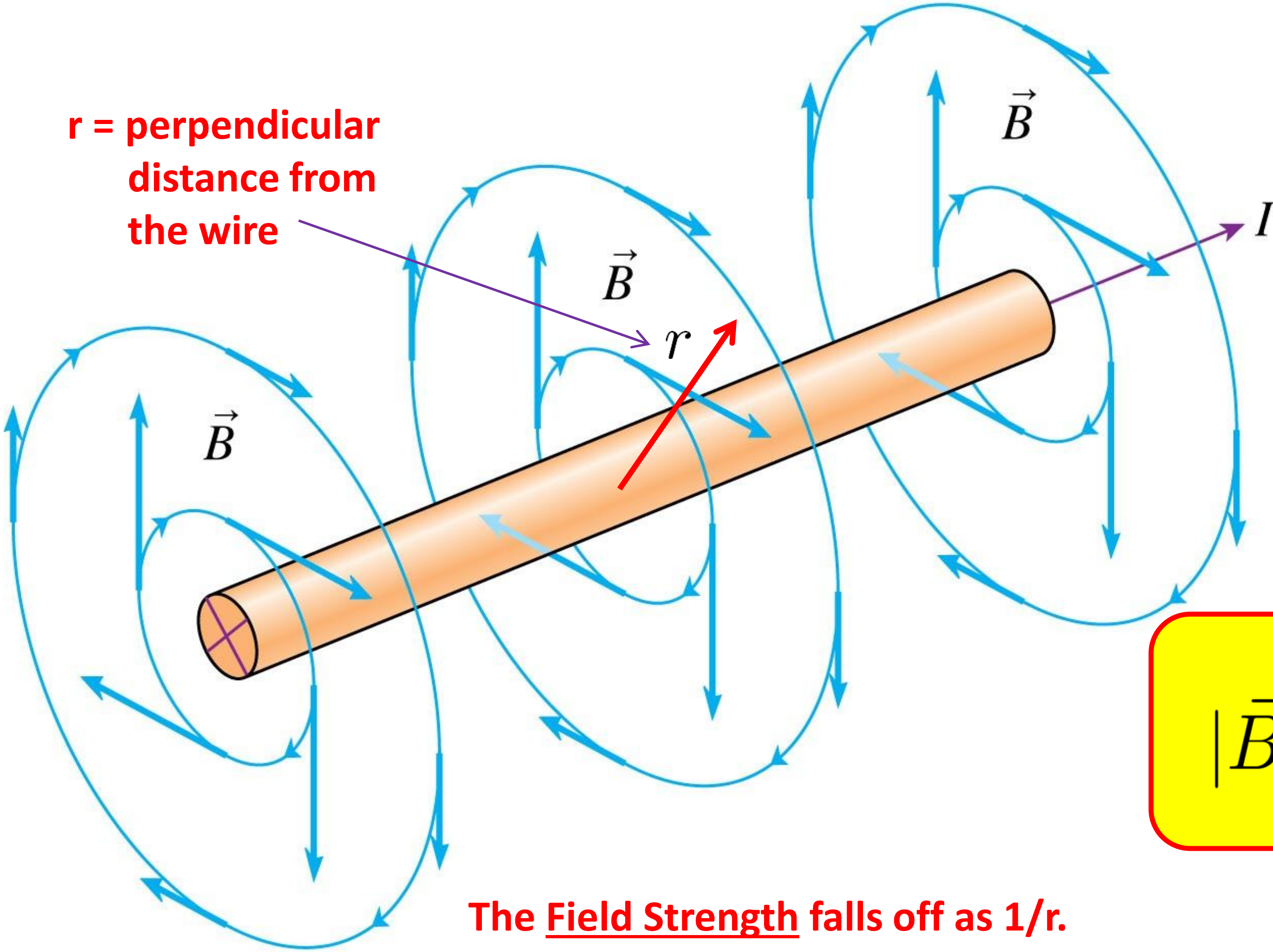
Answer:

$$|\vec{B}| = \frac{\mu_0 I y}{4\pi} \left[\frac{x}{y^2\sqrt{x^2+y^2}} \right]_{-L/2}^{+L/2} = \frac{\mu_0 I}{4\pi y} \left[\frac{L/2}{\sqrt{L^2/4+y^2}} - \frac{-L/2}{\sqrt{L^2/4+y^2}} \right]$$

$$= \frac{\mu_0 I}{4\pi y} \frac{L}{\sqrt{L^2/4+y^2}}$$

(what is this for L >> y?)

