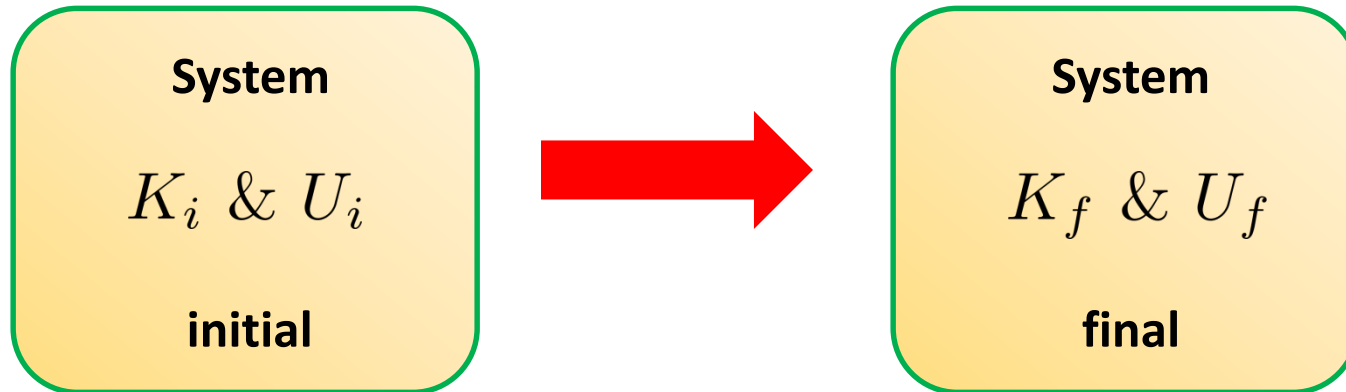


# 25: Electric Potential Energy

Back in PHY181, we learned how to solve dynamics problems using:

The Conservation of Mechanical Energy:



$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}$$

**Where:**  $E_{\text{mech}} = K + U$  = the total mechanical energy

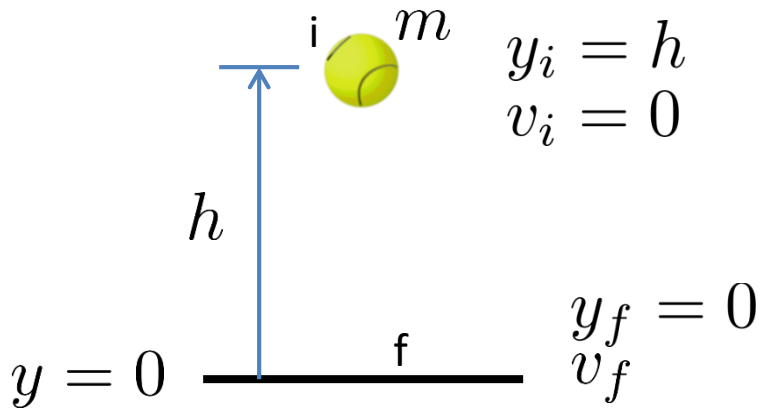
$K$  = kinetic energy

$U$  = potential energy associated with a conservative force  
where  $\Delta U = U_f - U_i = -(\text{work done by the force})$

$W_{nc}$  = work done by nonconservative forces  
(e.g. applied forces or friction)

# Whiteboard Problem: 25-1; 181 Flashback

A ball of mass  $m$  is released from rest at a height  $h$  above the ground. Find an expression for its speed when it hits the ground. (LC)



*I know that you can solve this easily using dynamics and kinematics, but don't do that. Use energy conservation instead.*

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) + mg(y_f - y_i) = 0$$

$$\frac{1}{2}mv_f^2 - mgh = 0$$

$$v_f = \sqrt{2gh}$$

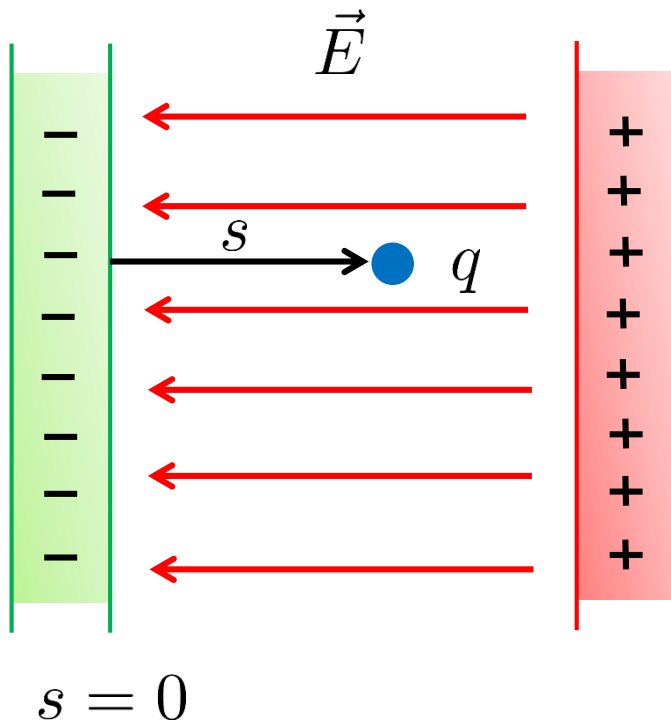
*Can we do something similar for the electric force?*

# Electric Potential Energy for a Uniform Field

The simple gravitational potential energy problem on the previous slide is for a uniform gravitational field (constant magnitude, always pointing down).

