



Currents  $I_1 \rightarrow I_5$  in assumed direction.

### Junction Equations:

$$\text{Junction } b: I_1 = I_2 + I_3 + I_4 \quad (1)$$

$$\text{Junction } a: I_1 = I_3 + I_5 \quad (2)$$

$$\text{Junction } c: I_5 = I_2 + I_4 \quad (3)$$

Note that these are not all independent.

$$\text{equ'n (1)} = \text{equ'n (2)} + \text{equ'n (3)}$$

### Loop Equations:

$$\text{left loop } \curvearrowright a: \mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0 \quad (4)$$

$$\text{middle loop } \curvearrowright b: -I_4 R_4 + I_3 R_3 = 0 \quad (5)$$

$$\text{right loop } \curvearrowright b: -I_2 R_2 - \mathcal{E}_2 + I_4 R_4 = 0 \quad (6)$$

NOTE: equations (1), (4), (5), & (6) constitute 4 unknown ( $I_1, I_2, I_3, \& I_4$ ) in four independent equations.

SOLVE THE FOUR EQUATIONS:

Subst. eqn (1)  $\rightarrow$  eqn (4):

$$\mathcal{E}_1 - (I_2 + I_3 + I_4)R_1 - I_3R_3 = 0 \quad (7)$$

Now, eqn (7), (5), & (6) have

3 equations and 3 unknown.

EQUATIONS (7), (5), & (6) with numbers:

$$9 - (I_2 + I_3 + I_4)6 - I_3 12 = 0 \quad (8)$$

$$-I_4 24 + I_3 12 = 0 \quad (9)$$

$$-I_2 10 - 15 + I_4 24 = 0 \quad (10)$$

REARRANGE:

$$9 = 6I_2 + 18I_3 + 6I_4 \quad (11)$$

$$0 = 12I_3 - 24I_4 \quad (12)$$

$$15 = -10I_2 + 24I_4 \quad (13)$$

$$\text{eqn (12)} \Rightarrow I_3 = 2I_4 \quad (14)$$

$$\text{eqn (13)} \Rightarrow I_2 = \frac{24I_4 - 15}{10} = 2.4I_4 - 1.5 \quad (15)$$

subst.  $I_3$  &  $I_2$  into eqn (11):

$$9 = 6(2.4I_4 - 1.5) + 18(2I_4) + 6I_4$$

$$\text{or, } 18 = 56.4 I_4 \Rightarrow \boxed{I_4 = 0.319 \text{ A}}$$

Subst.  $I_4$  into eqn (14) & (15):  $\frac{28-66}{3}$

$$I_3 = 2I_4 \Rightarrow I_3 = 0.638 \text{ A}$$

$$I_2 = 2.4I_4 - 1.5 \Rightarrow I_2 = -0.734 \text{ A}$$

Now, eqn (1):

$$I_1 = I_2 + I_3 + I_4 \Rightarrow I_1 = 0.223 \text{ A}$$

And, finally, eqn (3):

$$I_5 = I_2 + I_4 \Rightarrow I_5 = -0.415 \text{ A}$$

negative  $\Rightarrow I_5$  in left  $\rightarrow$  right

You CAN ALSO SET THIS UP ON THE  
PHET CIRCUIT SIMULATION:

The screenshot shows the PhET Circuit Construction Kit: DC simulation interface. The circuit consists of a 9.0 V battery on the left, a 15.0 V battery on the right, and a 0 Ω resistor at the top left. The top wire contains a 6.0 Ω resistor and a 10.0 Ω resistor. The bottom wire contains a 12.0 Ω resistor and a 24.0 Ω resistor. A light bulb is connected to the bottom wire. An ammeter is placed in the bottom wire, showing a current of 0.41 A. The simulation controls on the right include 'Show Current' (checked), 'Labels' (checked), and 'Values' (checked). The bottom of the screen displays 'Circuit Construction Kit: DC' and the PhET logo.