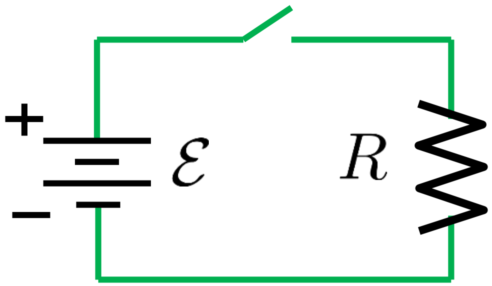
28-3: Resistor Capacitor (RC) Circuits

So far, we know how to handle resistors in a circuit:

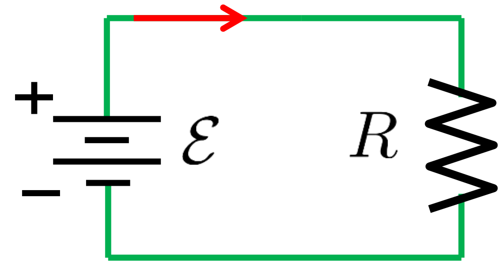


What happens when the switch is closed?

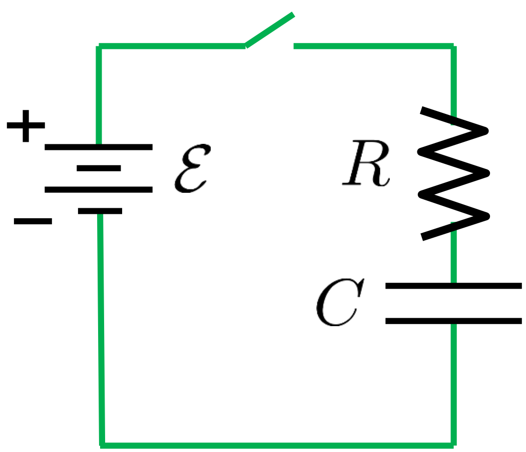
A **steady current** flows in the circuit.

(steady means constant in time)

$$I = \frac{\mathcal{E}}{R}$$



What happens if we also put a capacitor in the circuit with the resistor?



Now, close the switch; what happens?

Current can flow through the resistor, but it can't flow through the capacitor.

Before we do the analysis, let's take a look at a PhET demo of what happens.

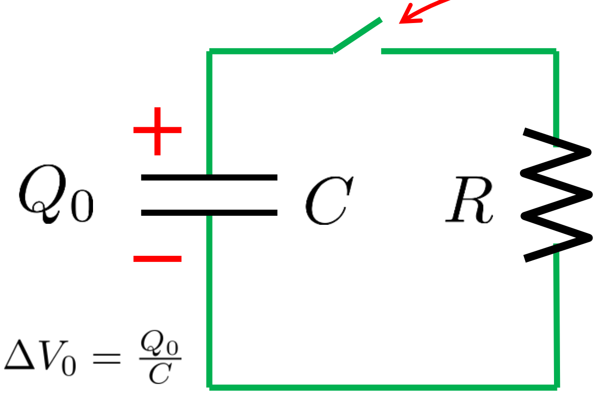
(You'll play with this PhET in HW.)

PhET

RC Circuits: The Details

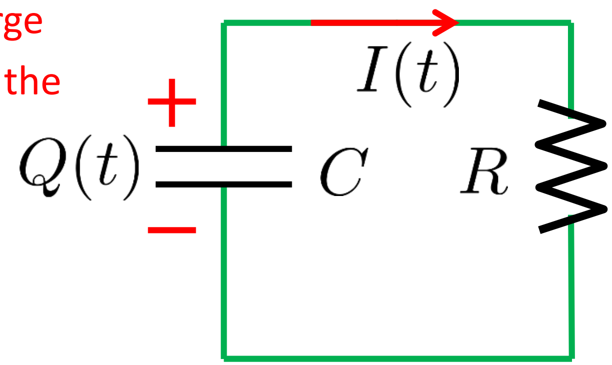
As you saw in the PhET demo, when you put a capacitor in a circuit, current can flow, but it is **time dependent**.

The Discharging Capacitor: Consider a fully charged capacitor:



When the switch is closed, charge from the **+** plate flows through the resistor to the **−** plate.

(*Really it's the* **−** *electrons flowing the other way*.)



