27: Electric Current

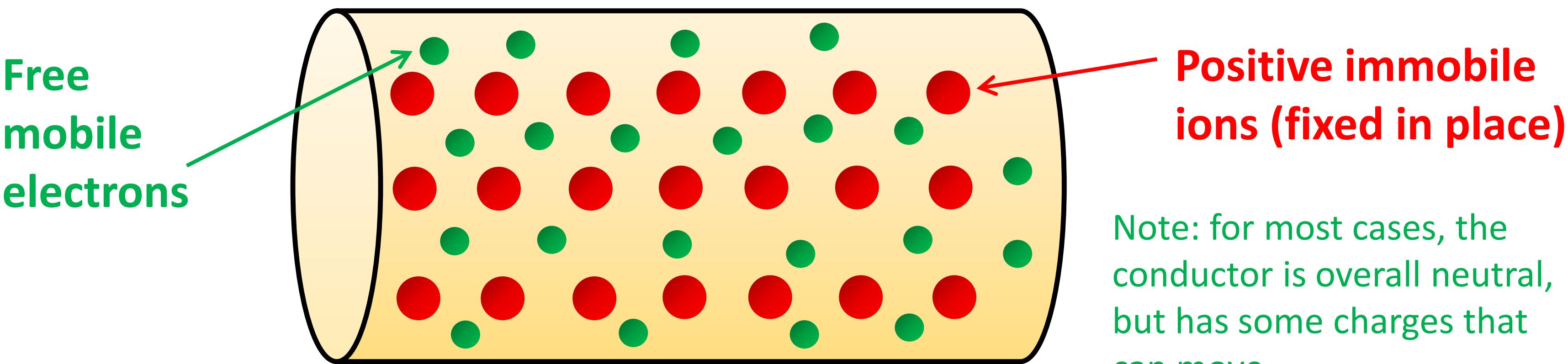
In chapter 24, we saw that for a **conductor in equilibrium**, any free excess charges move to the outside surfaces and the electric field inside the conductor is zero.

We now want to **lift this equilibrium condition** and examine what happens to the free charges (excess or not) when an **external electric field is applied**.

The brief answer is that the free charges move in response to an external field, and we call this motion of charge: **Electric Current**.

Metallic Conductors:

There are many types of conductors, some with positive mobile charges, some with negative, and some with both (e.g. a plasma). **The model of a conductor that we will concentrate on is a metal. When atoms of a metal come together to form a solid, each atom gives up one or more electrons that are free – these are the conducting charges:**

