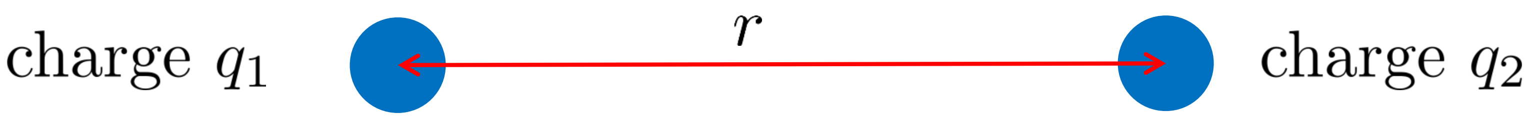
22-2: The Electric Field

So far, we have **Coulomb's Law** that gives the electric force between two charges:



$$\vec{F}_{(2\ on\ 1)} = -\vec{F}_{(1\ on\ 2)} = \left\{ K\frac{|q_1||q_2|}{r^2},\ \begin{matrix}\text{attractive for unlike charges}\\ \text{repulsive for like charges}\end{matrix} \right\}$$

Coulomb's Law is written in the language of **Newton** – i.e. **Forces**, which is OK.

However, as quickly as possible, we want to learn and adopt the language of Michael Faraday and James Clerk Maxwell – the Language of Fields.

What is a Field?

In mathematics, a field is a quantity (scalar, vector, or something else) that has some value at every point in space (x,y,z)

e.g. in this room, you might define the temperature field which would associate the value of the temperature (a scalar) with every point in space.

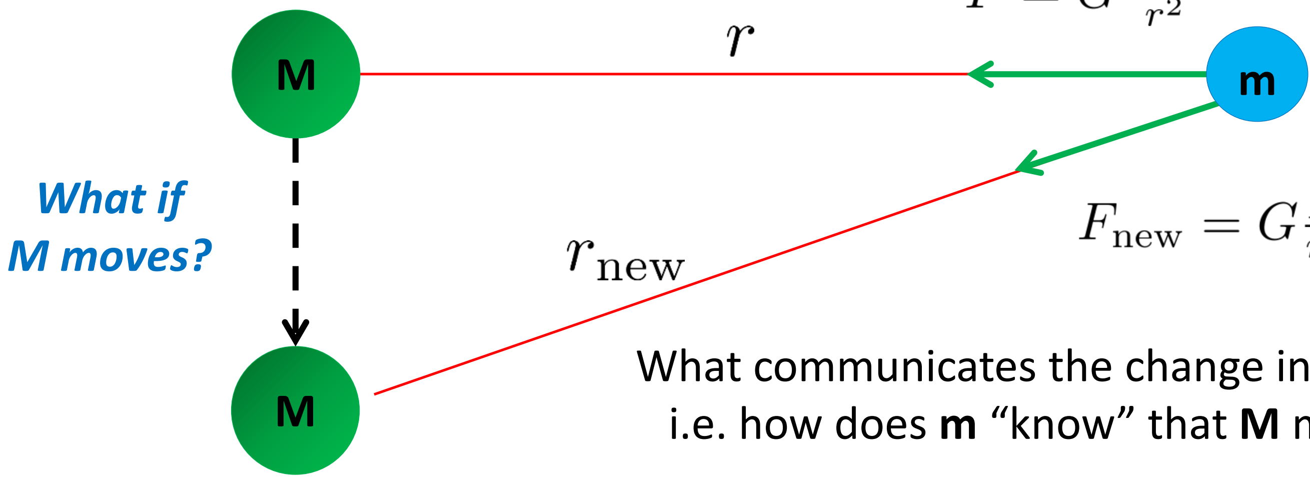
Or a fluid velocity field which would associate a vector (magnitude and direction) with every point in space.