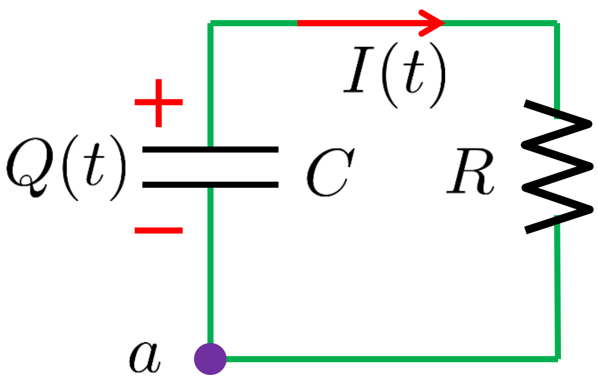
This is a time dependent current that will flow until the charge on the plates is zero.

Can we find the charge on the capacitor and the current in the circuit as functions of time, i.e. Q(t) and I(t)?

How in the world would we do this?

Write a Loop Equation!

The Discharging Capacitor



Loop from a  $\Delta V_c - IR = 0$

Now for a capacitor: $\Delta V_c = \frac{Q}{C}$

As charge on the capacitor decreases, the current increases: $I = -\dfrac{dQ}{dt}$

So the loop equation is: $\frac{Q}{C} + R\frac{dQ}{dt} = 0$ **Or** $\frac{dQ}{dt} + \frac{Q}{RC} = 0$

This is a differential equation for the charge, Q(t), on the capacitor.

Rearrange: $\dfrac{dQ}{Q} = -\dfrac{1}{RC}dt$

Integrate both sides:

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt \;\Rightarrow\; \ln(Q)\Big|_{Q_0}^{Q} = -\frac{t}{RC}\Bigg|_0^t$$

So:

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC} \;\Rightarrow\; Q(t) = Q_0 e^{-\frac{t}{RC}}$$

The Discharging Capacitor: The Results

Define the Time Constant: $\tau \equiv RC$ (Units = seconds)

So, the charge on a discharging capacitor is:

$$Q(t) = Q_0 e^{-t/\tau}$$

Note: *the* ***e*** *in this equation is not the electronic charge; it's the base of the natural logarithm!*

We can easily **get the current by differentiating the charge:**

$$I = -\frac{dQ}{dt} = -\frac{d}{dt} Q_0 e^{-t/\tau} = -Q_0 (-\tfrac{1}{\tau}) e^{-t/\tau} = \frac{Q_0}{RC} e^{-t/\tau}$$

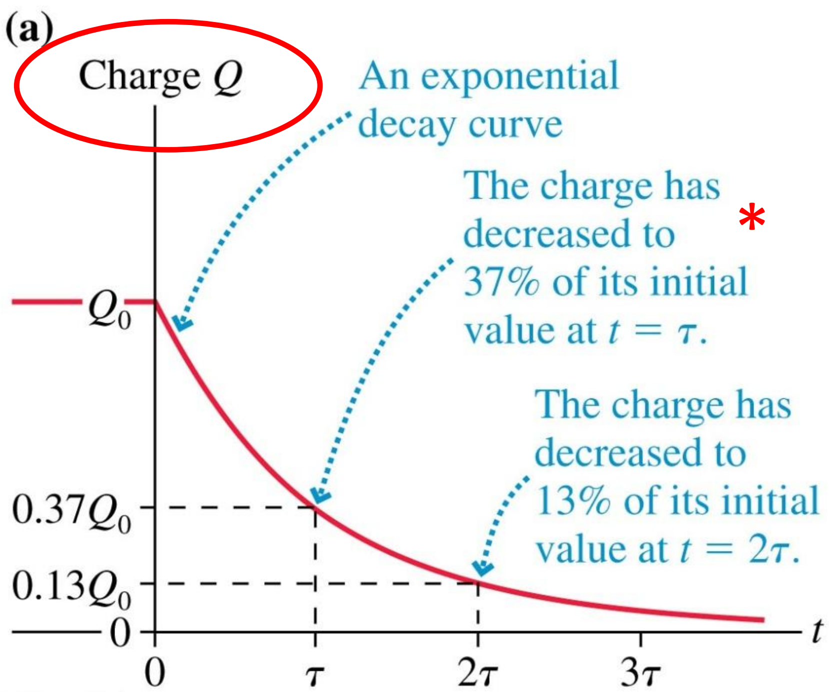
Now, $\frac{Q_0}{C} = \Delta V_0$ and $\frac{\Delta V_0}{R} = I_0$, the current at $t = 0$