**The electric force is also a conservative force.**

Consider a **Uniform Electric Field**:



As shown in your text, or just using an analogy with gravity, let's try this for the **potential energy of a charge in a uniform field**:

$$U_{\text{elec}} = U_0 + qEs$$

(can usually set  $U_0 = 0$  at  $s = 0$ )

**Note: this is only for uniform electric fields.**

**Also, note:**

Positive and negative charges “fall” in opposite directions, but they both “fall” to lower potential energy.

## Whiteboard Problem: 25-2

The electric field strength is 50,000 N/C inside a parallel-plate capacitor with a 2.0 mm spacing between the plates.

A proton is released from rest at the positive plate.

**What is the proton's speed when it reaches the negative plate? (LC) (Ignore gravity)**

*Again, I know that you can solve this easily using dynamics and kinematics, but don't!*

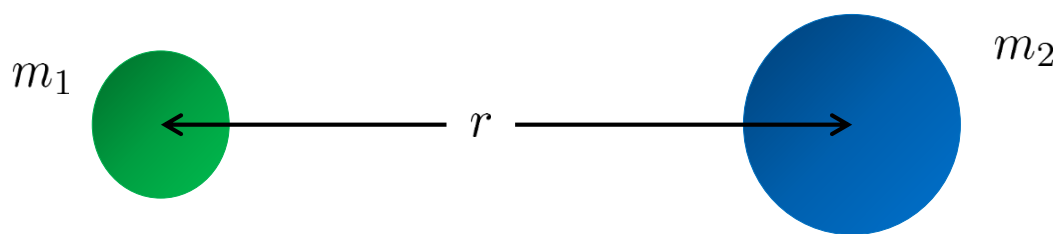
Use energy conservation with:

$$U_{\text{elec}} = \cancel{U_0} + qEs$$

**= 0**

# Electric Potential Energy of Groups of Point Charges

Also, back in PHY181, we learned how to find the **gravitational potential energy for groups of masses**; e.g. for two masses:

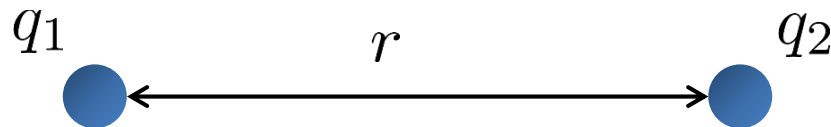


A diagram showing two masses,  $m_1$  and  $m_2$ , represented as circles.  $m_1$  is a green circle on the left, and  $m_2$  is a blue circle on the right. A horizontal double-headed arrow between them is labeled  $r$ , representing the distance between the centers of the two masses.

$$U_{\text{grav}} = -\frac{Gm_1m_2}{r}$$

where  $U_{\text{grav}} \rightarrow 0$  for  $r \rightarrow \infty$

Since the electric force is also an inverse square force, we expect something similar for two charges. Your author shows:



**The Electric Potential Energy  
of the two charges is:**

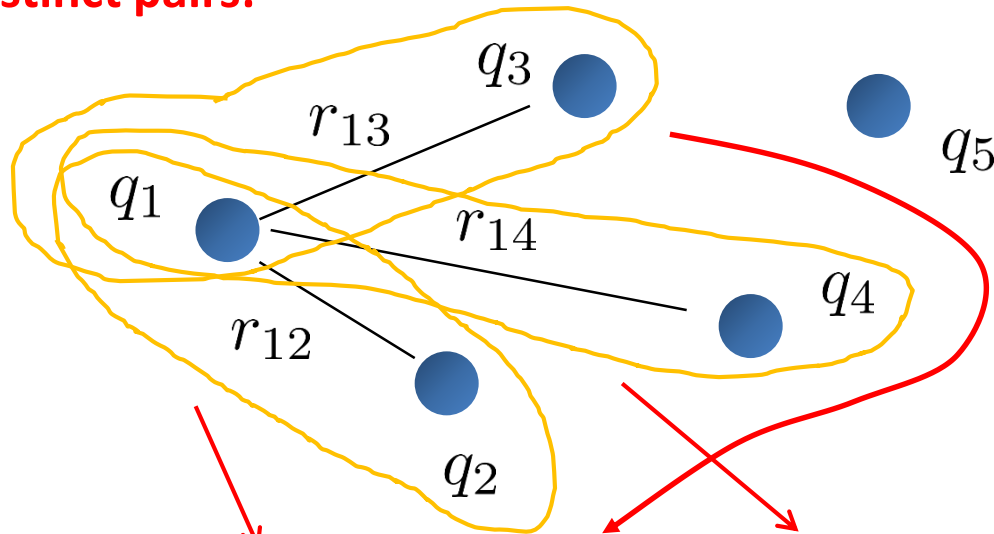
$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