In physics, some fields do this and more

The Electric Field

Consider a situation with **Newtonian Gravity**: *What if the mass M moves?*

$$F = G\frac{Mm}{r^2}$$

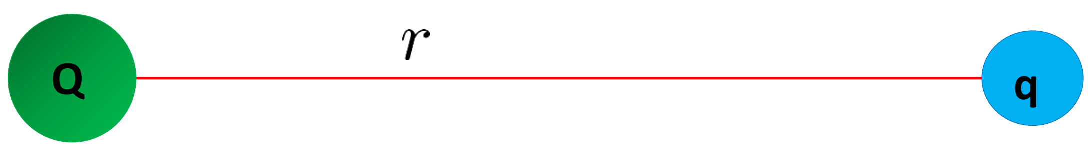


$$F_{\text{new}} = G\frac{Mm}{r_{\text{new}}^2}$$

What communicates the change in the gravitational force; i.e. how does **m** "know" that **M** moved?

Newton was aware of the problem, but it would wait for Einstein to solve it for gravity – **gravitational waves**!

For the **Electric Force**, we have the same problem:



$$F = K\frac{|Q||q|}{r^2}$$

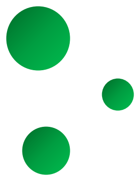
How is the force communicated?

Answer: The charge **Q** alters the space around it creating an **Electric Field** $\vec{E}$

The field then exerts a force on the charge **q**: $\vec{F}_{\text{on q}} = q\vec{E}$

The Electric Field, Definition

Source Charges (create a field in all space)



q_0, positive "probe charge" at (x,y,z)

$$\vec{F}_{\text{on } q_0}$$

(from all source charges)

Electric Field at (x,y,z)

$$\vec{E}(x,y,z) \equiv \frac{\vec{F}_{\text{on } q_0}(\text{at } x,y,z)}{q_0} \qquad [\text{Units: } \frac{N}{C}\ ]$$

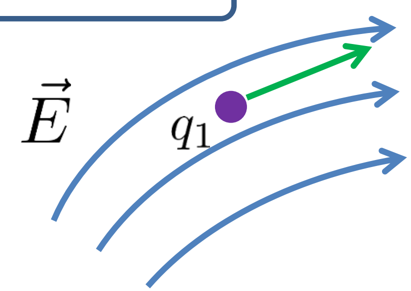
defined

Note: The field is created by the source charges and exists everywhere in space.
It associates a vector with every point in space.

The field exists even if the probe charge q_0 isn't there!

Also, if a field exists in space, and we put a charge in it.
We immediately know the force on the charge:

$$\vec{F}_{\text{on } q_1} = q_1 \vec{E}$$



(note: opposite charges feel oppositely directed forces in the same field)

In what sense is the electric field a real physical quantity?
(EM waves and quantum photons)

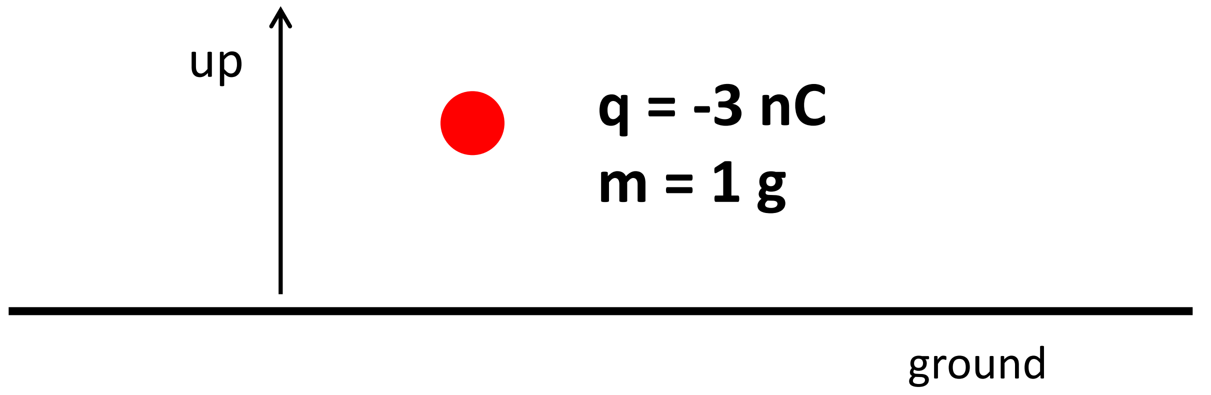
PhET antenna

Whiteboard Problem: 22-6

What are the **strength** and direction of an electric field that will balance the weight of a 1.0 g plastic sphere that has been charged To -3.0 nC?

a) **What is the direction of the electric field? (LC)**
b) **What is the strength of the electric field ? (LC)**

Note: Field “**Strength**” is the same as the field **Magnitude**.

