

# 28-2: The General Resistor Circuit Problem

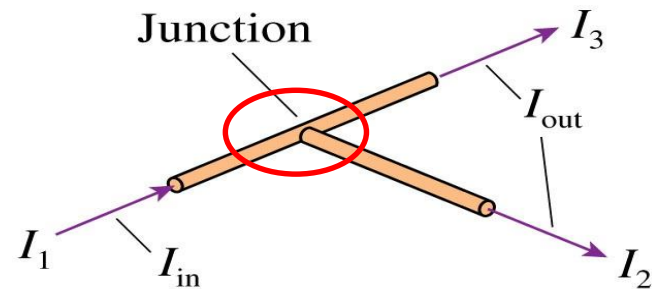
We're now ready to attack the **General Resistor Circuit Problem** that may have many sources of EMF, many resistors, and many current loops.

Remember, the basic laws that we'll need are the Kirchoff Laws:

Kirchoff Junction Law:

For Any Junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

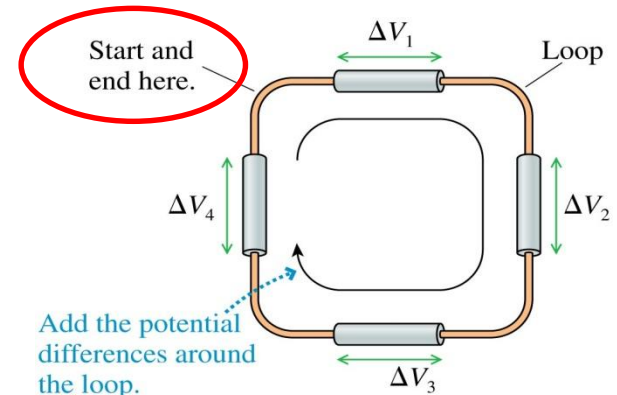


$$\text{Junction law: } I_1 = I_2 + I_3$$

Kirchoff Loop Law:

For Any Closed Circuit Loop:

$$\sum_{\text{around loop}} (\text{changes in potential}) = 0$$



$$\text{Loop law: } \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

We also have the rules for combining resistors in series and parallel – should we want to do that in a given problem.

From Your Textbook\*

**Resistor circuits**

(I have some issues with this)

**MODEL** Model wires as ideal and, where appropriate, batteries as ideal. **Yes**

**VISUALIZE** Draw a circuit diagram. Label all known and unknown quantities. **Yes**

**SOLVE** Base your mathematical analysis on Kirchhoff's laws and on the rules for series and parallel resistors. **Yes, and Ohm's Law too!**

- Step by step, reduce the circuit to the smallest possible number of equivalent resistors. **Maybe, but it's not necessary, and not always possible. Junction**
- Write Kirchhoff's loop law for each independent loop in the circuit. **Equations ?**
- Determine the current through and the potential difference across the equivalent resistors. **How? Solve the Loop & Junction equations and Ohm's Law.**
- Rebuild the circuit, using the facts that the current is the same through all resistors in series and the potential difference is the same for all parallel resistors.