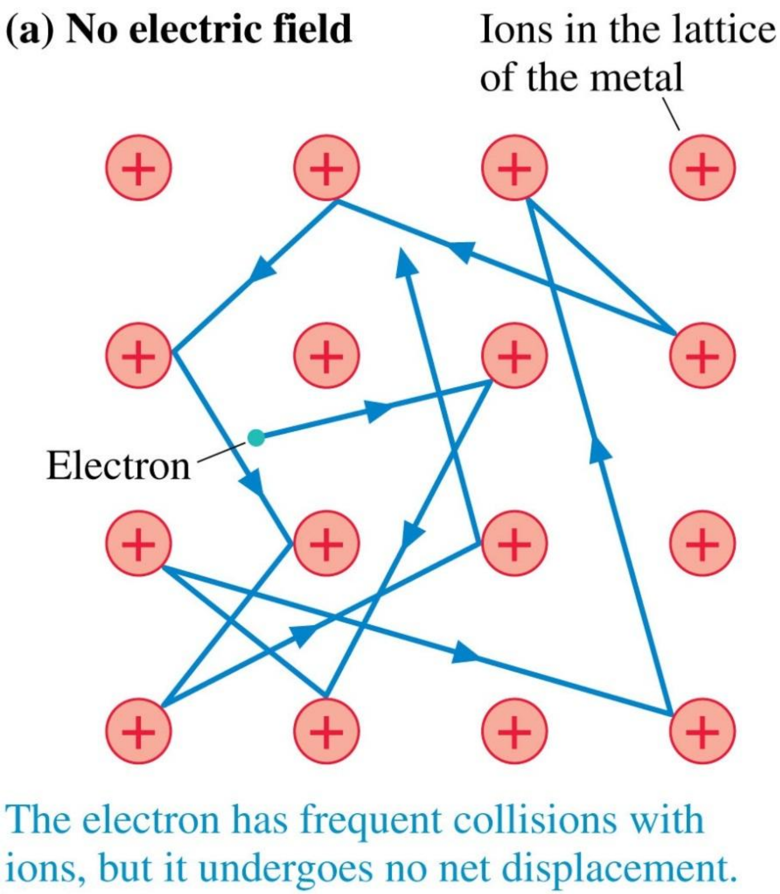


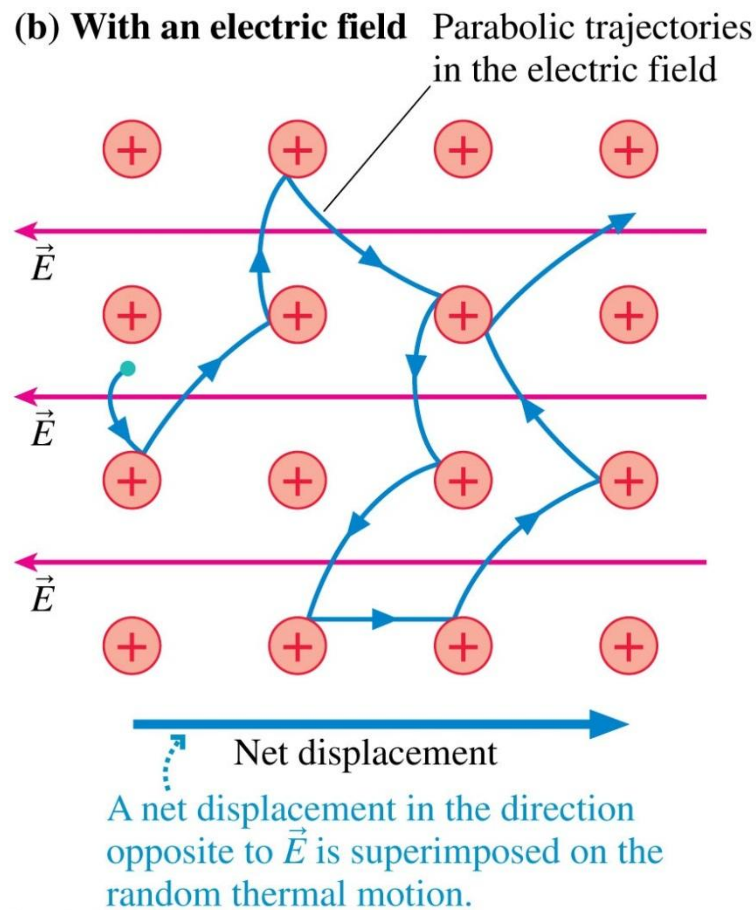
Note: for most cases, the conductor is overall neutral, but has some charges that can move.

Electric Current

If there is No External Field, the free electrons have random thermal motion; very much like molecules in a gas

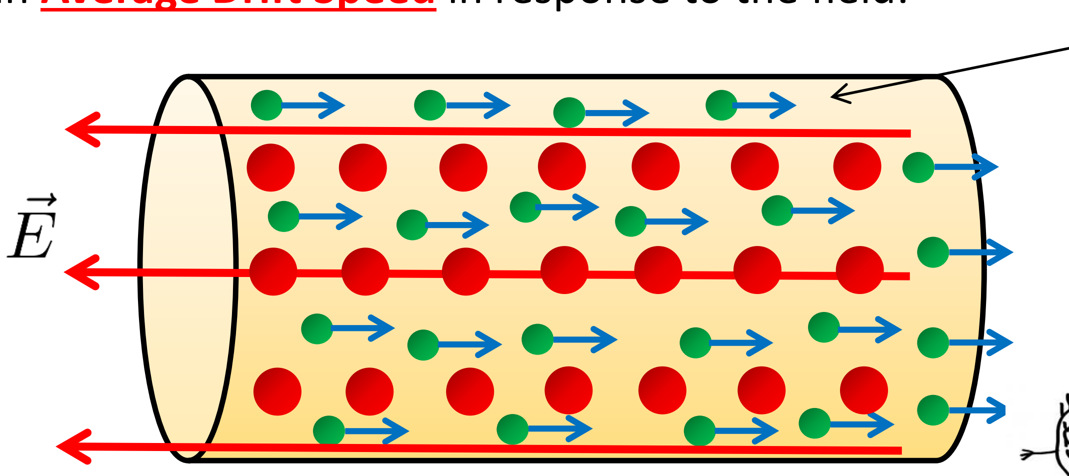
With an External Field, the electrons still have random thermal motion, but are accelerated in the opposite direction of the field in between collisions with ions.



