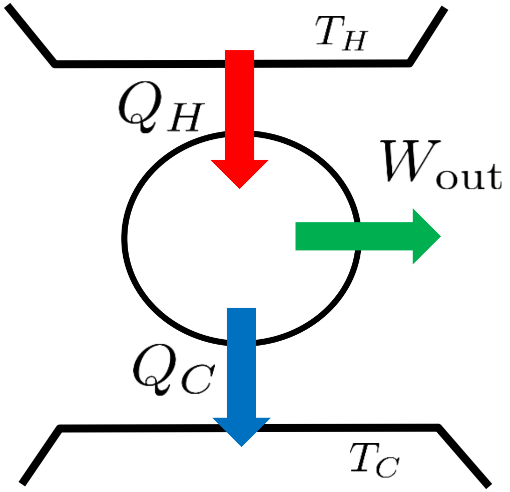
21-2: Heat Engine Efficiency

(a quick review from the last class)



Apply the First Law: the easiest way to do this is to say:

Energy into the system **=** **Energy out of the system**

$$Q_H = Q_C + W_{\text{out}}$$

Define: The Engine Efficiency:

$$\eta = \frac{\text{energy you get}}{\text{energy you pay for}} = \frac{\text{work output}}{\text{heat input}}$$

So:

$$\boxed{\eta = \frac{W_{\text{out}}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}}$$

The First Law just says that the efficiency can't be > 1.0, and if the efficiency = 1, then all of Q_H is converted into W_{out}. How close to 1.0 can the efficiency get?

What About the 2nd Law?

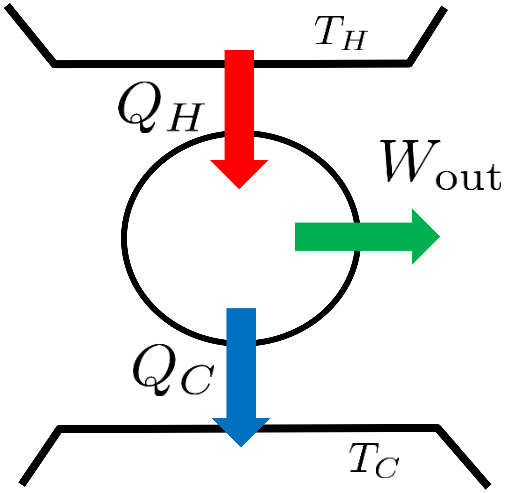
We have for the efficiency of a heat engine:

$$\eta = \frac{W_{\text{out}}}{Q_H}$$

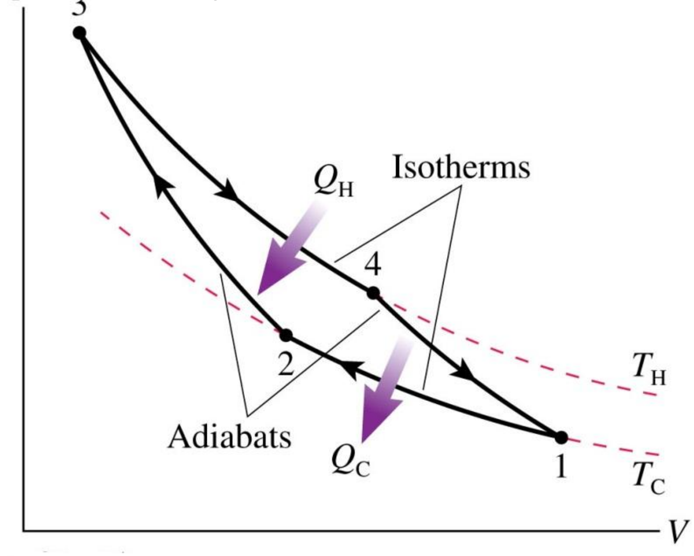
How large can the efficiency be? The First Law only says that it can't be greater than 1.

In the early 19th century, Sadi Carnot analyzed heat engines and showed that there is a theoretical upper limit on the efficiency that is governed, not by the 1st Law, but by the 2nd Law.