Note:  $U_{\text{elec}} \rightarrow 0$  for  $r \rightarrow \infty$

the sign of  $U_{\text{elec}}$  is determined by the signs of  $q_1$  and  $q_2$

# Electric Potential Energy of Groups of Point Charges

For more than two point charges, the total electric potential energy is just the sum over distinct pairs:



$$U_{\text{elec}} = \frac{K q_1 q_2}{r_{12}} + \frac{K q_1 q_3}{r_{13}} + \frac{K q_1 q_4}{r_{14}} + \dots$$

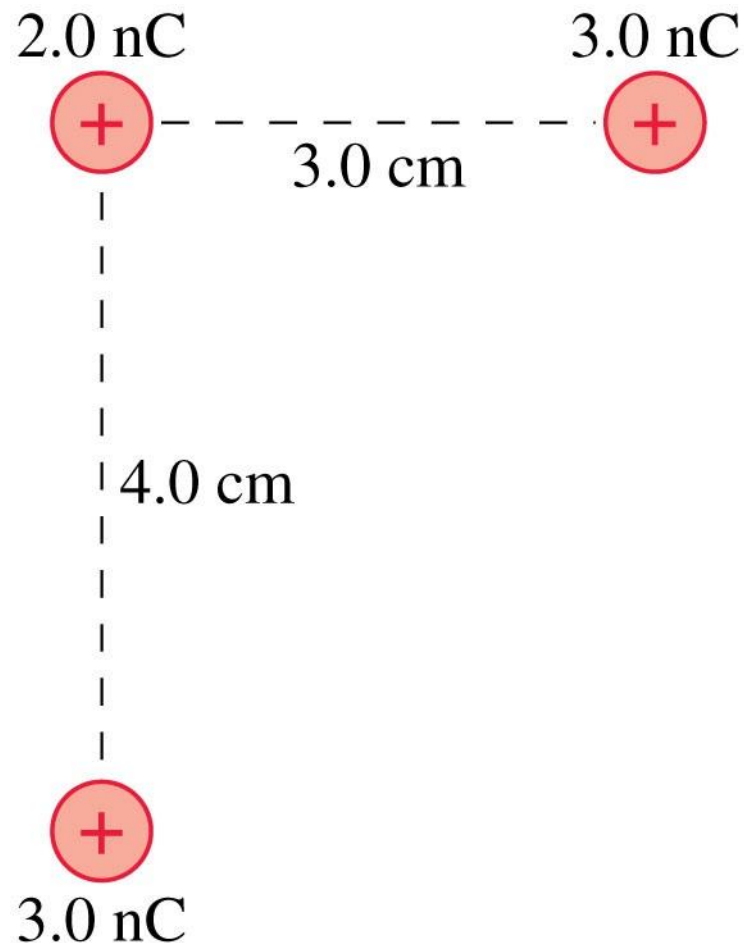
or

$$U_{\text{elec}} = \sum_{\text{distinct pairs}} \frac{K q_i q_j}{r_{ij}}$$

Hey, what's neat and different about this? **No Vectors!**

## Whiteboard Problem: 25-3

What is the electric potential energy of the group of charges in the figure below? **(LC)** *(This should only take ~2 minutes.)*

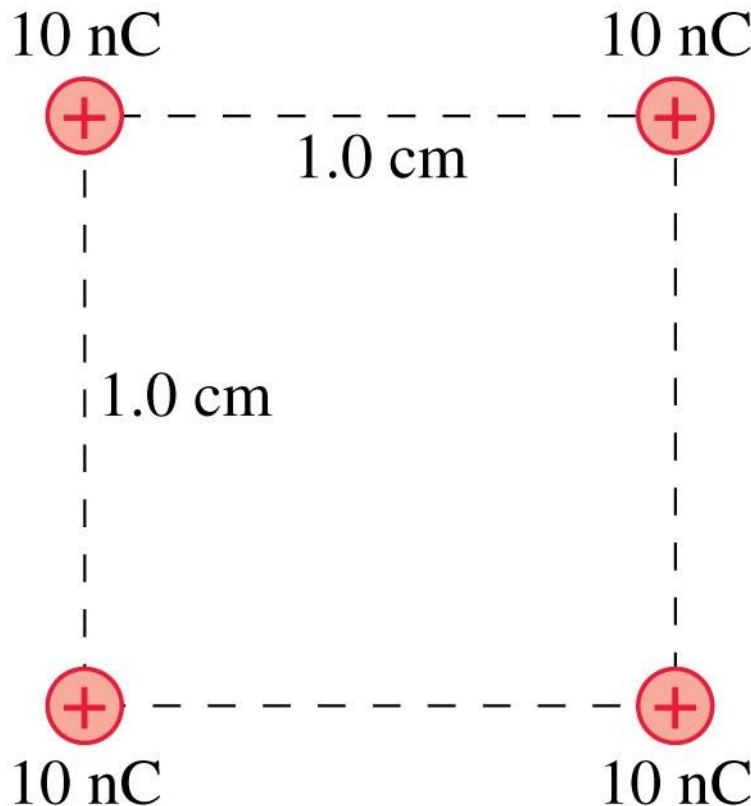


Something a little more fun:

## Whiteboard Problem 25-4

The four 1.0 g spheres shown in the figure below are released simultaneously and allowed to move away from each other.