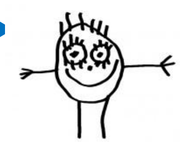


The Electron Current

Each free electron in a metal is accelerated in the opposite direction of the external field; however, interactions with the positive ions slow them down until they reach an **Average Drift Speed** in response to the field:



Remember, this drifting motion is superimposed on a much faster random thermal motion which is not shown in this figure.



Us . . a self portrait

If we just count the number of electrons passing through a cross section of the wire during some time, we have:

$$\text{Electron Current: } \boxed{i_e = n_e A v_d} \qquad \text{Units } [s^{-1}]$$

Where: $n_e =$ number density of free electrons

$A =$ cross sectional area of conductor

(Note: the electron current is not the current in Amperes that you may have seen before – they're related, and we'll get to that later today)

Whiteboard Problem: 27-1

1.0×10^{20} electrons flow through a cross section of a 2.0 mm diameter iron wire in 5.0 s.

What is the electron drift speed?(LC)

Some electron number densities in various metals:

TABLE 27.1 Conduction-electron density in metals

Metal	Electron density (m^{-3})
Aluminum	18×10^{28}
Iron	17×10^{28}
Copper	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

b.) Note how slow the drifting motion is. Let's assume* that the electrons have a thermal speed that obeys a Maxwell – Boltzmann distribution. **Calculate the rms thermal speed of the electrons for a temperature of 300K. (LC) How much larger is it than the drift speed?**

*(*turns out to be a poor assumption – electrons in a metal obey the quantum Fermi-Dirac statistics)*

Whiteboard Problem: 27-2

Using the value of the drift speed that you calculated in the previous problem, how long does it take an electron to go from the wall switch to the overhead lightbulbs in a typical room? **(LC)**

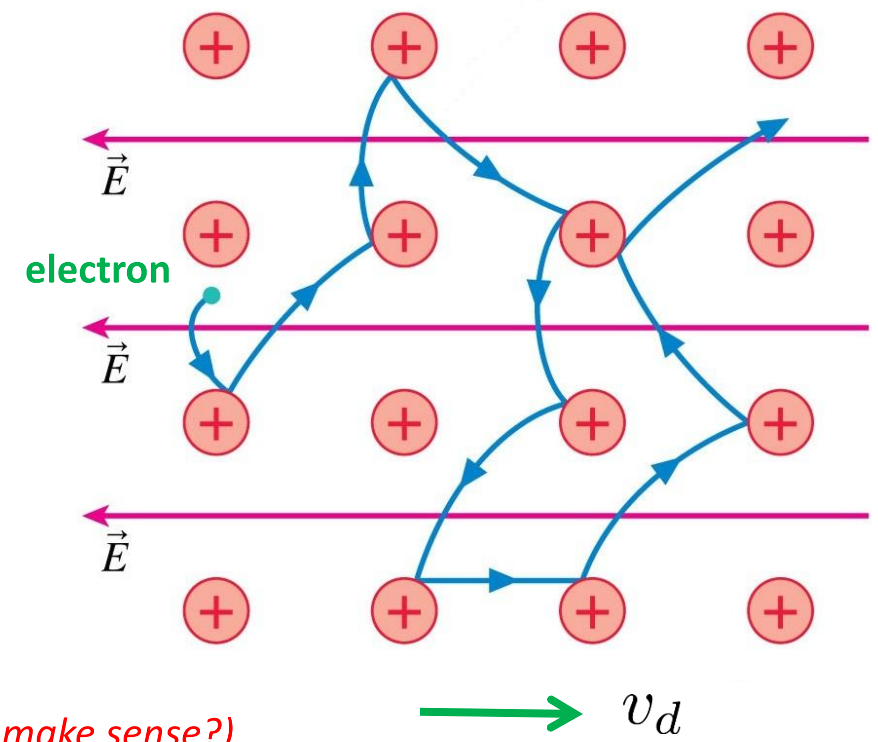
b.) Based on the number you got above, **why do the lights come on almost instantly when the switch is turned on?** Draw your explanation.