The Carnot Cycle is an **idealized reversible cycle** between some high temperature reservoir and a low temperature reservoir



For example: an Ideal Gas, two isotherms connected by two adiabats



The Carnot Efficiency

In your text, your author shows that for an **Ideal Gas Carnot Cycle**:

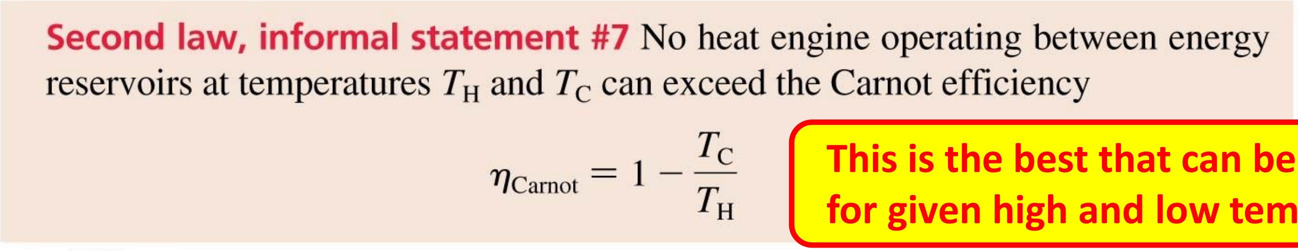
$$\eta = \underbrace{\frac{W_{\text{out}}}{Q_H} = 1 - \frac{Q_C}{Q_H}}_{\text{true for }\underline{\text{any}}\text{ cycle}} = \underbrace{1 - \frac{T_C}{T_H}}_{\text{true }\underline{\text{only}}\text{ for a Carnot cycle}}$$

So, we have the **Carnot Efficiency**:

Similarly for a Refrigerator Cycle,
Carnot Coefficient of Performance:

$$\boxed{\begin{aligned} \eta_{\text{Carnot}} &= 1 - \frac{T_C}{T_H} \\ K_{\text{Carnot}} &= \frac{T_C}{T_H - T_C} \end{aligned}}$$

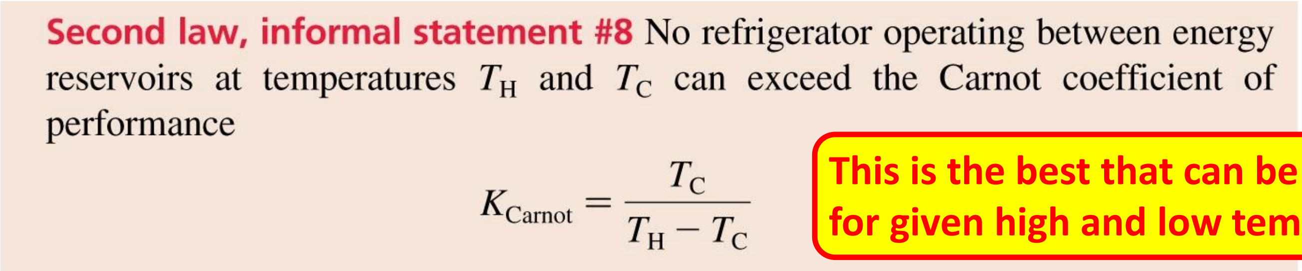
The 2nd Law in terms of Carnot Cycles:



Second law, informal statement #7 No heat engine operating between energy reservoirs at temperatures T_{H} and T_{C} can exceed the Carnot efficiency

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}}$$

This is the best that can be achieved for given high and low temperatures



Second law, informal statement #8 No refrigerator operating between energy reservoirs at temperatures T_{H} and T_{C} can exceed the Carnot coefficient of performance