**What is the speed of each sphere when they are very far apart?** (LC)



*Could you solve this problem using dynamics and kinematics?*

*Yes, in principle, but it would be very difficult! Conservation of Energy is much easier!*

*Draw the initial and final states. Determine the initial and final K and U, and apply energy conservation.*

Something even more fun:

## Hints for Homework Problem 25.40

(This is HW problem 25.40; use the hint below; spheres or cubes doesn't make any difference.)

Two small metal spheres with masses  $m_1 = 2.0$  g and  $m_2 = 4.0$  g are tied together by a  $d = 5.0$  cm long massless string and are at rest. Each is charged,  $q_1 = q_2 = q = 2000$  nC. **Ignoring gravity:**

- a) **What is the energy of the system? (LC)**
- b) **What is the tension in the string? (LC)**
- c) **The string is cut. What is the speed of each sphere,  $v_1$  and  $v_2$ , when they are very far apart (remember, ignore gravity)?**
  - i.  $v_1$ , speed of  $m_1$  (LC)
  - ii.  $v_2$ , speed of  $m_2$  (LC)

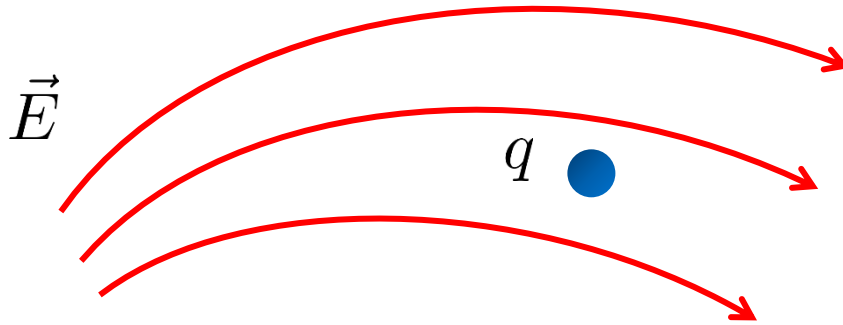
**Hint: There are two quantities that are conserved; you have to use both. What are they?  
Energy and momentum!**

*Why didn't we have to do this in Problem WB 25-4?*

*In WB 25-4, the symmetry (same charges and masses) means that the final velocities had to be the same; so you just needed to use only energy conservation. Here the masses are different.*

# Electric Potential

Consider a charge in an arbitrary external electric field created by some source charges:



The charge  $q$  in the field  $\vec{E}$  has some potential energy at its location,  $U(q \ \& \ \vec{E})$

**Define:** **Electric Potential\*** at the location of  $q \equiv \frac{\text{Potential Energy of } q \ \& \ \vec{E}}{q}$

$$\text{Or: } V = \frac{U(q \ \& \ \vec{E})}{q} \quad \text{Units} = \frac{J}{C} \equiv \text{Volt (V)}^+$$

**Note:** like the electric field, the potential is a property of the source charges and exists whether or not the charge  $q$  is present.

**Be careful:**  $V$  is the symbol used to represent electric potential and is also the abbreviation of its unit, the Volt (V). So you can end up with an absurd-looking equation like:  $V = 5V$  (which makes no sense in algebra).

\*Note: The electric potential ( $V$ ) and the electric potential energy ( $U$ ) are related, but different quantities, even though they both use the same word *potential* – *don't confuse them*.

## Whiteboard Problem: 25-5

What is the speed of an electron that has been accelerated from rest through a potential difference of 1000 V? (LC)

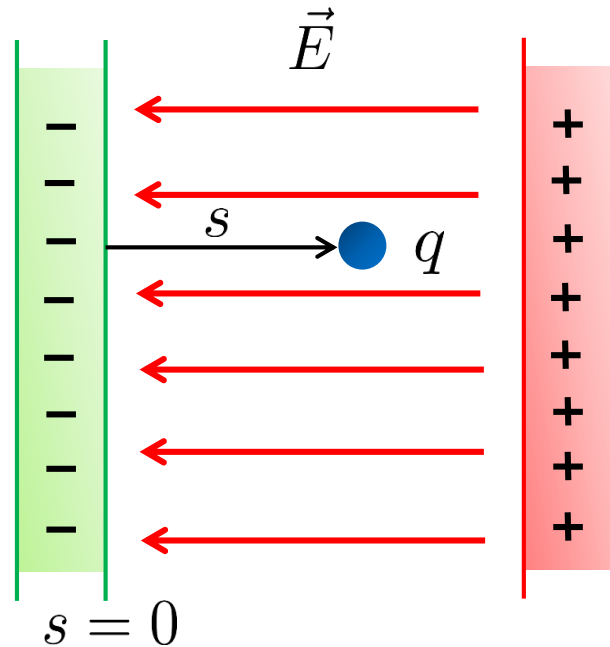
Having trouble getting the sign right,  
i.e. are you trying to take the square root  
of a negative number?