**Maybe, if you did the first step**

**ASSESS** Use two important checks as you rebuild the circuit.

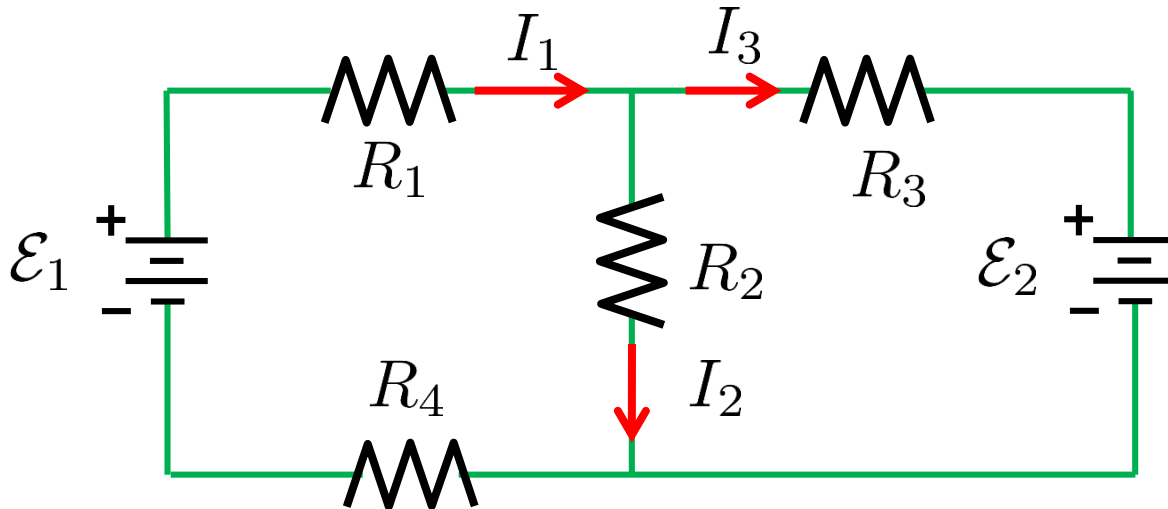
- Verify that the sum of the potential differences across series resistors matches  $\Delta V$  for the equivalent resistor.
- Verify that the sum of the currents through parallel resistors matches  $I$  for the equivalent resistor.

**\*There are lot of problems with these steps; let's do our own.**



# How to Solve Any Resistor Circuit

We will illustrate the steps with a generic 2-loop circuit. **Most of the time for a circuit, we know all of the voltages of the sources of emf and the resistances of all of the resistors, and the problem is to find all of the currents.**



## The Problem

Know: all  $\mathcal{E}$ 's and  $R$ 's

Find: all  $I$ 's

**Step 1:** OPTIONAL: Use parallel and series rule to simplify the circuit.

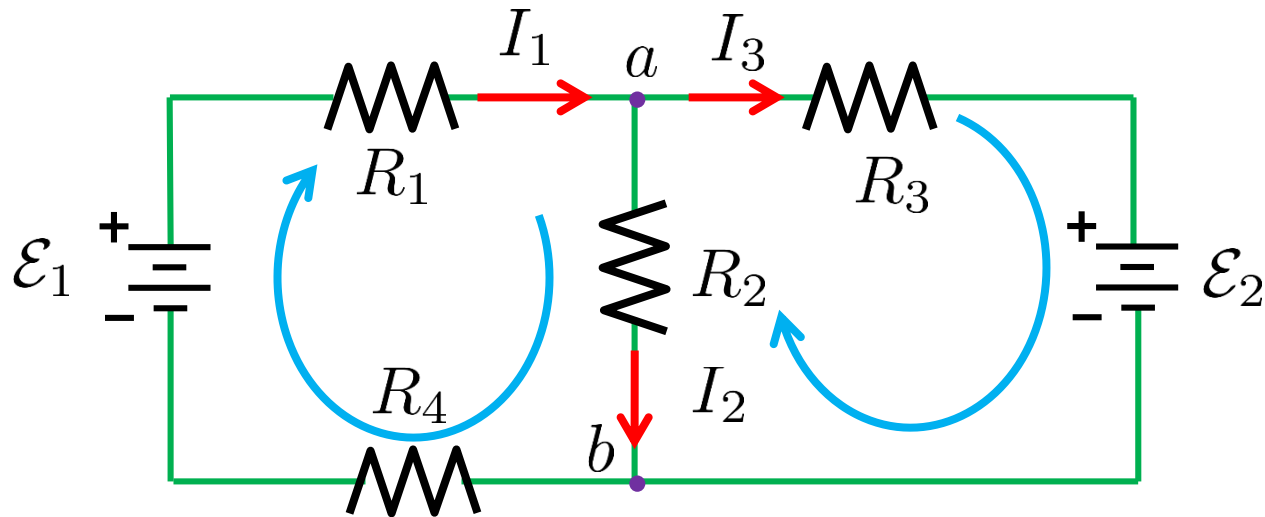
Can this be done with this circuit?