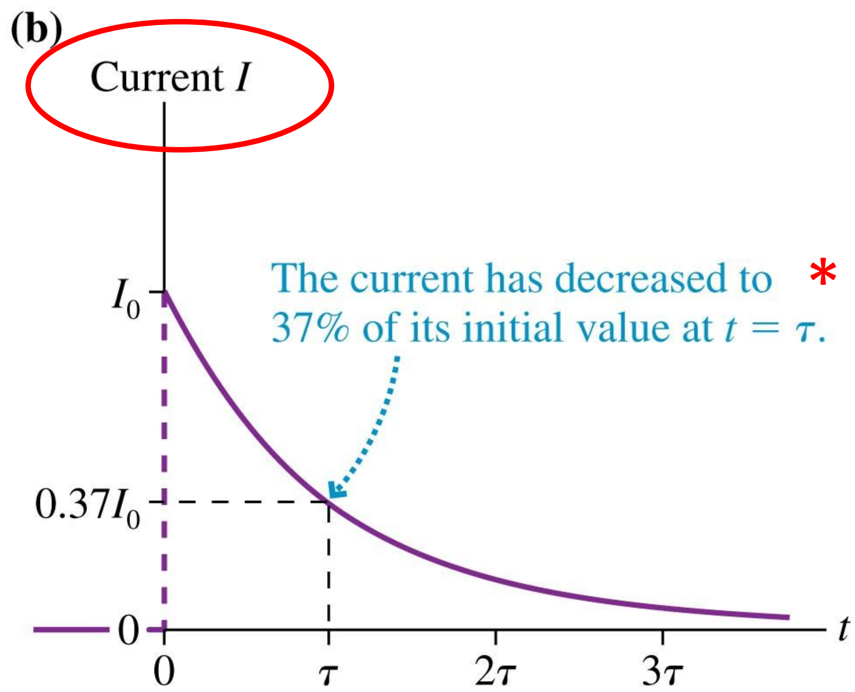
So, the current as a function of time for a discharging capacitor is:

$$I(t) = \frac{\Delta V_0}{R} e^{-t/\tau} = I_0 e^{-t/\tau}$$

The Discharging Capacitor: The Results

So both the charge on the capacitor and the current decay exponentially with time:





$$Q(t) = Q_0 e^{-t/\tau}$$

$$I(t) = I_0 e^{-t/\tau}$$

* Note: $e^{-1} \approx 37\%$

Whiteboard Problem: 28-9

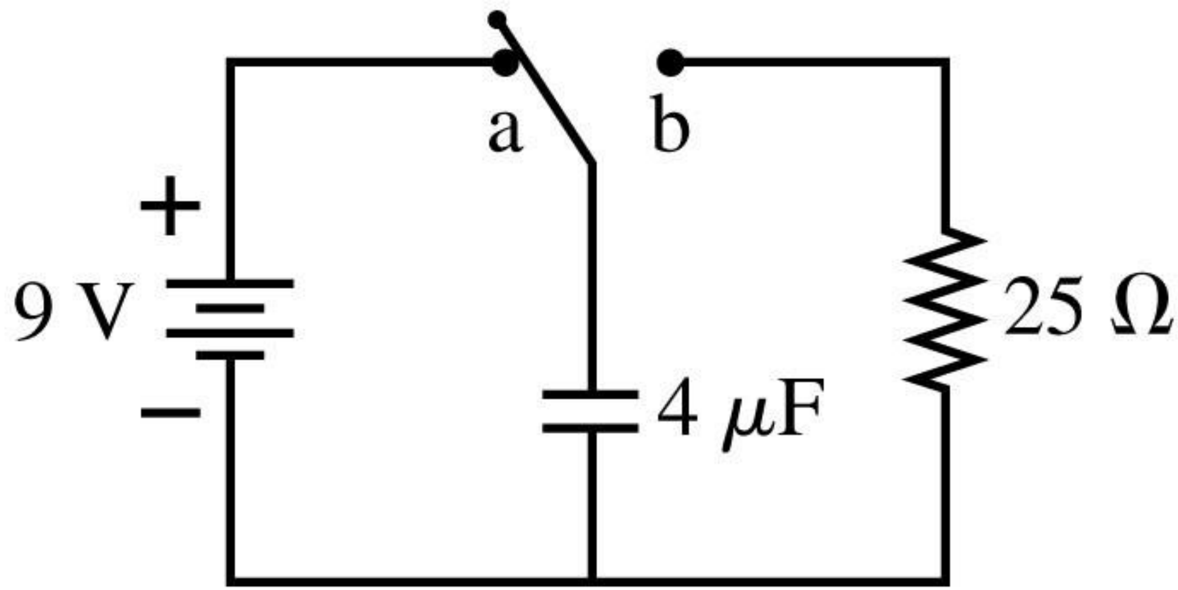
The switch in the figure has been in position **a** for a long time. This means that the capacitor is initially fully charged.