Try using conservation of energy carefully:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

and  $\Delta U = q\Delta V = -e\Delta V$  for an electron

# Potential for a Uniform Field



Previously, we had for the **potential energy for a uniform field**:

$$U = qEs \quad (\text{for } U_0 = 0)$$

So:  $V = Es$

**Only for a uniform field!**

**Note that the field points in the direction of decreasing potential.**

## Some important points:

A **positive charge** released at  $s$  “falls” to a **lower potential**.

A **negative charge** released at  $s$  “falls” to a **higher potential**.

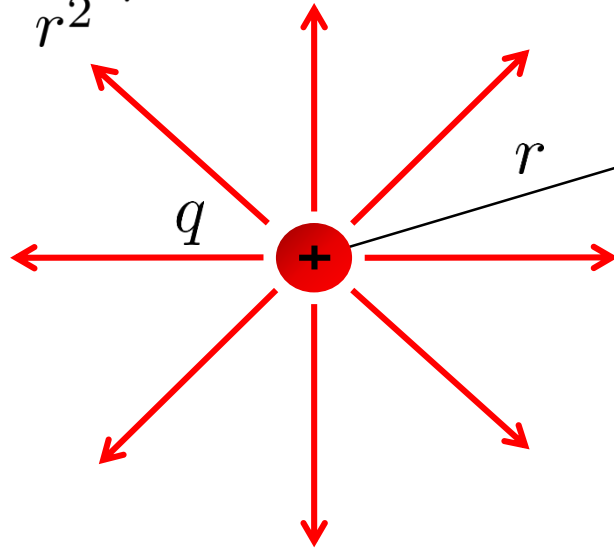
**But, they both “fall” to a lower potential energy.**

For a uniform field:  $E = \frac{V}{s} \Rightarrow \text{units of } E = \frac{N}{C} = \frac{V}{m}$

The units V/m are widely used for the electric field since both potential and distance are easy to measure in the laboratory. **From now on, we will use the units of V/m for the electric field.** 12

# Potential of a Point Charge Field

$$\vec{E} = \frac{Kq}{r^2} \hat{r}$$



So, at point P:

$$\vec{E} \text{ (for } q > 0 \text{)}$$

Your author argues (and we'll see how to derive this in chap 26) that **the Potential at P is:**

$$V = \frac{Kq}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

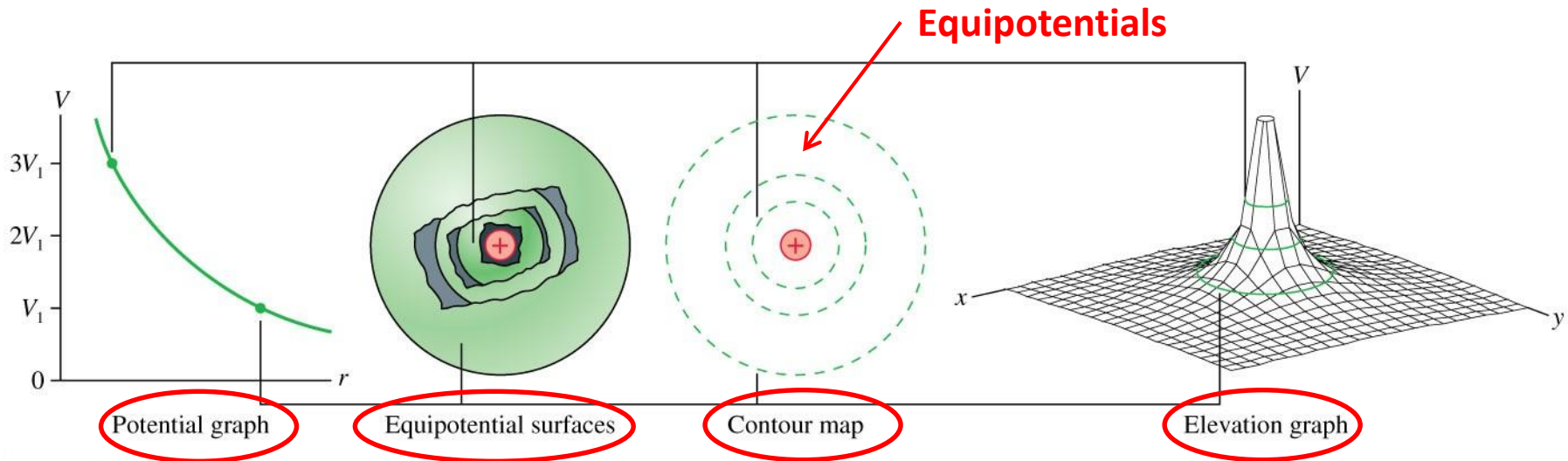
Note: the potential of a point charge decreases as  $\frac{1}{r}$   
as opposed to the electric field which decreases as  $\frac{1}{r^2}$   
The sign of  $q$  determines the sign of  $V$ .