$$K_{\text{Carnot}} = \frac{T_{\mathrm{C}}}{T_{\mathrm{H}} - T_{\mathrm{C}}}$$

This is the best that can be achieved for given high and low temperatures

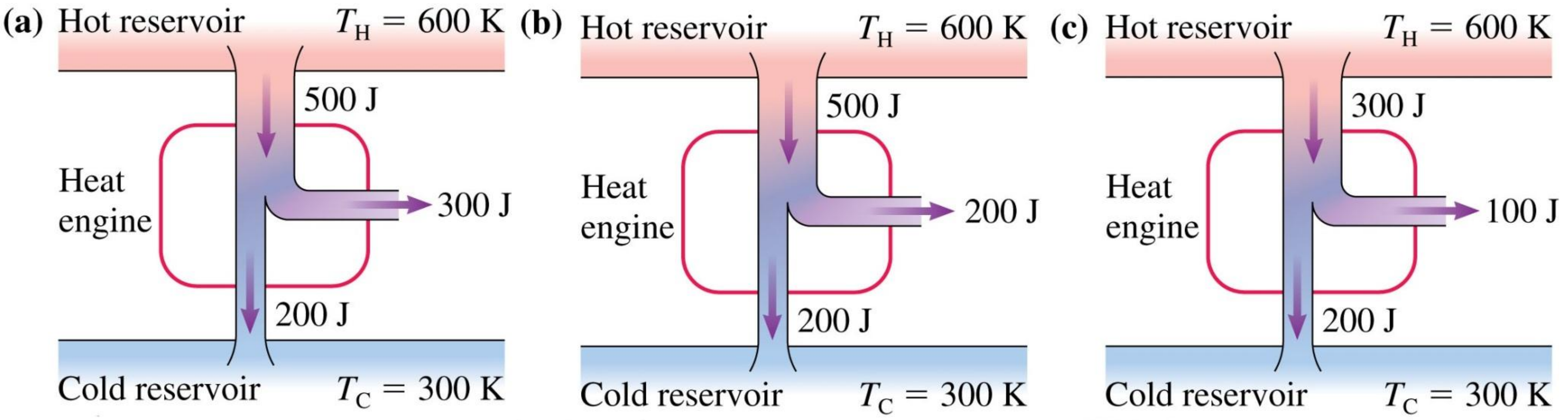
Whiteboard Problem 21-4

The following Heat Engines have been proposed.

Which engines are viable designs (LC)?

How are you going answer this?

*Check to see if the engine obeys **Both the 1st and 2nd Laws of Thermodynamics**.*



Tests:

$$1^{st}\ \text{Law: does } Q_H = Q_C + W_{\text{out}}?$$

$$2^{nd}\ \text{Law: } \eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

$$\text{is } \eta = \frac{W_{\text{out}}}{Q_H} \leq \eta_{\text{Carnot}}?$$

PROBLEM-SOLVING STRATEGY 21.1 Heat-engine problems

MODEL Identify each process in the cycle.

VISUALIZE Draw the pV diagram of the cycle. **(always, always!)**

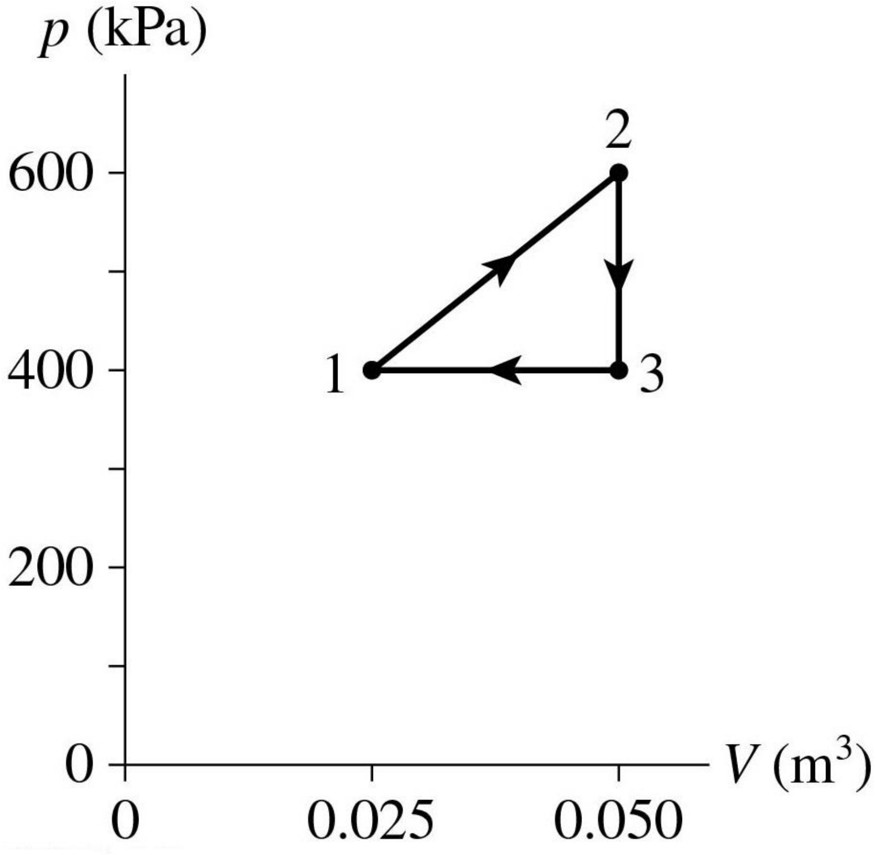
SOLVE There are several steps in the mathematical analysis.