*No, here it wouldn't help.*

**Step 2:** Identify each distinct current\* and assume a direction for each. Draw and label on the diagram. You don't have to know the actual direction of the current; that will come out in the solution.

\*the current only changes at a junction


# How to Solve Any Resistor Circuit




**Step 3: Write a Junction Equation for each independent junction:**

Junction at a:  $I_1 = I_2 + I_3$  (the junction at b gives the same equation, i.e. it's not independent)

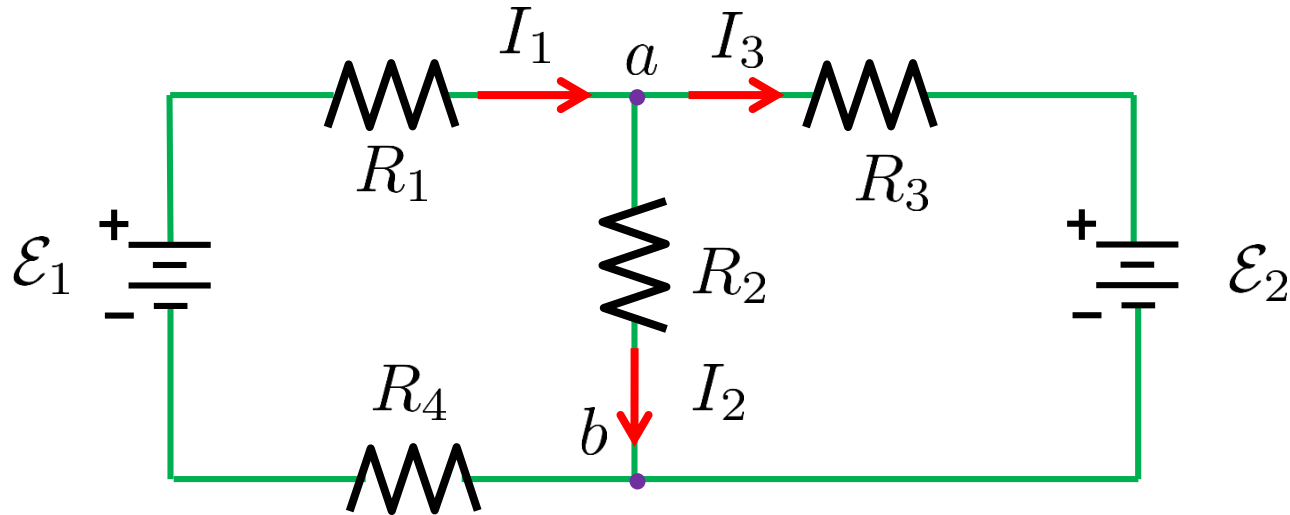
**Step 4: Write a Loop Equation for each independent loop following the sign conventions:**

Left loop from a   $-I_2R_2 - I_1R_4 + \mathcal{E}_1 - I_1R_1 = 0$

Right loop from a   $-I_3R_3 - \mathcal{E}_2 + I_2R_2 = 0$

**What about the outside loop?** It's not independent; the equation you get from it, is the sum of the two equations above.

# How to Solve Any Resistor Circuit



**Step 5:** Check your equations to see that:

**number of unknowns = number of independent equations.**

Our Equations:

$$I_1 = I_2 + I_3$$
$$-I_2 R_2 - I_1 R_4 + \mathcal{E}_1 - I_1 R_1 = 0$$
$$-I_3 R_3 - \mathcal{E}_2 + I_2 R_2 = 0$$

So, three unknown currents, three independent equations.