The switch is changed to position b at t = 0 s.

What are the charge Q and current I (LC) through the resistor

a) **Immediately after the switch is closed? (LC)**

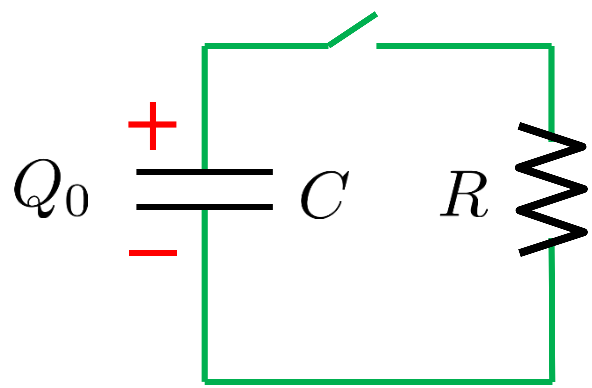
b) **At t = 50 micro-seconds? (LC)**



Whiteboard Problem: 28-10

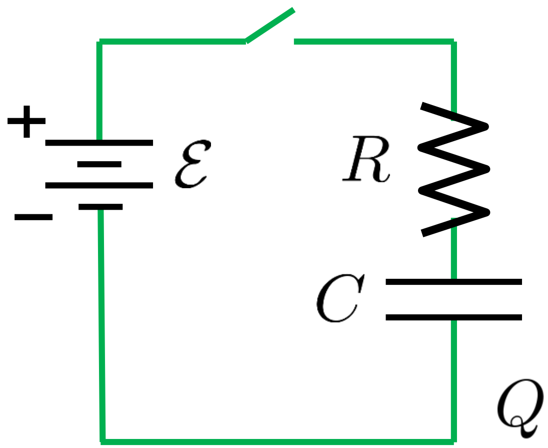
For the RC circuit shown below,

Find the resistance, R, of the resistor that will discharge a 1.0 micro-Farad capacitor to 10% of its initial charge in a time of 2.0 milliseconds? (LC)



The Charging Capacitor

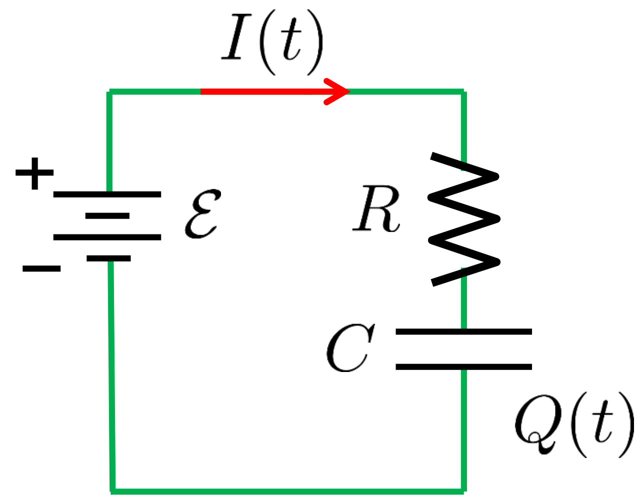
Suppose we have an uncharged capacitor in a circuit with a battery and a resistor:



$$Q(0) = 0$$

When the switch is closed, current flows, and charge collects on the capacitor until:

$$\Delta V_c = \mathcal{E}$$



You find Q(t) and I(t) for the charging capacitor the same way as for the discharging case: **write a loop equation**, here use:

$$I = +\frac{dQ}{dt}$$

and solve the differential equation for Q(t).