- Use the ideal-gas law to complete your knowledge of n, p, V, and T at one point in the cycle. **(sometimes just ratios)**
- Use the ideal-gas law and equations for specific gas processes to determine p, V, and T at the beginning and end of each process. **(sometimes just ratios)**
- Calculate Q, W_s, and ΔE_{th} for each process. $W_{\text{out}} = \sum W'_{s}\text{s}$
- Find W_{out} by adding W_s for each process in the cycle. If the geometry is simple, you can confirm this value by finding the area enclosed within the pV curve.
- Add just the *positive* values of Q to find Q_{H}. $Q_H = \sum|+\text{Q's}|$ $Q_c = \sum|-\text{Q's}|$
- Verify that $(\Delta E_{\text{th}})_{\text{net}} = 0$. This is a self-consistency check to verify that you haven't made any mistakes.
- Calculate the thermal efficiency η and any other quantities you need to complete the solution. How does η compare to η_{Carnot}?

ASSESS Is $(\Delta E_{\text{th}})_{\text{net}} = 0$? Do all the signs of W_s and Q make sense? Does η have a reasonable value? Have you answered the question?

Whiteboard Problem: 21-5

The PV diagram shows a heat engine cycle that uses 2.0 mol of a monatomic gas.



a) **Calculate the temperatures, T_1, T_2, and T_3 .**
(enter T_3 into LC)

b) **Starting at point 1, go around the cycle calculating the work (W_s), the heat (Q), and the change in thermal energy (ΔE_{th}) for each process.**
(enter $Q_{2\rightarrow 3}$ into LC)

c) **Determine the engine's thermal efficiency.** (LC)

Whiteboard Problem: 21-6

(a really good problem)

A heat engine uses a diatomic ideal gas and goes through the following closed cycle:

- **Isothermal compression until the volume is halved.**
- **Isobaric expansion until the volume is restored to its initial value.**
- **Isochoric cooling until the pressure is restored to its initial value.**

a) Sketch the cycle on a PV diagram. (LC)

b) Now, calculate the thermal efficiency of this engine. (LC, 3 point shot! *Everyone at the table work together*!)

c) What is the thermal efficiency of a Carnot engine operating between the highest and lowest temperatures reached by this engine (LC)?

Note! *This is a real scary problem. It wants a numerical answer, but not a single number is given! All we know is that the volume is reduced by a factor of 2. Hint: analyze it and be brave!* ***Try to get all the W_s's and Q's in terms of T_1.***

