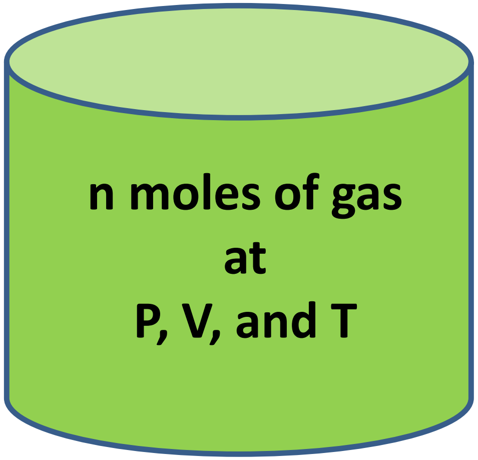
The Ideal Gas Law

Consider a system that is a gas in a container:



n moles of gas
at
P, V, and T

During the 18th century it was discovered experimentally that the state variables of a gas are related to each other.

You may have seen these relations in a chemistry class – remember **Charles' Law, Boyle's Law, and Gay Lussac's laws**?

All of these laws can be combined together into **The Ideal Gas Law***:

$$\boxed{PV = nRT}$$

PhET
Gas Properties

Where R = Universal Gas Constant

$$R = 8.314 \ \frac{J}{mol \ \ K} \ \ (\text{MKS units})$$

Where should you put this number?
Store it in your calculator!
Note: you may be used to seeing R as a different number. That's a different set of units. We'll mostly use MKS.

**sometimes you see this referred to as an "Equation of State" since it relates the state variables of the system.*

Some Important Things about the Ideal Gas Law (IGL)

Anytime you use the IGL, both the temperature and the pressure must be absolute temperature and pressure!

This means that when doing problems, we want to make sure that the **temperature is in Kelvins and the pressure is the absolute pressure, not the gauge pressure. And, if we multiply or divide by R using R = 8.314 J/mol K, we must also use pressure in Pascals.**

The IGL is not valid for all gases in all situations. It is reasonably valid if:

1. The gas density is low *(how low is low? We'll see in chap 20)*
2. The temperature is large compared to the condensation temperature

Whiteboard Problem 18-6

A rigid container holds 2.0 mol of gas at a pressure of 1.0 atm and a temperature of 30°C.

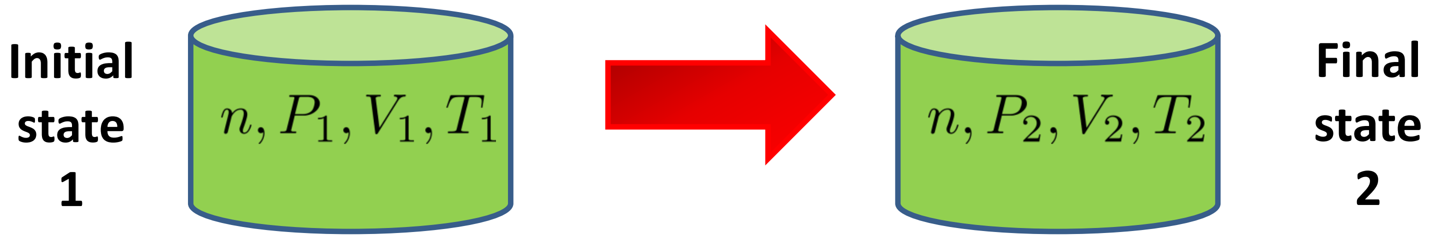
a) What is the container's volume? (LC)

b) If the temperature of the gas is raised to T_2 = 130°C, what is its new pressure, P_2? (LC)

How do we use the IGL?

Of course, we can use the IGL to find an unknown state variable if we know all of the other ones.

In most problems that we'll do, the state of the system will be changing from some initial state to some final state. Most of the time the quantity of gas will be fixed (n = constant), and we can use the IGL to find how the other state variables change:



Initial state 1

n, P_1, V_1, T_1

n, P_2, V_2, T_2

Final state 2

e.g. suppose that both n and the temperature,T, are constant:

$$PV = nRT = \text{constant} \Rightarrow P_1V_1 = P_2V_2$$

or, suppose that both n and the pressure,P, are constant:

$$PV = nRT \Rightarrow \frac{V}{T} = \frac{nR}{P} = \text{constant} \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

You can work out the others for constant volume (see WB 18-6b) or when only n is constant.