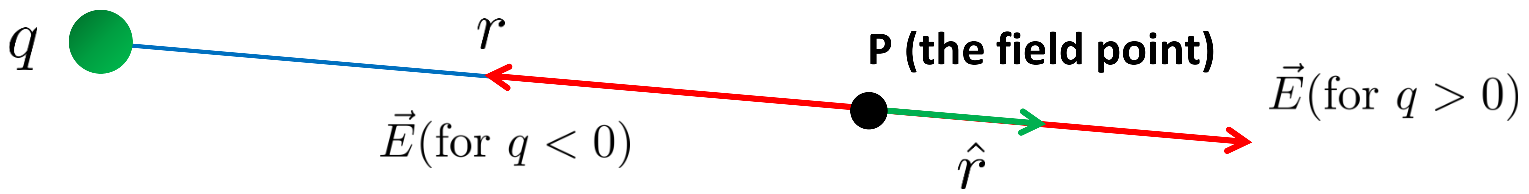


The Electric Field of a Point Charge

In the next several chapters, one of our major concerns will be the problem:

How to find the electric field for different distributions of charges.

The most important field that we'll use is the **field of a point charge**. Using the definition of the field and Coulomb's Law, it is straightforward to show – see your text:



$$\vec{E}(\text{at P}) = K\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Where: r = distance from q to the point P

$\hat{r}$ = unit vector at the point P **radially away** from q

This equation works for both q > 0 and q < 0. For either, the **field magnitude** is given by:

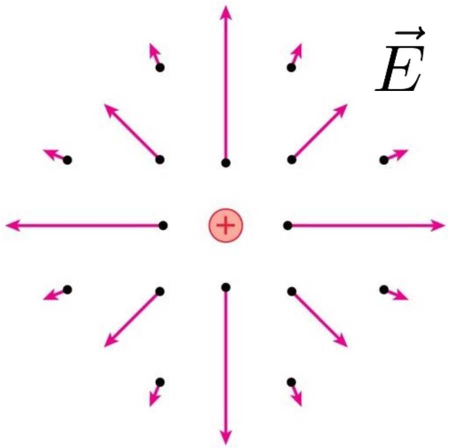
$$|\vec{E}| = \frac{K|q|}{r^2}$$

But the field is away from q if q > 0, and towards q if q < 0.