Answer: The wire is already full of electrons.

(also see signal circuit) PhET

How is the electron current related to the electric field?

The electric field accelerates the electrons in the opposite direction of the field, but collisions with the ions slow them down. So the electrons slowly drift in the opposite direction of the field.



Your author derives an expression for **The Electron Drift Speed**:

$$v_d = \frac{eE}{m}\tau$$

(Does this equation make sense?)

Where: $\tau = \text{mean time between collisions}$ $\quad m = \text{electron mass}$

Combining this with the **electron current**:

$$i_e = \frac{n_e e \tau A}{m} E$$

Whiteboard Problem: 27-3

How often do electrons moving in a current, collide with ions?

A 2.0 X 10^{-3} V/m electric field creates a 3.5 X 10^{17} electrons/s electron current in a 1.0 mm diameter aluminum wire.

For the electrons in the wire;

What is the mean time between collisions with the ions? (LC)

Some electron number densities in various metals:

TABLE 27.1 Conduction-electron density in metals

Metal	Electron density (m^{-3})
Aluminum	18×10^{28}
Iron	17×10^{28}
Copper	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

The Conventional Current

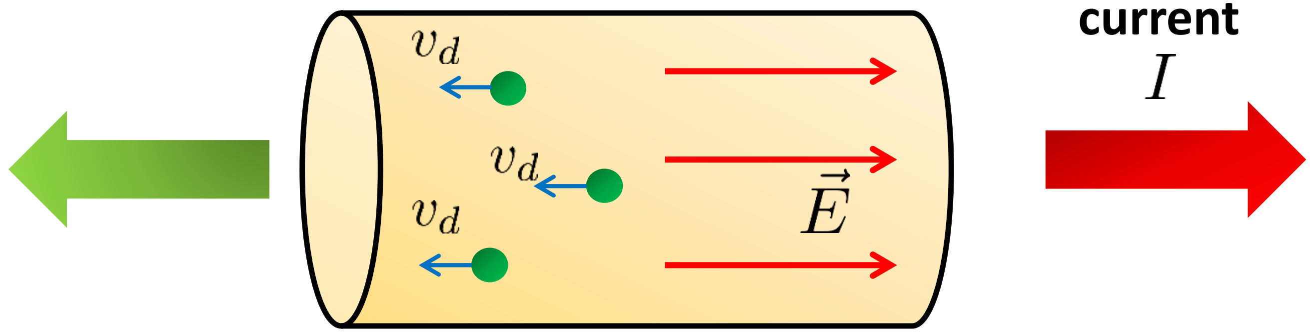
So far, we have:

Electron current

$$i_e = n_e A v_d$$

Conventional current

I



Current, resistance, and circuits had been fully used and studied long before anyone knew that the **free charge carriers in a metal are actually negative elections**. Hence, the **current was defined as the rate of flow of positive charge in the direction of the field.** We call this the **Conventional Current** (*we'll also call it just the current*):

$$I = \frac{dQ}{dt} \quad \text{(in the direction of } \vec{E}\text{)}$$

$$\text{[Units: } 1\tfrac{C}{s} = \text{Ampere (A)]}$$

This historical accident seems odd, but

"The flow of negative electrons in the opposite direction of the field is physically the same as the flow of positive charge in the direction of the field."

So, all of our calculations and devices will work fine using the conventional current even though we know what actually is going on at the microscopic level.