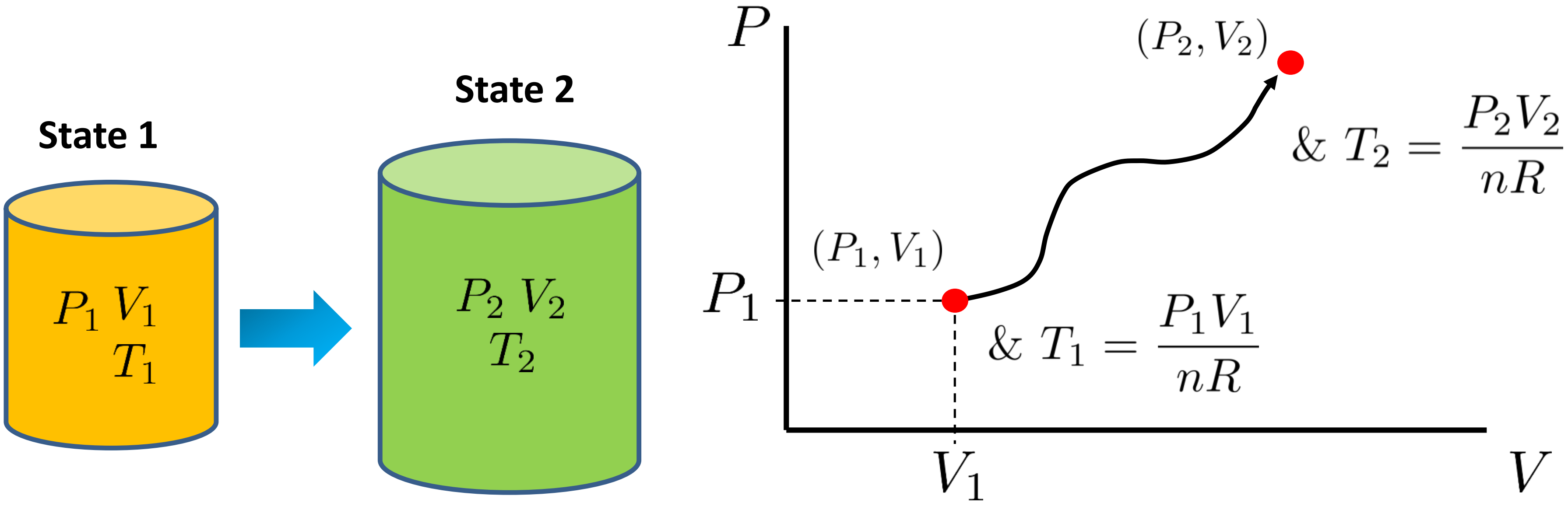
Whiteboard Problem 18-7

A compressed-air cylinder is known to fail if the pressure exceeds 110 atm. A cylinder that was filled to 25 atm at 20°C is stored in a warehouse. Unfortunately, the warehouse catches fire and the temperature reaches 950°C.

Does the cylinder blow up? (LC)

The PV Diagram

For a fixed amount of gas (n = constant), we can represent the state of the system in PV phase space:

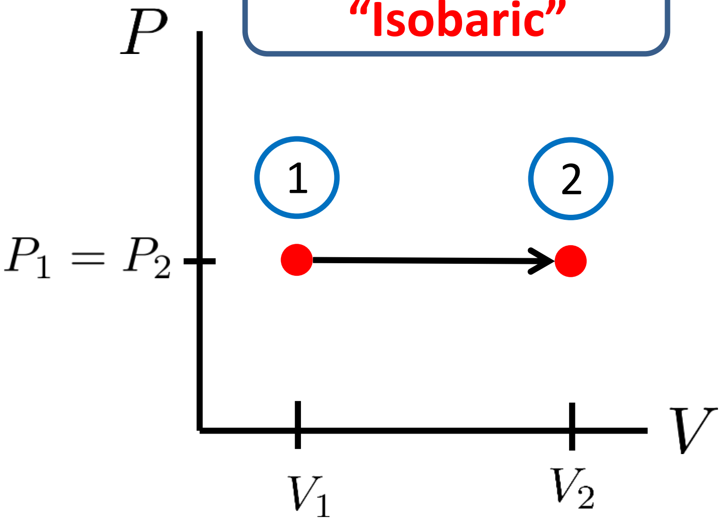


If the system changes slowly from state 1 to state 2, it follows a path in PV space.