# Graphical Representations of the Point Charge Potential

The potential of a point charge is spherically symmetric so that all points at the same distance from the charge have the same potential

$$V = \frac{Kq}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

There are many ways to represent this graphically:



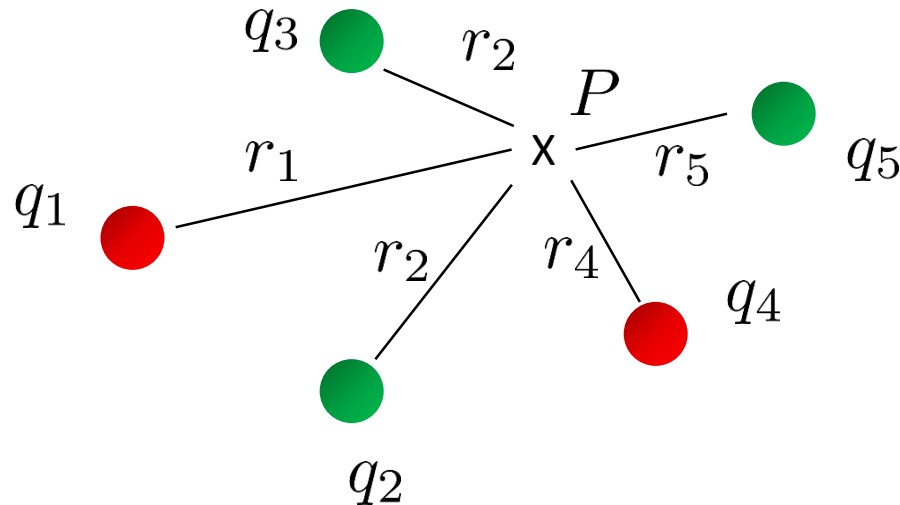
**3D**

**2D slice**

**+ charges are the mountains,  $V > 0$   
- charges are the valleys,  $V < 0$**

# Potential for a Group of Point Charges

The potential for multiple point charges is just the sum of the individual potentials for each charge:



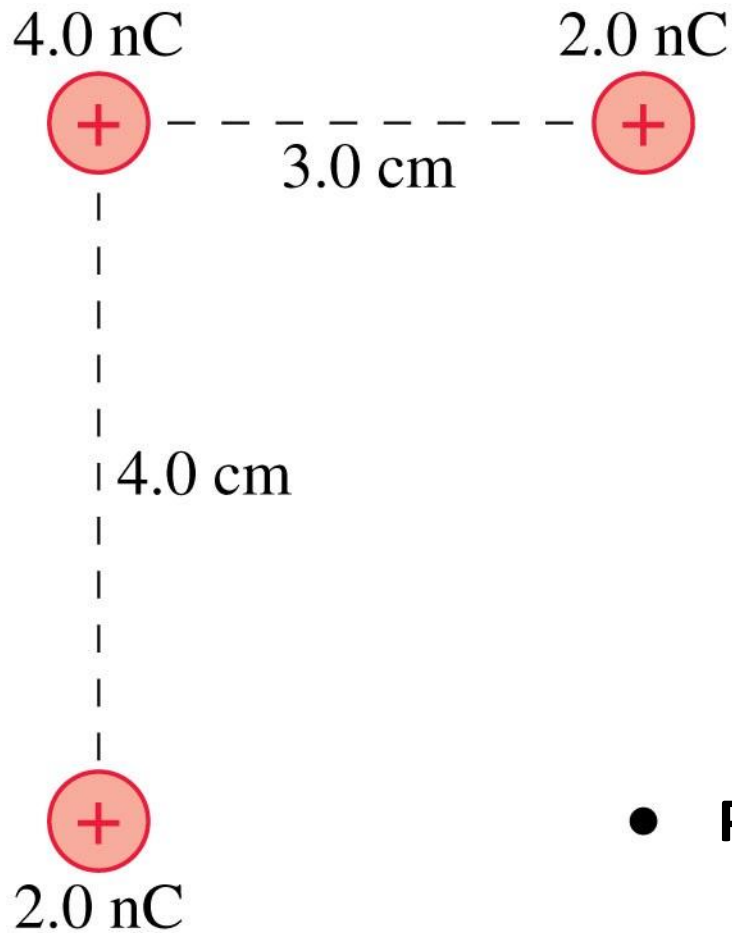
Here, we can just add up the potential for each charge at the point P:

$$\text{Total Potential at P: } V = \sum_i \frac{K q_i}{r_i}$$

**Note that:** the positive charges contribute a positive potential and the the negative charges contribute a negative potential.