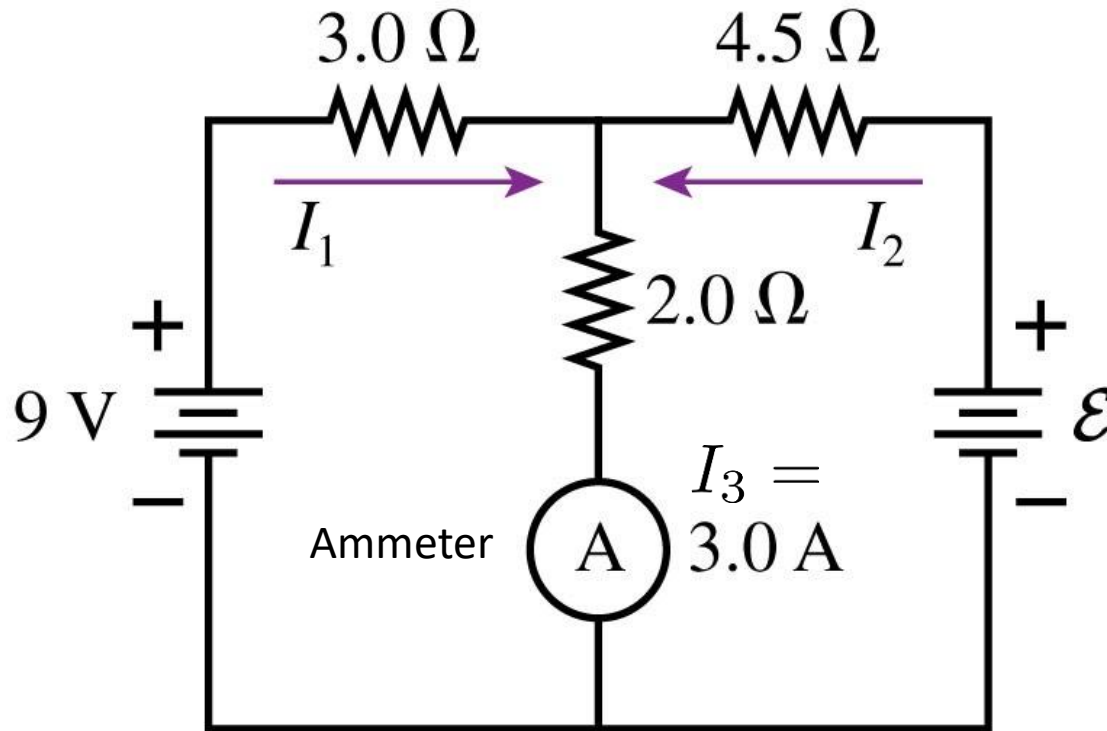
**Step 6:** Solve the set of simultaneous equations for the unknown currents.

(If a current is negative, the magnitude is correct, but the assumed direction is wrong; however, use it as negative in any subsequent calculations.)

**Step 7:** IF YOU DID OPTIONAL STEP 1: rebuild the original circuit using parallel and series rules for potentials and currents.