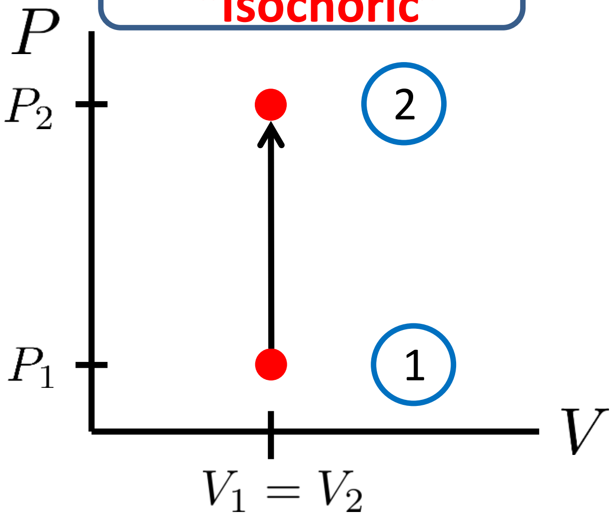
This is really in a three dimensional PVT space, but that gets a little hard to draw!

Special Process or Paths on the PV Diagram

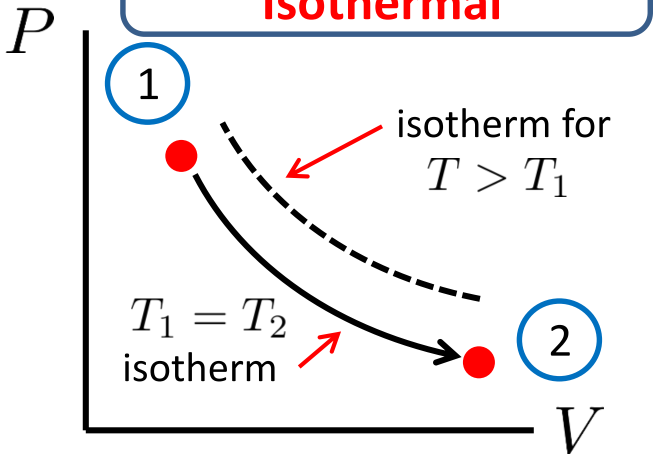
Constant Pressure
"Isobaric"



Constant Volume
"Isochoric"



Constant Temperature
"Isothermal"



Note: for T = constant:

$$PV = nRT = \text{constant}$$

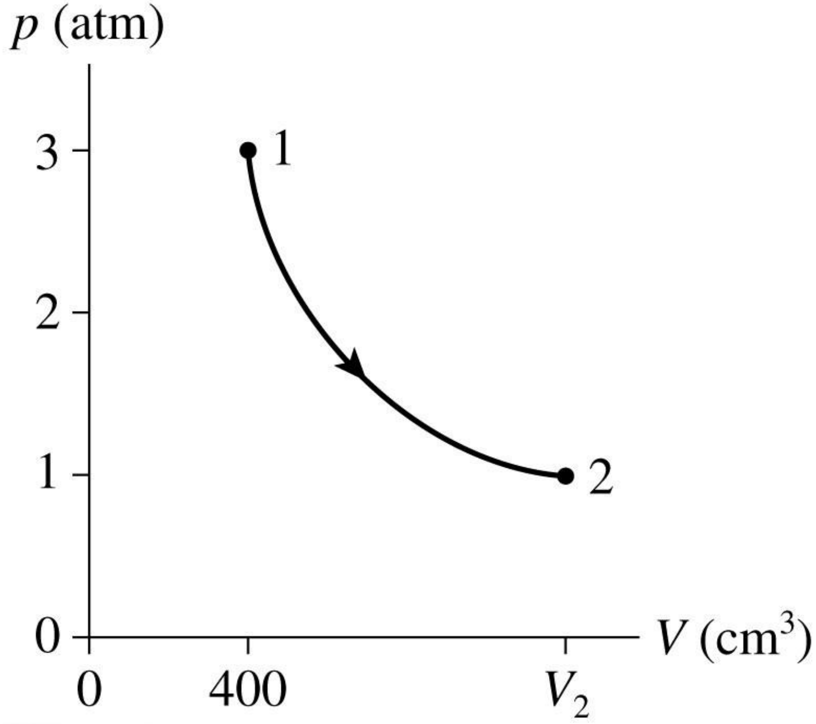
$$\Rightarrow P = \frac{\text{constant}}{V} \propto \frac{1}{V}$$