# Whiteboard Problem: 25-6

Part a) What is the electric potential at the point P indicated with the dot? (LC)



**Part b)** What is the potential energy of an electron placed at the dot? (you have 10 seconds!) (LC)

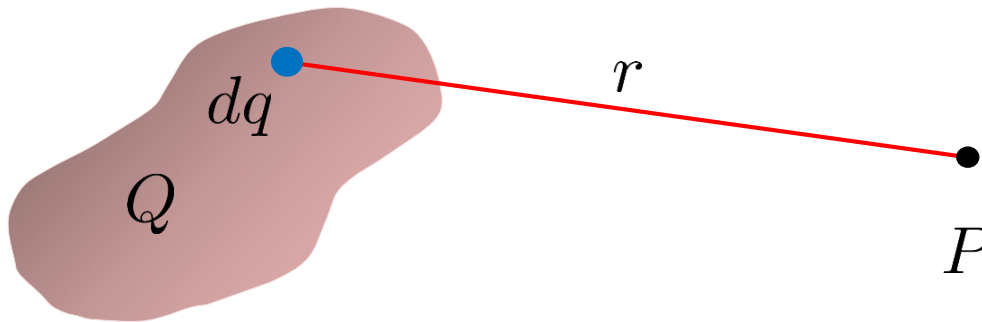
If you know  $V$  at some point in space, and you put a charge  $q$  there, you immediately know the potential energy of the charge:

$$U = qV$$

# Potential for Continuous Distributions of Charge

Back in chapter 23, we learned how to find the electric field of a continuous distribution of charge. We treated the distribution as a collection of an infinite number of point charges, each with infinitesimally small charge,  $dq$ . Then we just added up the fields produced by all the  $dq$  – i.e. we integrated over the total charge.

**We'll do the same thing here for the potential: e.g. in 3D:**



The point charge  $dq$  creates a potential at the point P:

$$dV = \frac{K dq}{r}$$

So, the total potential at P is:  $V(\text{at } P) = \int_Q dV = \int_Q \frac{K dq}{r}$

**The Difficult Part**: just as it was for the electric field, we have to express  $dq$  in terms of  $Q$ , and we have to express  $r$  in terms of the coordinates of  $dq$  and P; then we can integrate.

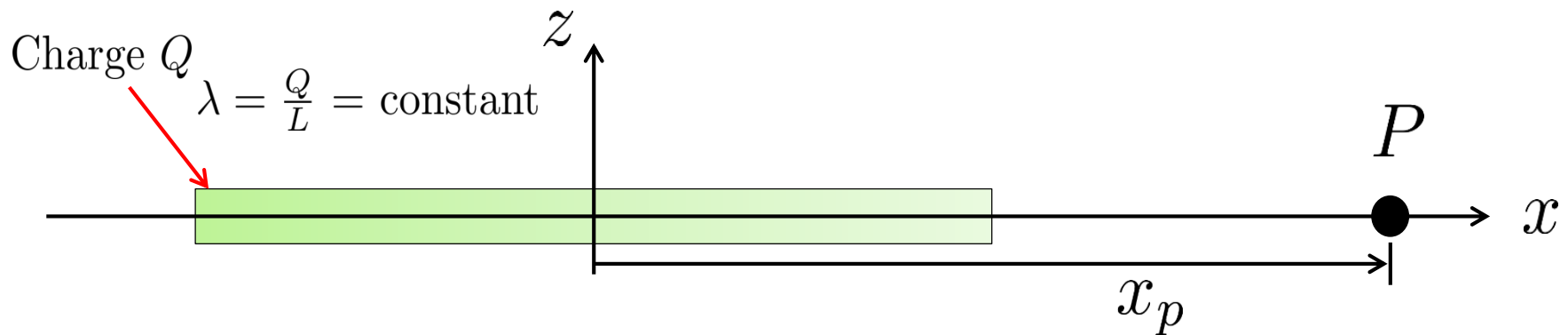
*OK, we can use all the steps that we learned in calculating the electric field, but in what way is this much easier?*

**All Scalars, no Vectors!**

## Example Problem: The Potential on the Axis of a Charged Rod

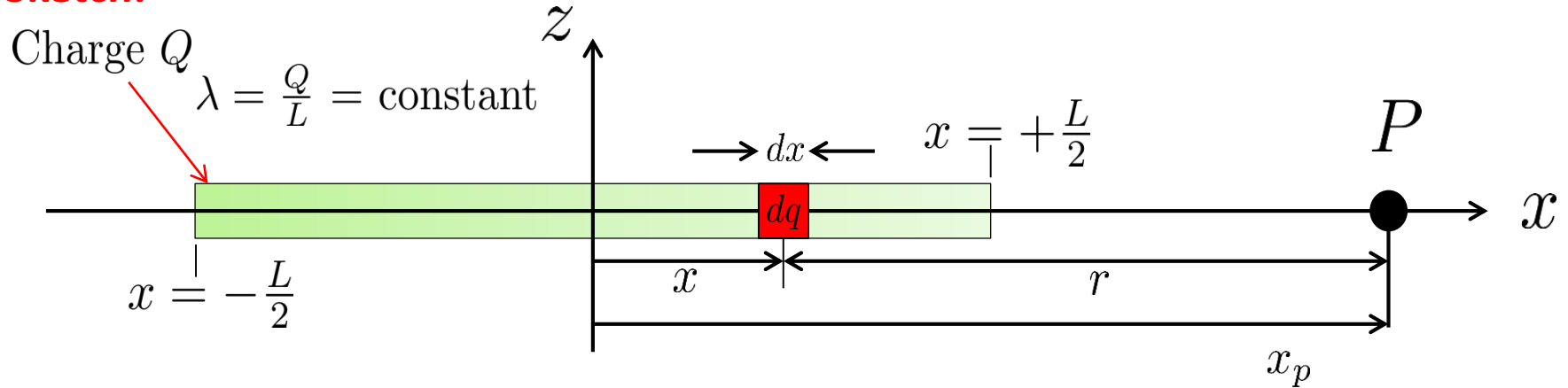
The figure below shows our thin rod of length  $L$  and total charge  $Q$  uniformly distributed on the rod.