Also, the electron and conventional currents are in opposite directions, but they are related:

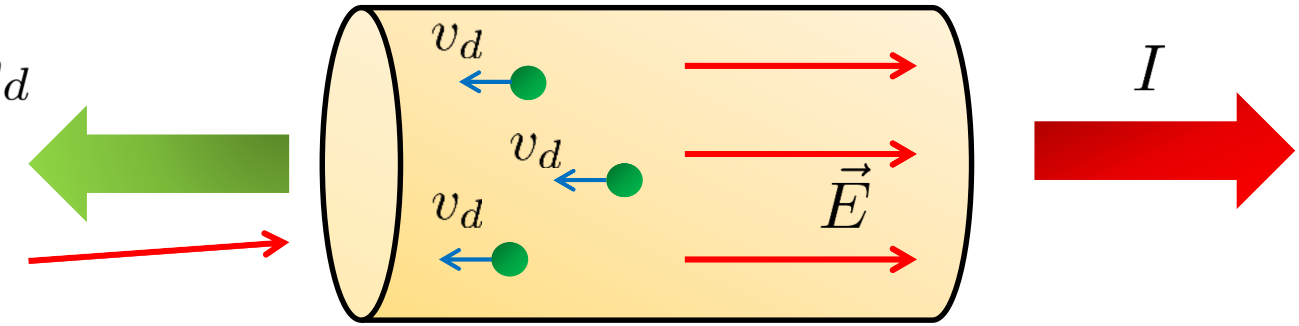
$$I = e i_e$$

<u>Whiteboard Problem: 27-4</u>

The current in an electric hair dryer is 10.0 A. While you dry your hair for 5 minutes,
how many electrons flow through the hair dryer. (LC)

Current Density

$$i_e = n_e A v_d$$



Cross section, A

Define: **The Current Density**: $J = \frac{I}{A}$ [Units: $\frac{A}{m^2}$]

So: $J = \frac{I}{A} = \frac{e i_e}{A} = \frac{e n_e A v_d}{A} \Rightarrow J = n_e e v_d$

Also, from above: $v_d = \frac{eE\tau}{m} \Rightarrow J = (\frac{n_e e^2 \tau}{m})E$

Or, $J = \sigma E$ where the Conductivity, $\sigma = \frac{n_e e^2 \tau}{m}$ [Units: $\Omega^{-1} m^{-1}$]

Note: since the **conductivity is a property of the metal**, this says that the **current density is proportional to the applied electric field**.

We will also see this in terms of **Resistivity**: $\rho = \frac{1}{\sigma} \Rightarrow J = \frac{E}{\rho}$

Whiteboard Problem: 27-5

A 0.50 mm diameter silver wire carries a 20 mA current. In the wire, **what are**

a) **The electric field? (LC)**
b) **The electron drift speed? (LC)**

Some Conductivities, Resistivities, and Number Densities:

TABLE 27.1 Conduction-electron density in metals

Metal	Electron density (m^{-3})
Aluminum	18×10^{28}
Iron	17×10^{28}
Copper	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

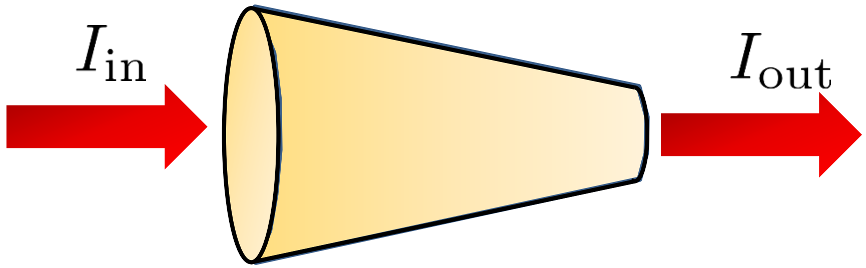
TABLE 27.2 Resistivity and conductivity of conducting materials

Material	Resistivity (Ω m)	Conductivity (Ω^{-1} m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires.