What shape is this? It's a hyperbola!

There's one more special process that we'll see in chapter 19, the **adiabatic process**.

Whiteboard Problem 18-8

0.020 moles of a gas undergoes the isothermal process shown below.



a) What is the final temperature, T_2, in ^{o}C? (LC)
b) What is the final volume, V_2? (LC)

Whiteboard Problem 18-9

An ideal gas starts at point 1 and goes through the following
1 --> 2 --> 3 --> 1 closed cycle:

- **Isothermal compression until the volume is halved to point 2.**
- **Isobaric expansion to point 3 until the volume is restored to its initial value.**
- **Isochoric cooling until the pressure is restored to its initial value.**

Sketch and label the cycle on a PV diagram on your Whiteboard, and then try to reproduce it on LC – do the best you can.

Now for some fun*: Whiteboard Problem 18-10

A 50 kg lead piston floats in equilibrium on 0.12 mol of compressed air. (Assume that the piston is circular, as most pistons are.)

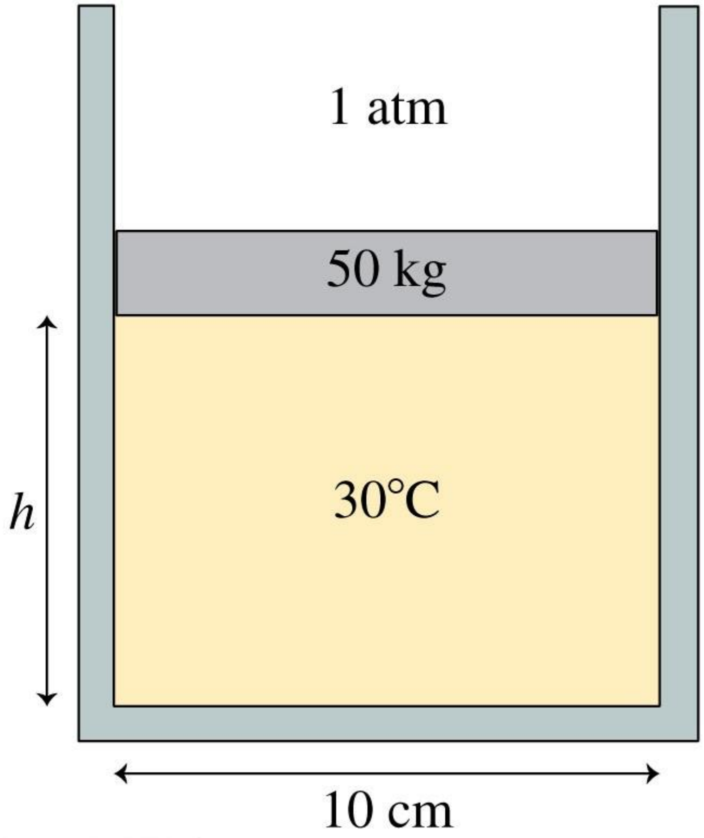
a) **What is the piston height h if the temperature is 30°C? (LC)**

b) **How far does the piston move if the temperature is increased by 100°C? (LC)**