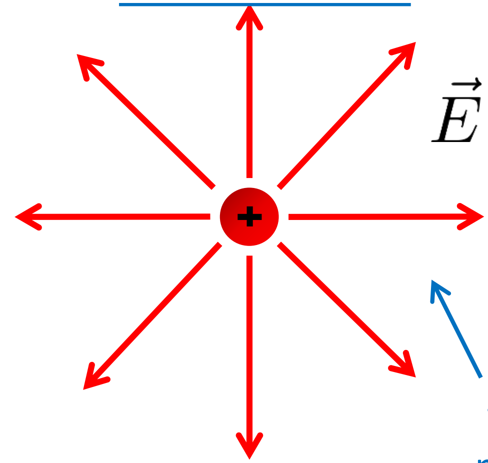
Drawing the Point Charge Field

Knight's accurate, but difficult way:
The Field Vectors

An easier, but less accurate, way:
The Field Lines

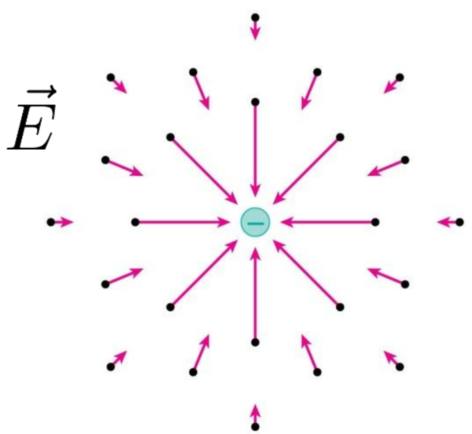


$q > 0$

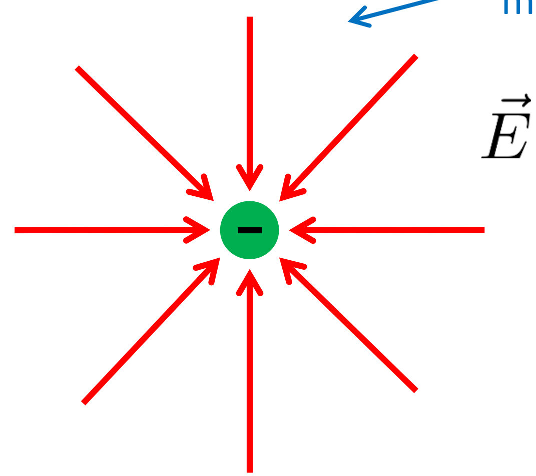


Both of these are a 2D slice of a 3D field.

Field lines further apart means weaker magnitude.



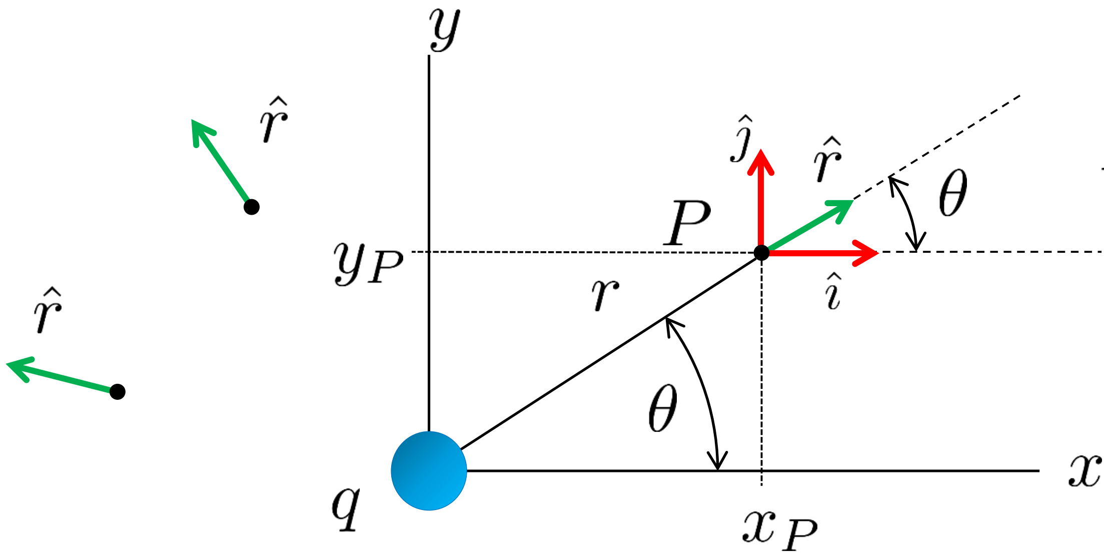
$q < 0$



For both, remember that the field strength decreases with the square of the distance, and the field comes out of positive charge and goes into negative charge.

What about r-hat $\hat{r}$?

$\hat{r}$ **(r-hat)** bothers some people because it points in different directions at different points in space since it always is radially away from the charge q.



$$\vec{E}(\text{at P}) = K\frac{q}{r^2}\hat{r}$$

In all of the problems that we do, r-hat will be able to be expressed in terms of the coordinate unit vectors, **e.g. for point P above:**

$$\hat{r} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath} \qquad \text{Note: } |\hat{r}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\text{Also: } \cos\theta = \frac{x_p}{r} \text{ and } \sin\theta = \frac{y_p}{r}$$

$\hat{r}$ **(r-hat)** provides a compact way to write a vector expression for the electric field of a a point charge. In problems, we can use it or just define coordinates and write the field in component form for the chosen coordinates.