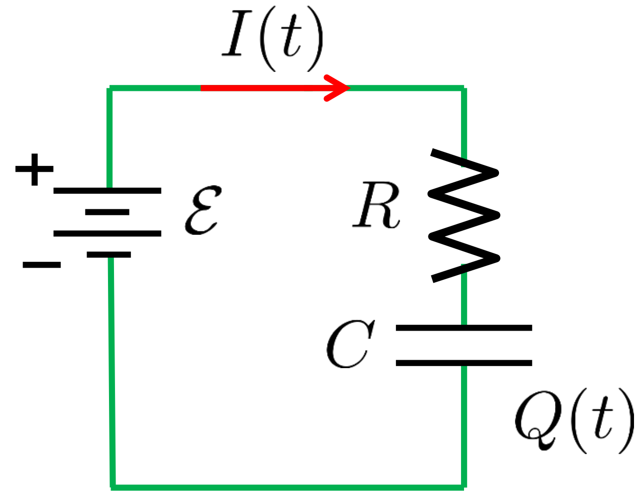
(this one is a bit more difficult than the discharging case; we'll go right to the solution)

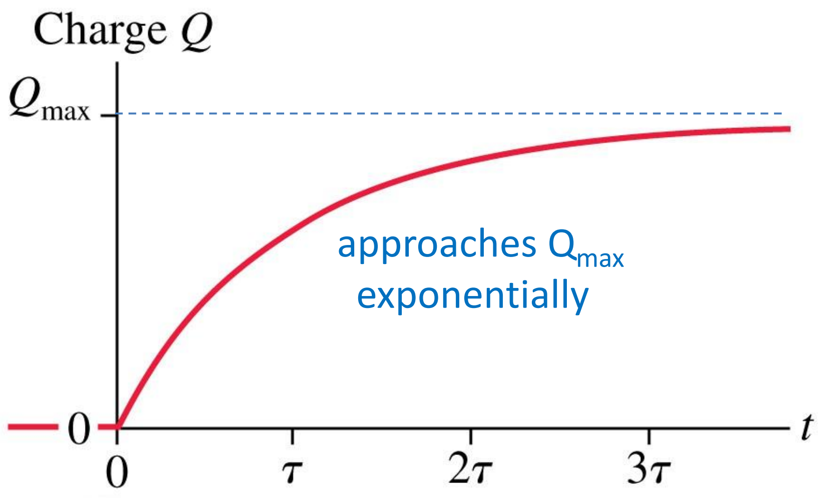
The Charging Capacitor: Results

The charge on the capacitor is:

$$Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$$
$$= Q_{\max}(1 - e^{-t/\tau})$$

where: $\tau = RC$





The potential across the capacitor is:

$$\Delta V_c = \frac{Q(t)}{C} = \mathcal{E}(1 - e^{-t/\tau})$$

And the current is:

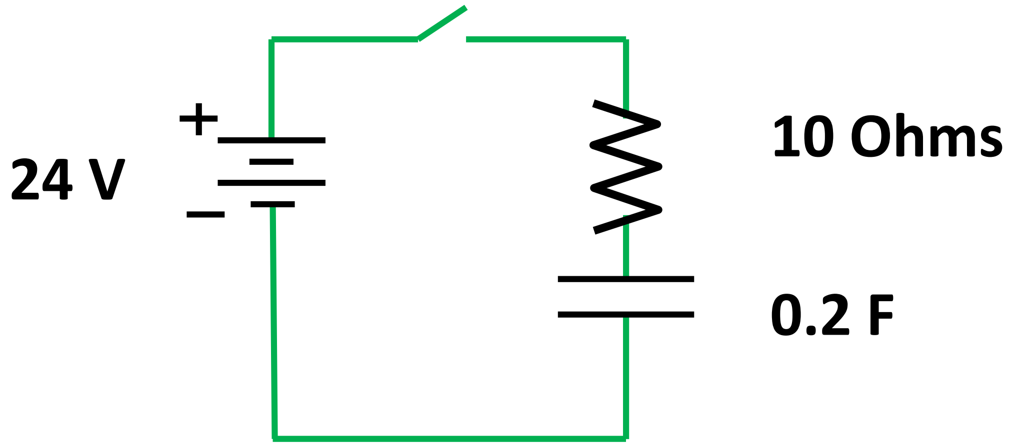
$$I(t) = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

Exponential decay

Whiteboard Problem: 28-11

For the RC circuit shown below, the charge on the capacitor is zero when the switch is closed.