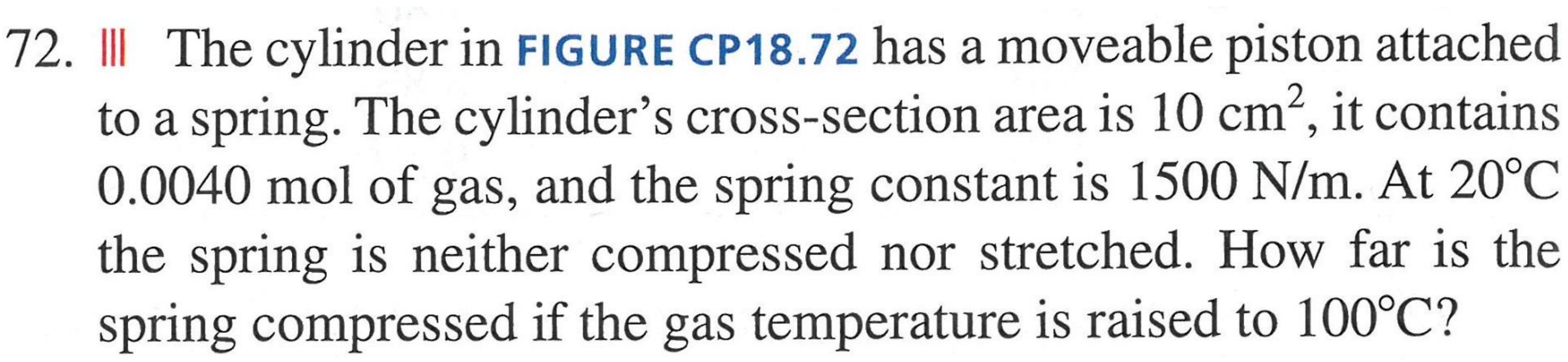
Hint for part a: Find the pressure first – you might want to draw a FBD!

Hint for part b: does the pressure change?

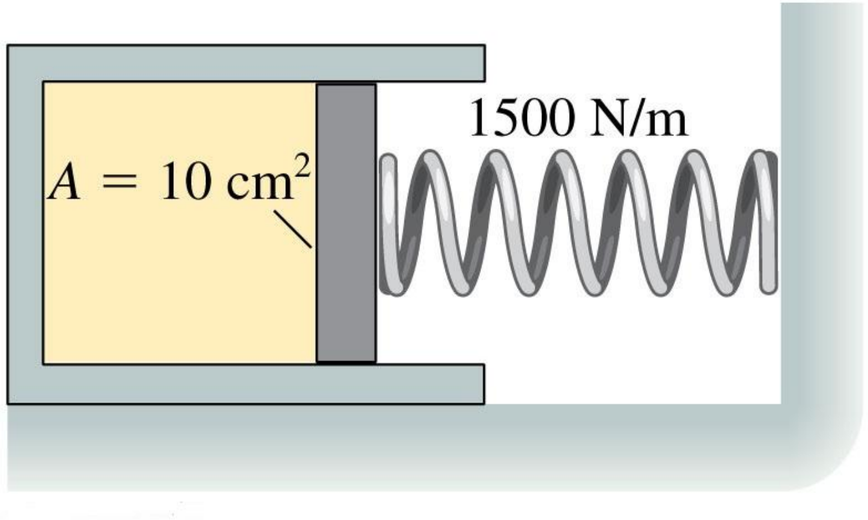
**wherein we use some of the great stuff that we learned in PHY181!*



Homework Hints: Problem 18-73 (5th ed.)



72\. ||| The cylinder in **FIGURE CP18.72** has a moveable piston attached to a spring. The cylinder's cross-section area is 10 cm^2, it contains 0.0040 mol of gas, and the spring constant is 1500 N/m. At 20°C the spring is neither compressed nor stretched. How far is the spring compressed if the gas temperature is raised to 100°C?



This is a fairly tough problem. It is similar to WB18-10, but there are some important differences:

Instead of gravity pushing back on the pressure, it's the spring force which is not constant. Gravity is not important in this problem.

Draw good initial and final sketches and FBDs, and don't be surprised if you have to solve a quadratic equation somewhere in your solution.