Conservation of Current

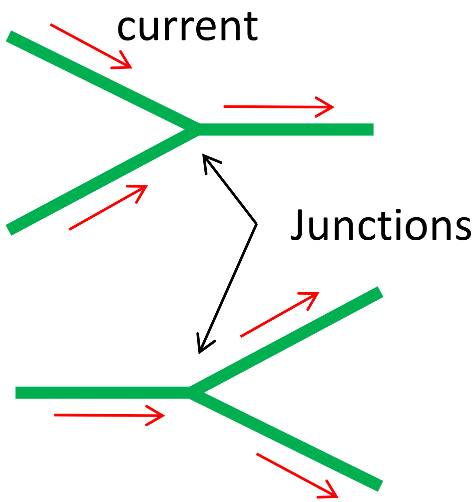
Law of conservation of current The current is the same at all points in a current-carrying wire. **(Means: Current In = Current Out)**



$I_{\text{out}} = I_{\text{in}}$ and I is the same at all points along conductor

But, $J = \frac{I}{A}$ is not constant

Junctions:



At a Junction:

Net Current In = Net Current Out

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

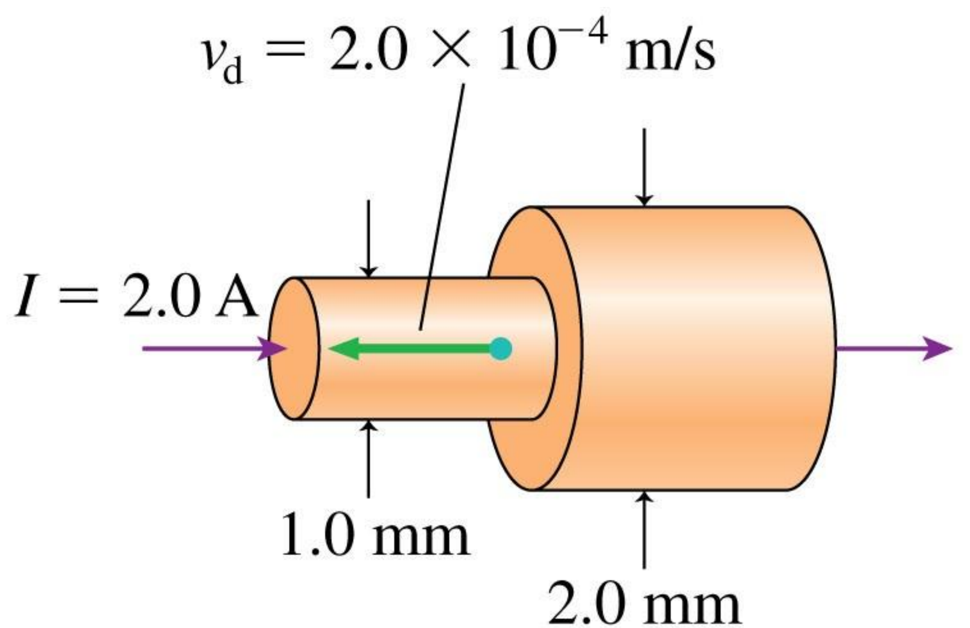
Whiteboard Problem: 27-6

The two wires shown in the figure are made of the same material.

In the 2.0 mm diameter segment of the wire, what are

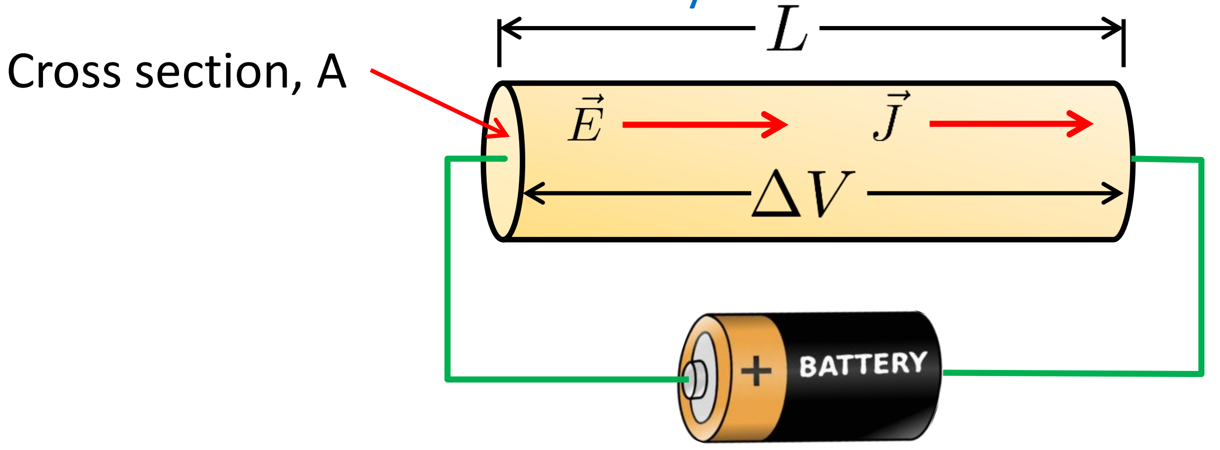
a) **The current? (LC – you have 5 seconds)**