Find the time for the capacitor to charge to one half its maximum charge. (LC)



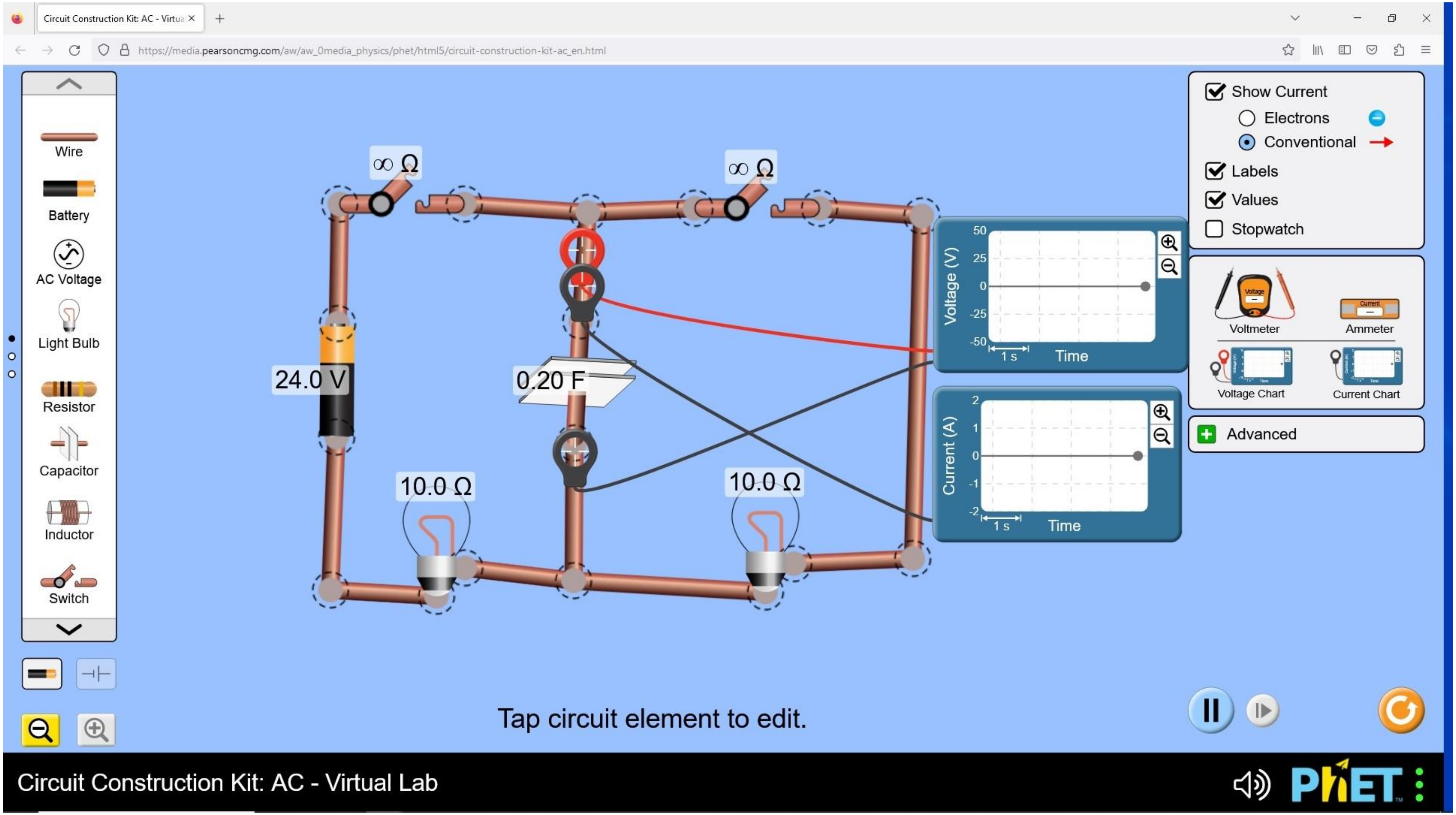
Equations from the previous slide

$$Q(t) = C\mathcal{E}(1 - e^{-t/\tau}) = Q_{\max}(1 - e^{-t/\tau}) \quad \text{where: } \tau = RC$$

PhET

Circuit for charging and discharging on the PhET Demo

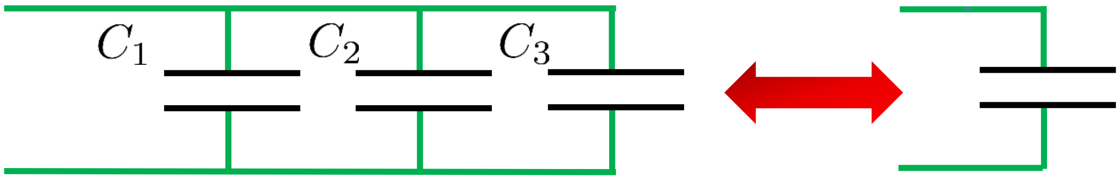
If you want to play with this charging and discharging RC circuit shown here; go to https://phet.colorado.edu/en/simulations/circuit-construction-kit-ac-virtual-lab and build it as shown below.