**Find an expression for the electric potential a distance  $x_p$  away from the center of the rod on the axis of the rod.  
(in terms of  $K$ ,  $Q$ ,  $L$ , and  $x_p$ )**



# Example Problem: Solution

**Sketch:**



**Select a dq:**

Note:  $x_p$  = coordinate of the field point P

$x$  = coordinate of the charge element dq

**So, at P:**  $dV = \frac{Kdq}{r}$  where:  $dq = \lambda dx$  and  $r = x_p - x$

**So, the total potential at P:**

$$V = \int_Q dV = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{K\lambda dx}{(x_p - x)} = K\lambda \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{(x_p - x)}$$

## Example Problem: Solution

**Integrate:** (for this one, you don't even need tables – or you can use the table)

$$\begin{aligned} V &= K\lambda \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{(x_p - x)} = K\lambda(-1) \ln(x_p - x) \Big|_{x=-\frac{L}{2}}^{x=+\frac{L}{2}} \\ &= -K\lambda \left\{ \ln\left(x_p - \frac{L}{2}\right) - \ln\left(x_p + \frac{L}{2}\right) \right\} \\ &= K\lambda \left\{ \ln\left(x_p + \frac{L}{2}\right) - \ln\left(x_p - \frac{L}{2}\right) \right\} \end{aligned}$$

**Final Answer:**

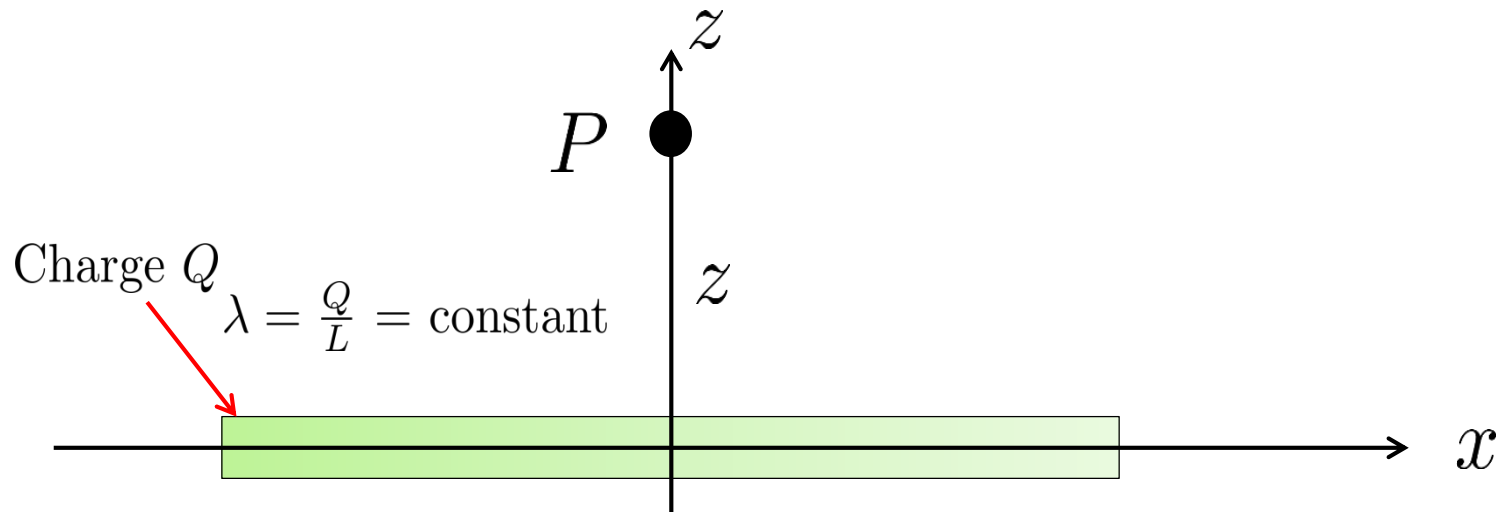
$$V = \frac{KQ}{L} \ln \left\{ \frac{x_p + \frac{L}{2}}{x_p - \frac{L}{2}} \right\}$$

*Now, that wasn't so bad! Are you ready to try one?*

## Whiteboard Problem: 25-7

The figure below shows our same thin rod of length  $L$  and total charge  $Q$  uniformly distributed on the rod.

**Now, Find an expression for the electric potential a distance  $z$  away from the center of the rod on the line that bisects the rod. (in terms of  $K$ ,  $Q$ ,  $L$ , and  $z$ ) (LC, 2 points)**



***This one is only a little harder than the example; follow all of the steps that we did in that solution.***

***Anyone want an Integral Table?***