## Whiteboard Problem: 28-6

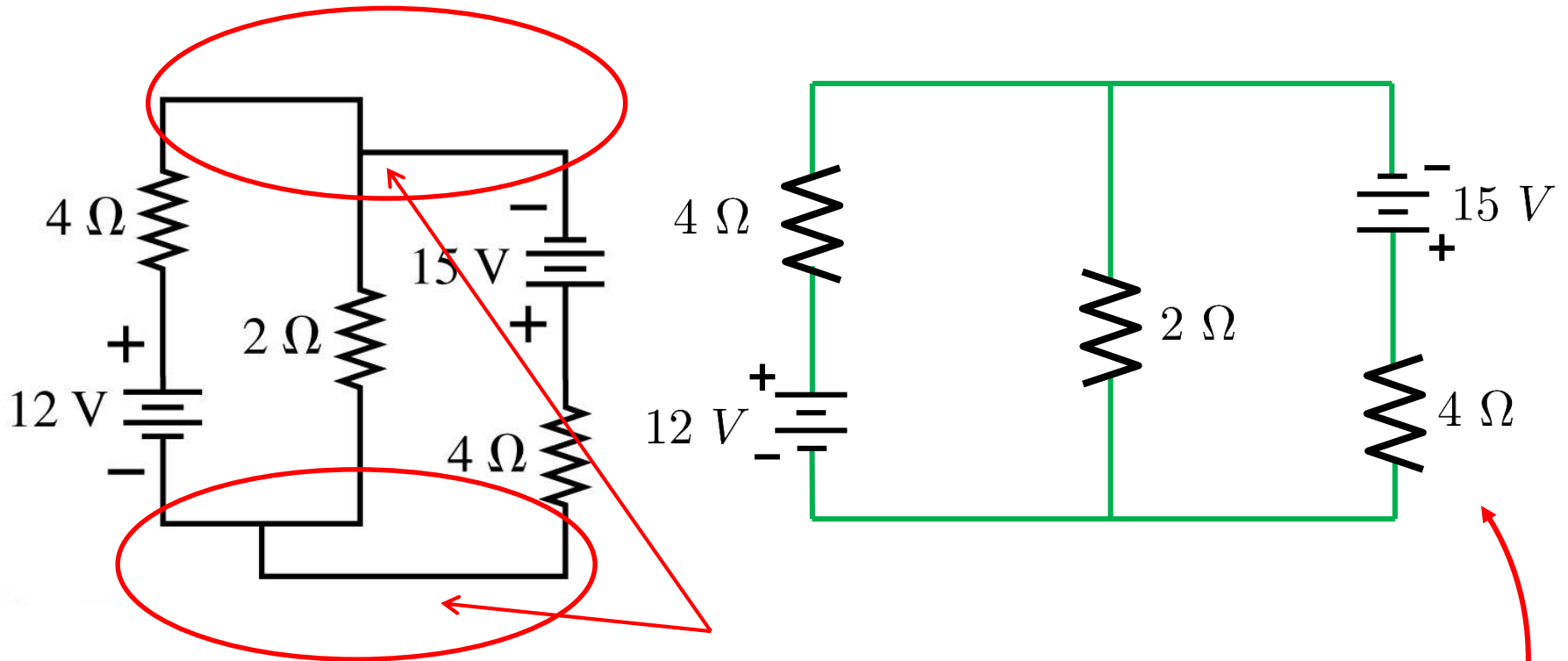
The ammeter in the Figure reads  $3.0\text{ A}$ . Find  $I_1$ ,  $I_2$ , and  $\mathcal{E}$ .  
Enter your value of  $\mathcal{E}$  in LC.



This problem is a little different than a general problem. Here, we know one of the currents, and we're told to assume a particular direction for some. Follow the rest of the steps.

# Example WB Problem: (A full blown circuit problem)

Determine the current through each resistor in the circuit below.