Combinations of Capacitors (from Chapter 26)

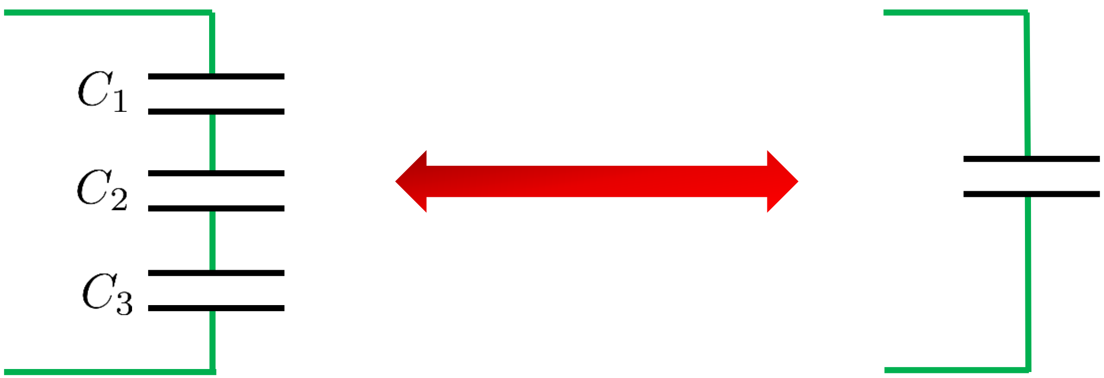
As we have for resistors, sometimes there is more than one capacitor in a circuit, and some combinations can be combined into a single capacitor:

Capacitors in Parallel (**parallel means the same potential**), e.g. for three capacitors:



$$C = C_1 + C_2 + C_3$$

Capacitors in Series (**series means the same charge**), e.g. for three capacitors:



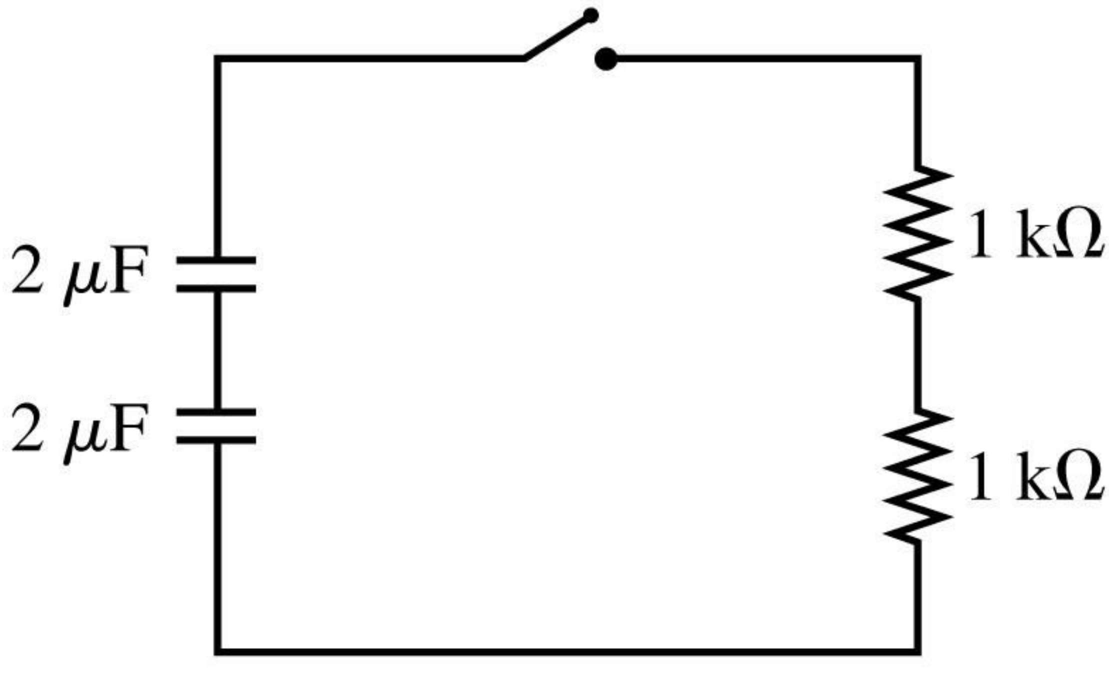
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Caution: these are the opposite of the resistor combinations. For capacitors, parallel is the direct sum, and series is the reciprocal sum.