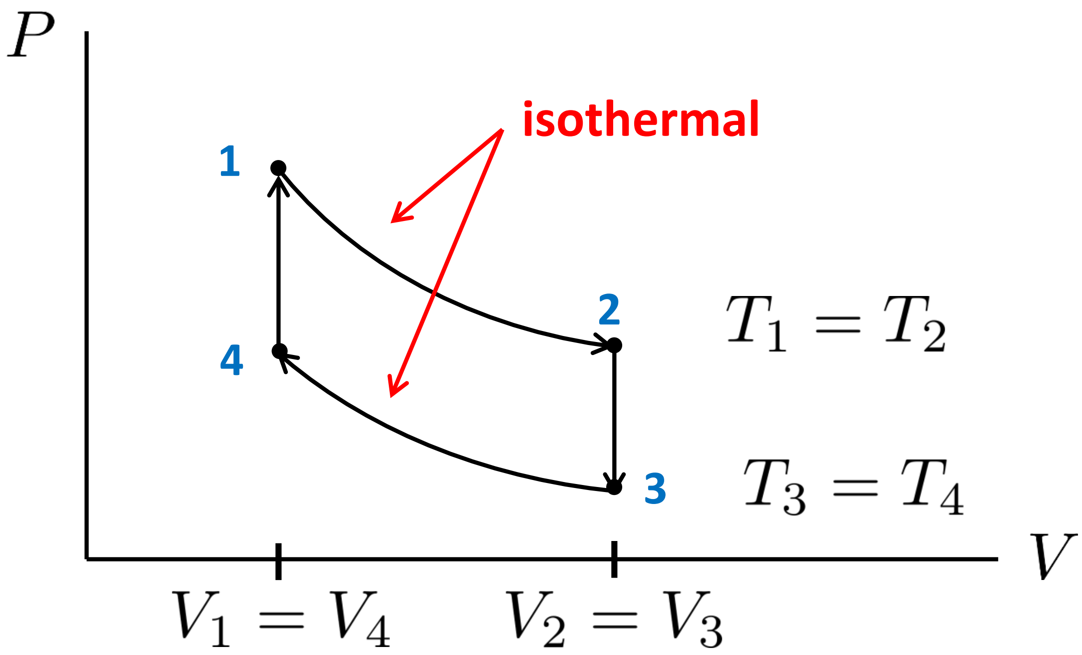
The Stirling Engine Cycle

The **Stirling Engine** is an extremely versatile engine cycle where you only need a source of heat (or cold?), i.e. no boiling or combustion.

[Let's look at some actual Stirling Engines.]

(active link to a video demo)

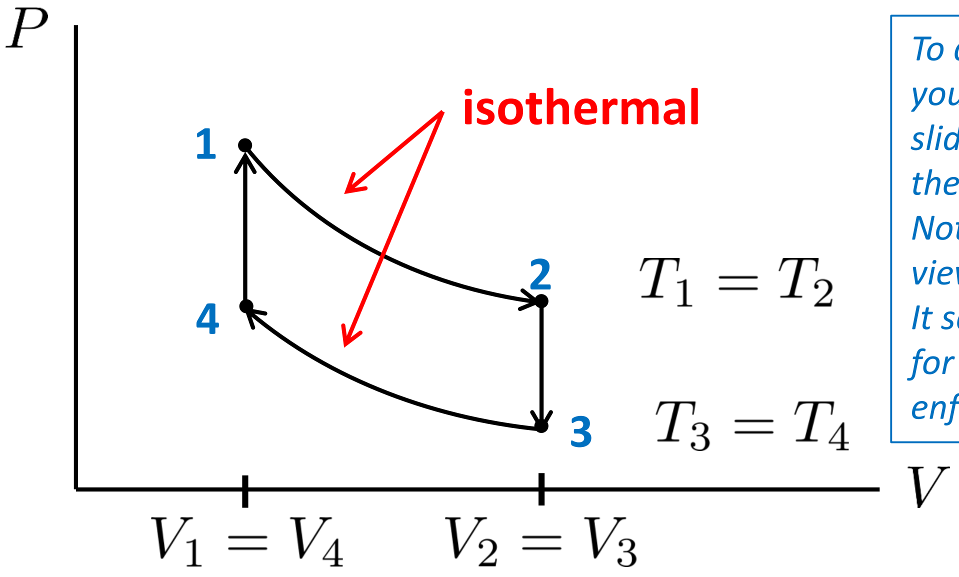
PV Diagram for A Stirling Engine



Whiteboard Problem: 21-7 (Probably as HW)

(another really good problem)

a) For a Stirling Engine using air, find an expression for the efficiency in terms of the high and low temperatures and the maximum and minimum volumes, i.e. in terms of T_1 & T_4 and V_1 & V_2. (LC, 2 points)



To do this as a HW problem, you should refer back to this slide, do the problem, and then enter your answers in MP. Note on MP, as a hint, you can view a video of the solution. It says there's a point deduction for using the hint, but it's not enforced.

b) The Coffee Cup Stirling Engine that we have operates between the T_1 ~ boiling temperature of water and T_4 ~ room temperature, and its compression ratio is V_2/V_1 ~ 4. What is its efficiency? (LC)

c) What is the thermal efficiency of a Carnot engine operating between the highest and lowest temperatures reached by the Coffee Cup Stirling Engine? (LC)