b) **The electron drift speed? (LC)**



Resistance

From what we just did with current, if we take a sample of conducting material; and connect its ends to a battery:



This produces a potential difference, ΔV, between the ends.

If there's a ΔV, then there's also an electric field, $\vec{E}$, where for a uniform field:

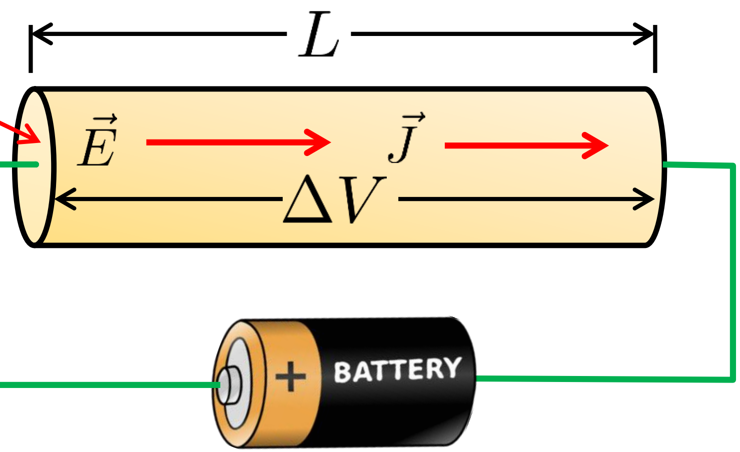
$$E = \frac{\Delta V}{L}$$

The field, $\vec{E}$, then pushes a current with current density:

$$J = \sigma E$$

Resistance

Cross section, A



conductivity

$$J = \sigma E \quad \Rightarrow \quad \frac{I}{A} = \frac{1}{\rho}\frac{\Delta V}{L}$$

$$\frac{I}{A} \qquad \frac{1}{\rho} \qquad \frac{\Delta V}{L}$$

resistivity

So: $$I = \left(\frac{A}{\rho L}\right)\Delta V$$ **Or:** $$I = \left(\frac{1}{R}\right)\Delta V$$

Ohm's "Law"

Current is directly proportional to the potential

Where: $$R = \frac{\rho L}{A}$$ is the **Resistance of the sample**

Units of resistance: $$1\ \frac{V}{A} \equiv 1\ \Omega$$ (pronounced "ohm")

Some Comments on Resistance & Ohm's Law

Resistivity is a property of the material; whereas **Resistance** depends on the material **and** the geometry of the sample.

Ohm's "Law" is not really a Law of Nature. It just expresses the fact that **for some materials the current is proportional to the applied potential** (**ohmic materials**). There are some materials (e.g. semiconductors) that do not obey Ohm's Law (**non-ohmic materials**).

Why should we use Ohm's Law as $I = \Delta V/R$ instead of $V = IR$?

The potential difference causes the current, not the other way around. *(That's why I went so slow on the first two slides above.)*

In Chapter 28 on Circuits, we will work with three types of ohmic materials:

Metallic Wires have a very low resistivity, so that to a good approximation we can assume that they have zero resistance (*unless the wire is very long and/or very thin, like in the next WB problem*).

Resistors are made of material with a fairly high resistivity, and are used to control the flow of current.