The Field for an Infinite Straight Wire: Results



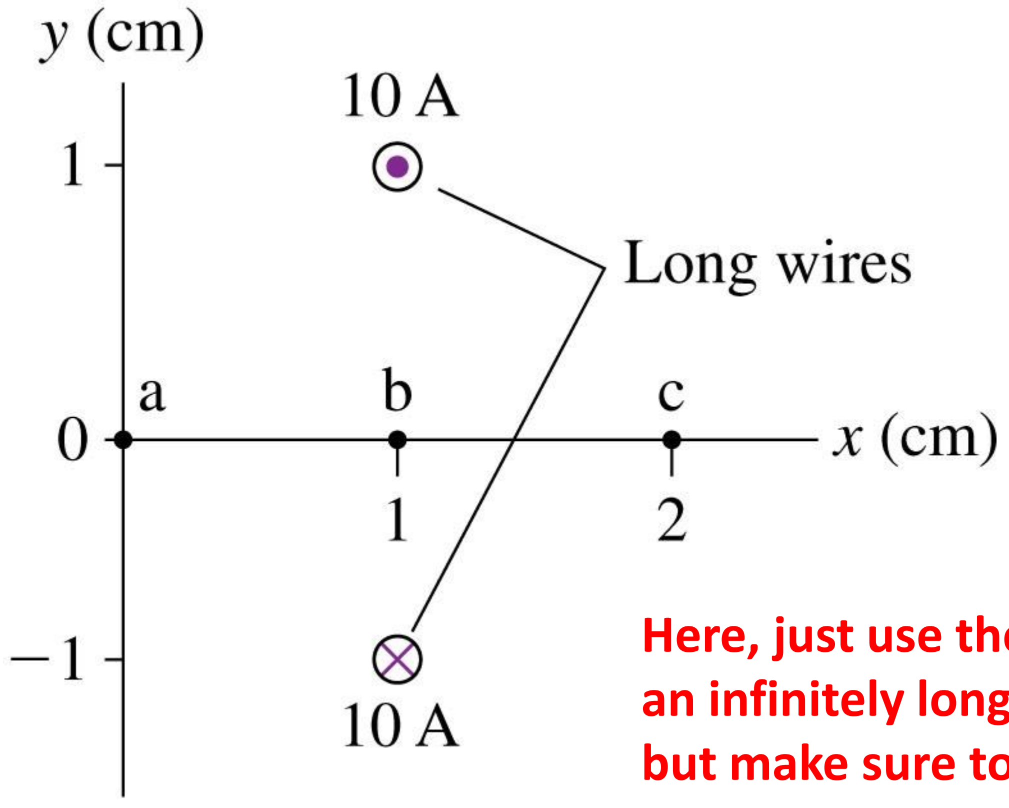
$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

The Field Strength falls off as 1/r.
Direction is from the Right Hand Rule: thumb in direction of the current, fingers wrap in the direction of the field.

(This is an equation that we will use many many times.)

Whiteboard Problem: 29-4

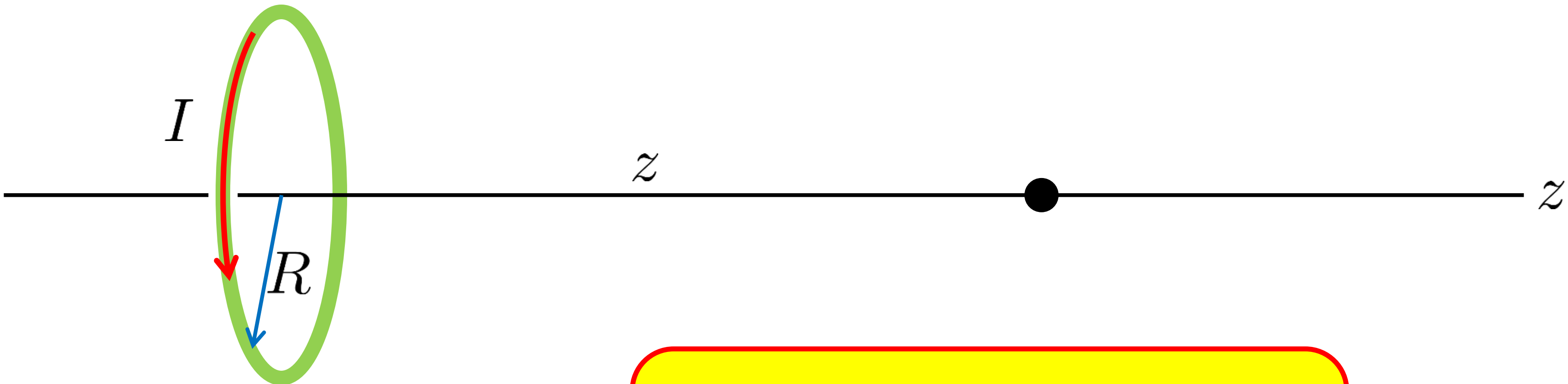
What is the magnetic field at point a in the figure? Find your answer in component form, and enter the field magnitude in LC.