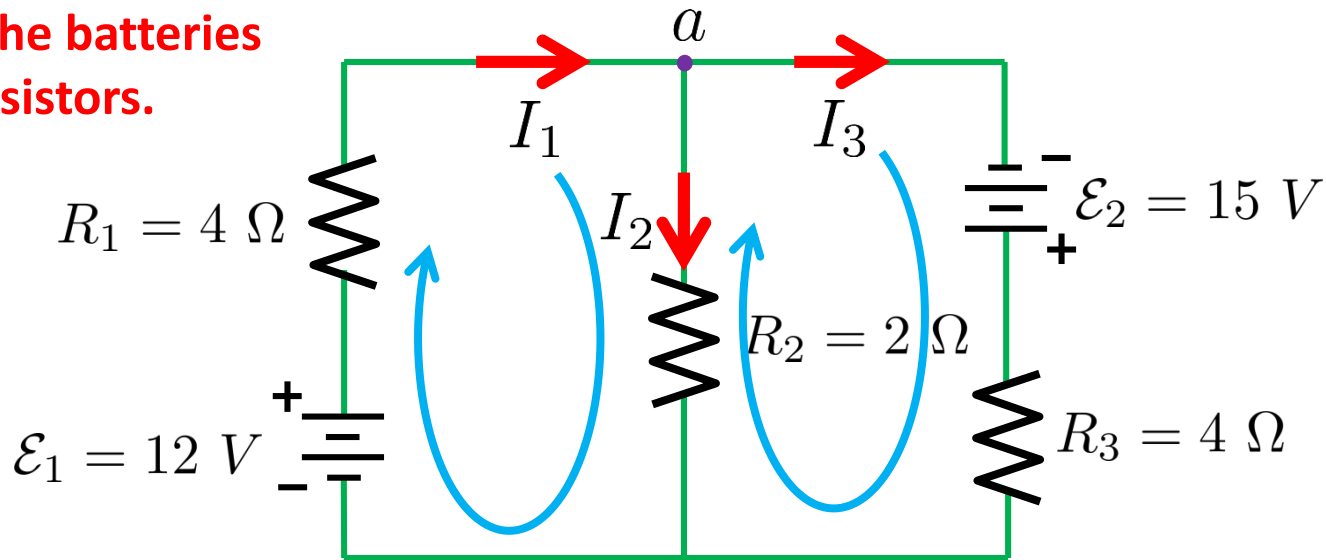


*Whoa! What's this sh . . . I mean stuff?*

All points on the sections of wires circled are at the same potential, so the shapes of the wires don't matter, as long as the junctions are preserved. Hence, we are free to redraw the circuit like this, which is a little easier to analyze.

# WB Example Problem


Name the batteries and resistors.




Choose currents and assume a direction for each.

Write out Junction and Loop Equations:

Junction at a:  $I_1 = I_2 + I_3$  (1)

Left loop from a   $-I_2 R_2 + \mathcal{E}_1 - I_1 R_1 = 0$  (2)

Right loop from a   $\mathcal{E}_2 - I_3 R_3 + I_2 R_2 = 0$  (3)

Three equations and three unknowns – solve.

*(Note, the equation numbers are a good idea)*

# WB Example Problem

$$I_1 = I_2 + I_3 \quad (1)$$

$$-I_2 R_2 + \mathcal{E}_1 - I_1 R_1 = 0 \quad (2)$$

$$\mathcal{E}_2 - I_3 R_3 + I_2 R_2 = 0 \quad (3)$$

**Substitute equation (1) into equation (2):**

$$-I_2 R_2 + \mathcal{E}_1 - (I_2 + I_3) R_1 = 0 \quad (4)$$

**Equations (3) and (4) with the numbers put in:**

$$2I_2 - 4I_3 = -15 \quad (3)$$

$$-6I_2 - 4I_3 = -12 \quad (4)$$

**Take equation (3) – equation (4):**  $8I_2 = -3 \Rightarrow I_2 = -\frac{3}{8} \text{ A} = -0.375 \text{ A}$

(note the negative sign means our assumed direction is wrong, but don't change the sign in the rest of the calculations.)

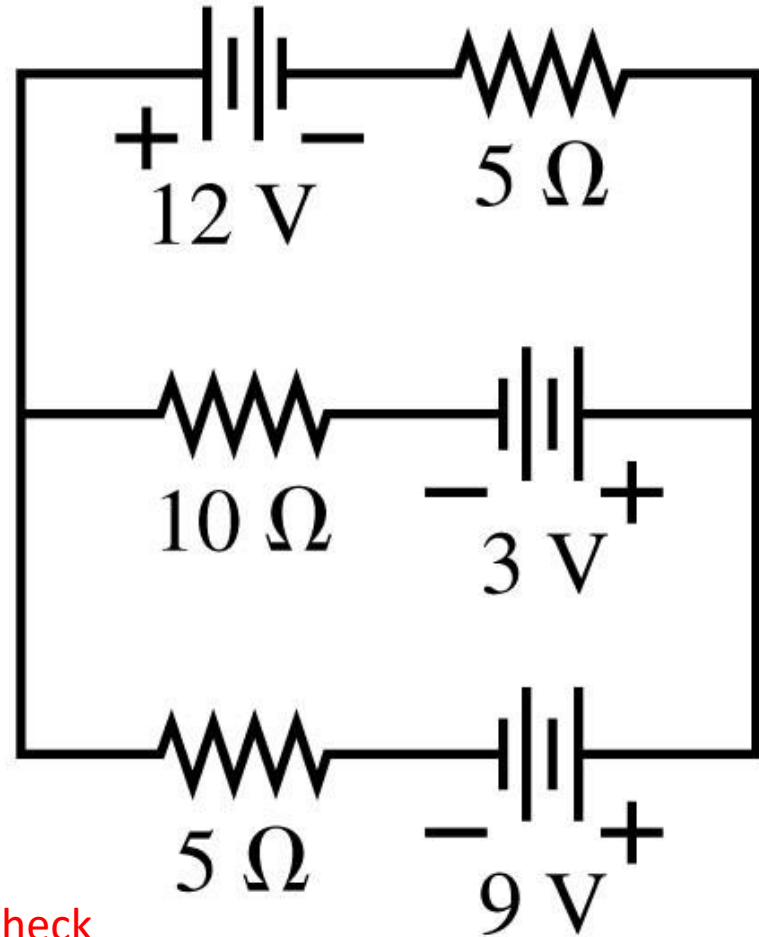
**Equation (3):**  $I_3 = \frac{2I_2 + 15}{4} \Rightarrow I_3 = 3.562 \text{ A}$