Whiteboard Problem: 22-7

A charge **q = +12 nC** is located at the origin of an x-y coordinate system.

a) At the LC prompt, show the charge and sketch the electric field vector at the point P at (x,y) = (5.0cm, 0cm). (LC)
Find the electric field vector in component form at the point. (LC)

a) At the LC prompt, show the charge and sketch the electric field vector at the point P at (x,y) = (-5.0cm, 5.0cm). (LC)
Find the electric field vector in component form at the point. (LC)

Instructions for entering a vector in component form on LC:

For the vector:

$$\vec{E} = -5.32 \times 10^4 \hat{\imath} + 1.78 \times 10^3 \hat{\jmath}$$

Use only two significant figures and the pull down menu for powers of ten if necessary; put the components in parentheses, and use just i and j for unit vectors – no hats!

For the vector above, enter: **$(-5.3 \cdot 10^4)$i + $(1.8 \cdot 10^3)$j**

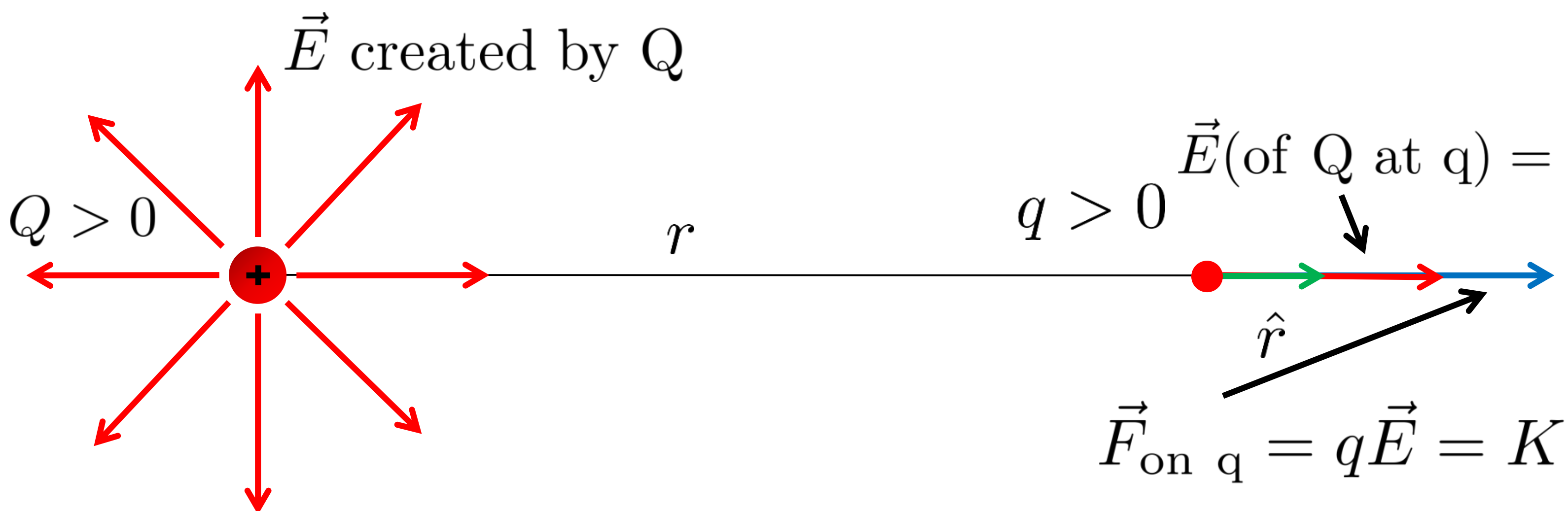
Before we continue: Where Did Coulomb's Law Go?

Now, that we're beginning to learn the language of fields, some students get nostalgic for Coulomb's Law. ***After all, it is familiar and easy to work with. Where did it go?***

<u>Here's how we think using Coulomb's Law</u>:

$$F_{\text{on q}} = K\frac{|Q||q|}{r^2}$$

<u>Now, we think in terms of Fields</u>:



$$\vec{E}(\text{of Q at q}) = \text{K}\frac{\text{Q}}{\text{r}^2}\hat{\text{r}}$$

$$\vec{F}_{\text{on q}} = q\vec{E} = K\frac{Qq}{r^2}\hat{r}$$

Coulomb's Law is contained in the equation of a point charge field.