Insulators are material with very high resistivities, and hence we can assume that their resistance is infinite.

Whiteboard Problem: 27-7

The terminals of a 0.70 V watch battery are connected by a 100 m long gold wire with a diameter of 0.10 mm.

What is the current in the wire? (LC)

Some necessary data

TABLE 27.2 Resistivity and conductivity of conducting materials

Material	Resistivity (Ω m)	Conductivity (Ω^{-1} m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires.

Whiteboard Problem: 27-8

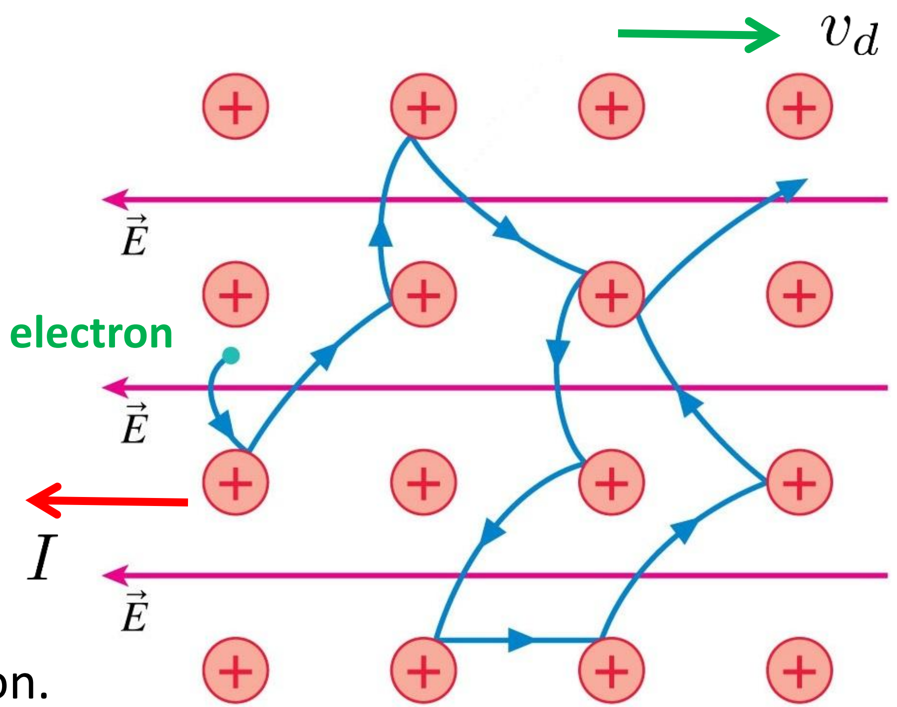
Wires 1 and 2 are made of the **same metal**. Wire 2 has twice the length and twice the diameter of wire 1.
What are the following ratios?

a) the ratio of the resistivities, ρ_2/ρ_1 **(LC, 5 seconds)**

b) the ratio of the resistances, R_2/R_1 **(LC)**

What Causes Resistance?

- As the electrons move through the lattice, they interact with the ions.
- This impedes the flow of electrons, which causes the resistance.
- The energy lost by the electrons is transferred to the lattice ions and shows up as thermal energy of vibration.



For most materials, the resistivity depends on the temperature. For higher temperature, the ions vibrate more vigorously and the electrons interact with them more readily; hence, higher resistance.

Early in the 20th century, it was discovered that for some conductors, **the resistivity goes to zero at some nonzero temperature called the critical temperature, T_c; these materials are called Superconductors.**

Superconductivity is a quantum phenomenon, so we won't treat it in PHY182, but its potential for practical applications is tremendous. **Here's an optional brief overview by Arvin Ash that is really good.**

Note: We might not have time to do this problem in class. Give it a try; the solution will be posted, and there is a HW problem that is similar to this one.

Whiteboard Problem 27-9: Charge in a Battery

How much charge is moved by a household battery before it "*runs out*?"

A 1.5 V battery is connected to a wire with a resistance of 3 Ohms. The graph below shows that the potential supplied by the battery is constant for two hours, and then decreases linearly with time to zero in another hour.

How much total charge did the battery supply? (LC)