**Equation (1):**  $I_1 = I_2 + I_3 \Rightarrow I_1 = 3.187 \text{ A}$

**Here's circuit on PhET:**  
(Feel free to use the [PhET circuit simulation](#) to check your HW calculations.)

## Whiteboard Problem: 28-7

Determine the current through each resistor in this circuit (follow all of the steps as in the previous example).

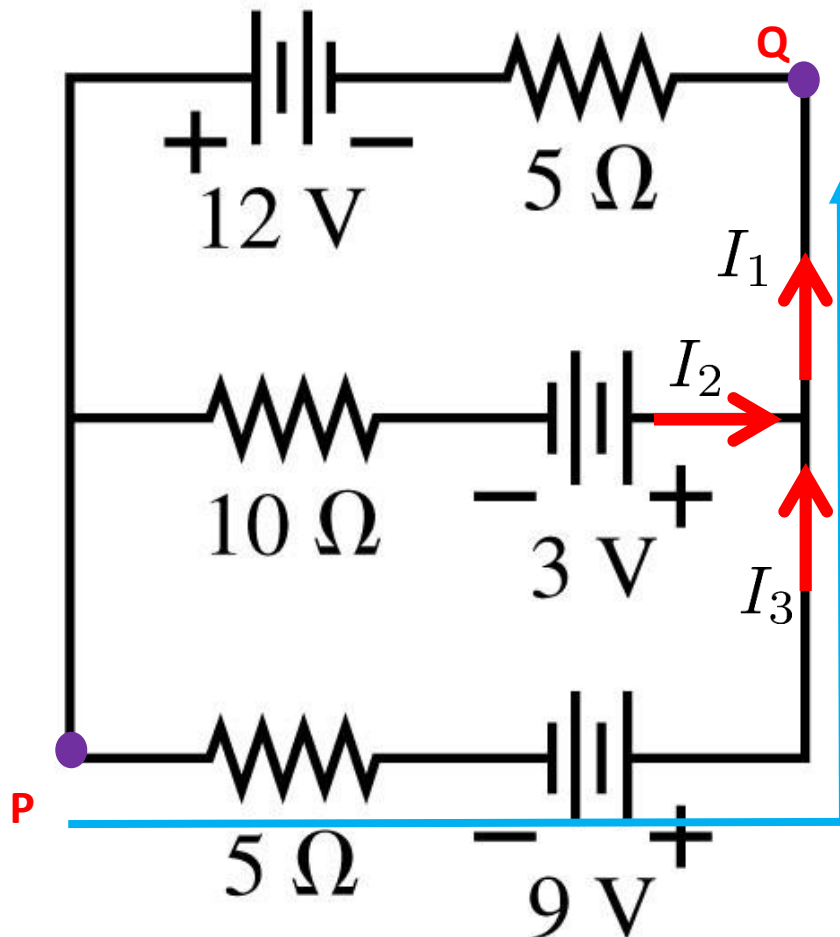


When you have values for the three currents, check your answers on the PhET simulation; click here for [HTML version](#) or use the link sent out in the announcement. Start up the simulation, build the above circuit, and see if your solution is correct. (Make sure to select the conventional current.)

When your two solutions agree, enter the magnitude of the current through the bottom 5 Ohm resistor into LC – a 3-point shot.