Here, just use the equation for an infinitely long straight current, but make sure to treat each field as a vector.

The Magnetic Field of a Current Loop

Your author uses the Biot-Savart Law to find the magnetic field **on the axis of a current loop**:



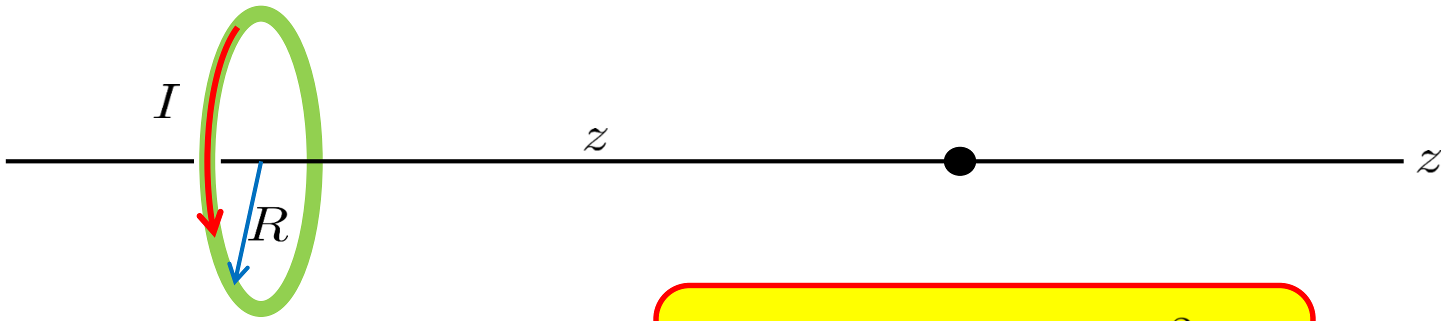
On the Axis:

$$B(z) = \frac{\mu_0}{2} \frac{I R^2}{(z^2 + R^2)^{3/2}}$$

Direction is by the RHR: fingers wrap around in direction of current, thumb points in direction of the field.

Whiteboard Problem: 29-5

At what distance on the axis of a current loop is the magnetic field half the strength of the field at the center of the loop? Give your answer as a multiple of R, i.e. some number times R. (LC)