Also note: we illustrated this for three capacitors, but the formulae hold for any number of capacitors.

Whiteboard Problem 28-12

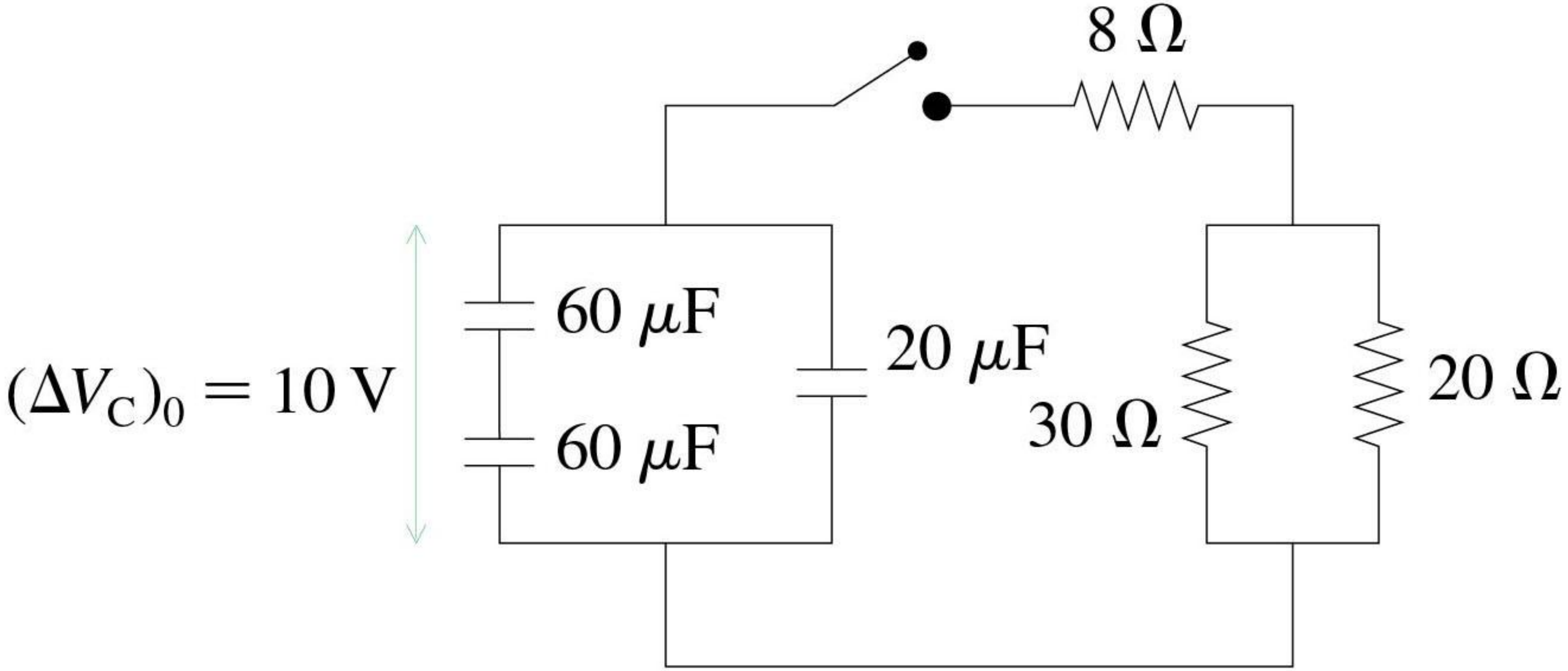
What is the time constant for the discharge of the capacitors in the figure below? (LC)



Hint: *Combine* and Conquer!

Whiteboard Problem 28-13

The capacitors shown in the circuit below are fully charged.
The switch is closed at t = 0.
At what time will the current in the 8 Ohm resistor be half of the value it had immediately after the switch was closed? **(LC)**



Hint: As in the previous problem,
Combine and Conquer!