# How to find the potential between any two points in a circuit

Once, we know all of the currents in a circuit, we can easily find the potential difference between any two points in the circuit. **Consider the WB problem that we just did:**



$$I_1 = 2.28 \text{ A}, I_2 = 0.36 \text{ A}, I_3 = 1.92 \text{ A}$$

**For example**, if I want to know the potential between points P and Q, write a **partial loop equation** along any path from the point P to point Q:

$$V_P - I_3(5) + 9 = V_Q$$

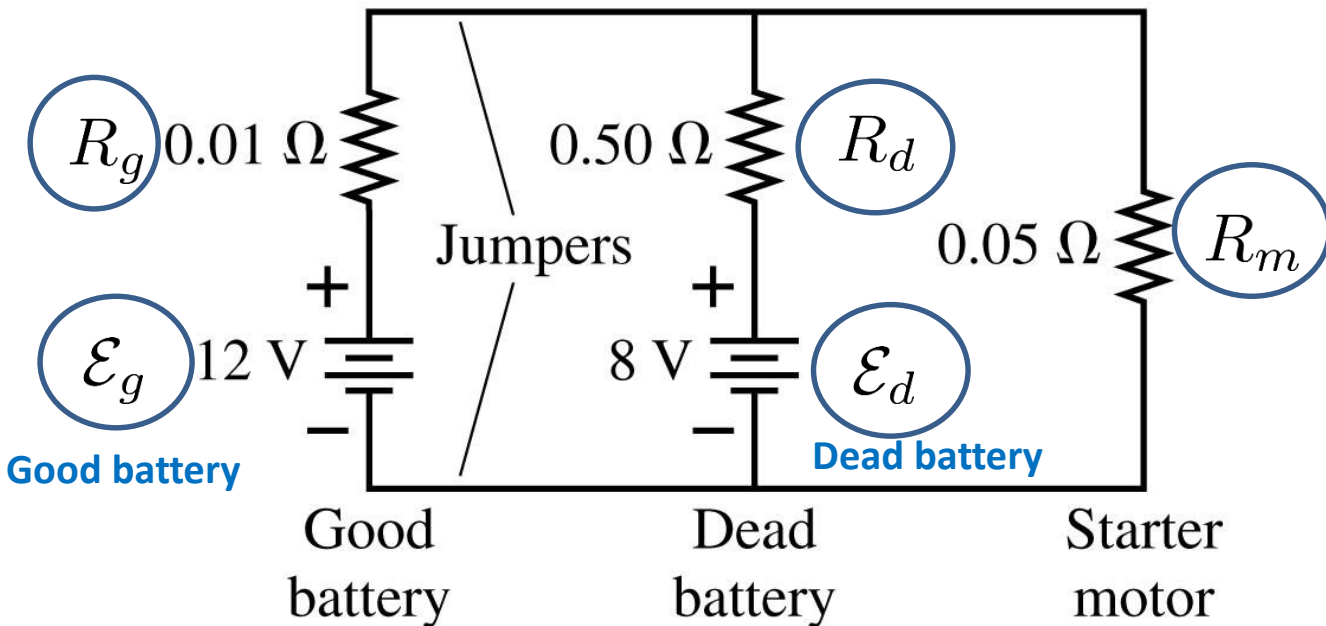
$$\begin{aligned} \text{So: } \Delta V_{PQ} &= V_P - V_Q \\ &= 5I_3 - 9 \\ &= 0.6 \text{ V} \end{aligned}$$

Let's check this with the voltmeter on the **PhET simulation**.

# Whiteboard Problem: 28-8

A 12 V car battery dies not so much because its voltage drops but because chemical reactions increase its internal resistance. A good battery connected with jumper cables can both start the engine and recharge the dead battery. Consider an automotive circuit below. *(Use the variable names circled below.)*

- How much current could the good battery alone drive through the starter motor? (LC)
- How much current is the dead battery alone able to drive through the starter motor? (LC)
- With the jumper cables attached, how much current passes through the starter motor, and how much through the dead battery and in which direction? (Enter the magnitude of the current though the starter motor into LC – a 3-point shot)



The HTML PhET simulation won't work for this circuit – resistors don't go low enough. Here's the circuit in the old PhET Java version; it works, but not perfectly.