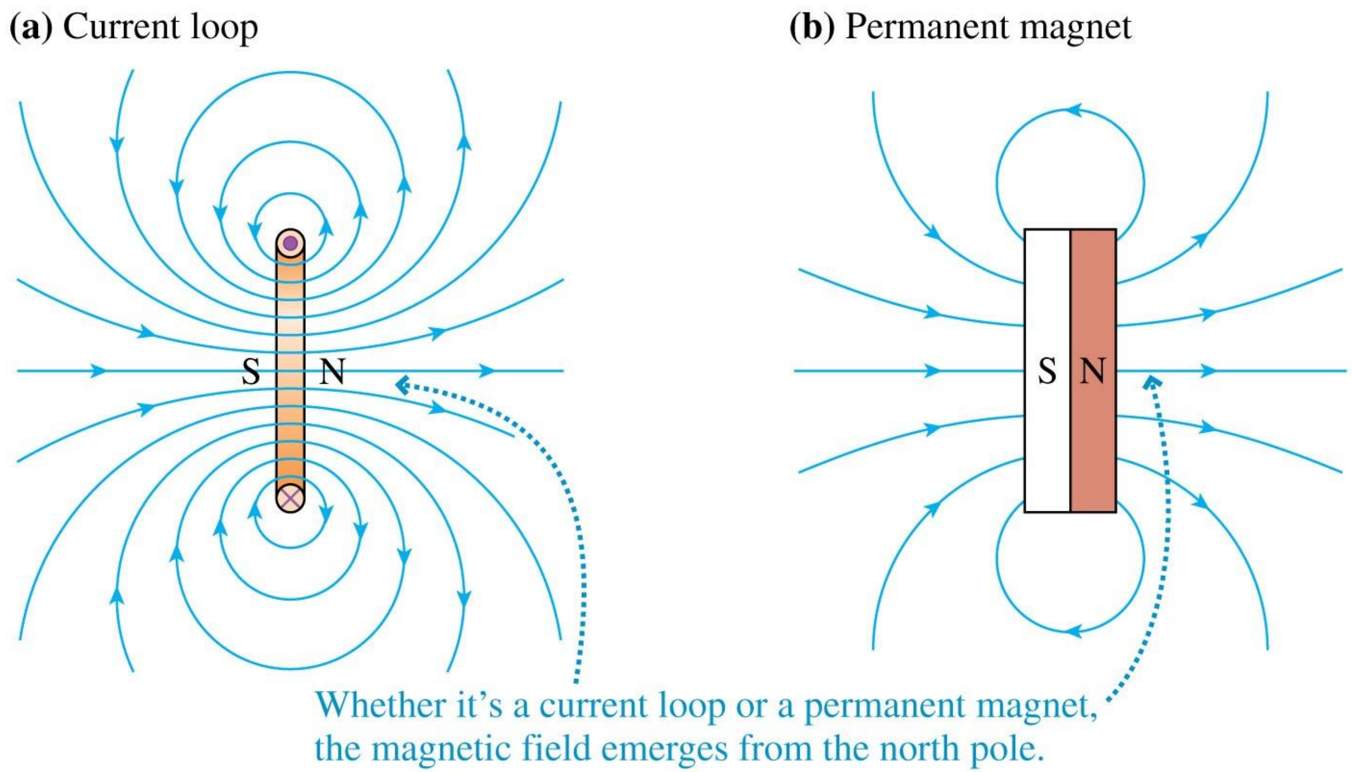
On the Axis:

$$B(z) = \frac{\mu_0}{2} \frac{I R^2}{(z^2 + R^2)^{3/2}}$$

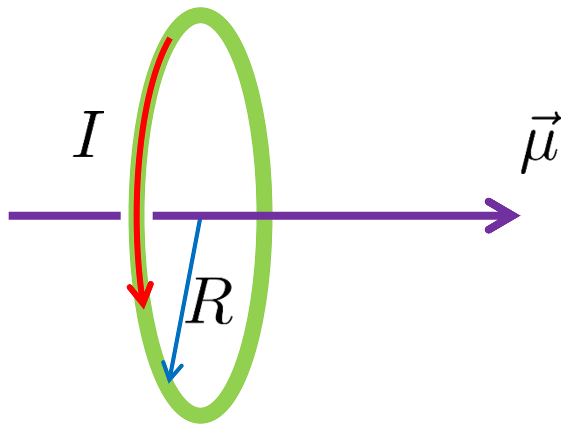
Direction is by the RHR: fingers wrap around in direction of current, thumb points in direction of the field.

The Dipole Field

As you will show in a lab activity, **both a bar magnet and a current loop have a dipole field:**



Dipole Moment of a Current Loop



Using the equation for the field on the axis of a loop **for z >> R**:

$$\vec{B}_{\text{dipole}} \approx \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$$

(watch the different mu's!)

Where the **magnetic dipole moment** is defined by:

$$\vec{\mu} = (AI, \text{South pole to North pole (RHR)})$$

Where: A = area of the loop; I = current

Important Point: A magnetic dipole moment behaves in an external magnetic field just like an electric dipole moment in an external electric field.

Whiteboard Problem: 29-6

The Earth's magnetic dipole moment is 8.0×10^{22} A m^2.

a) **What is the magnetic field strength on the surface of the Earth at the Earth's north magnetic pole?** You can assume that the current loop is deep inside the Earth. **(LC)**
Look up a value for the magnetic field strength at the surface of the Earth; is your result close?

b) Astronauts discover an Earth sized planet without a magnetic field. To create one with the same strength as the Earth's, they propose running a current through a wire around the equator. **What size current would be needed? (LC)**