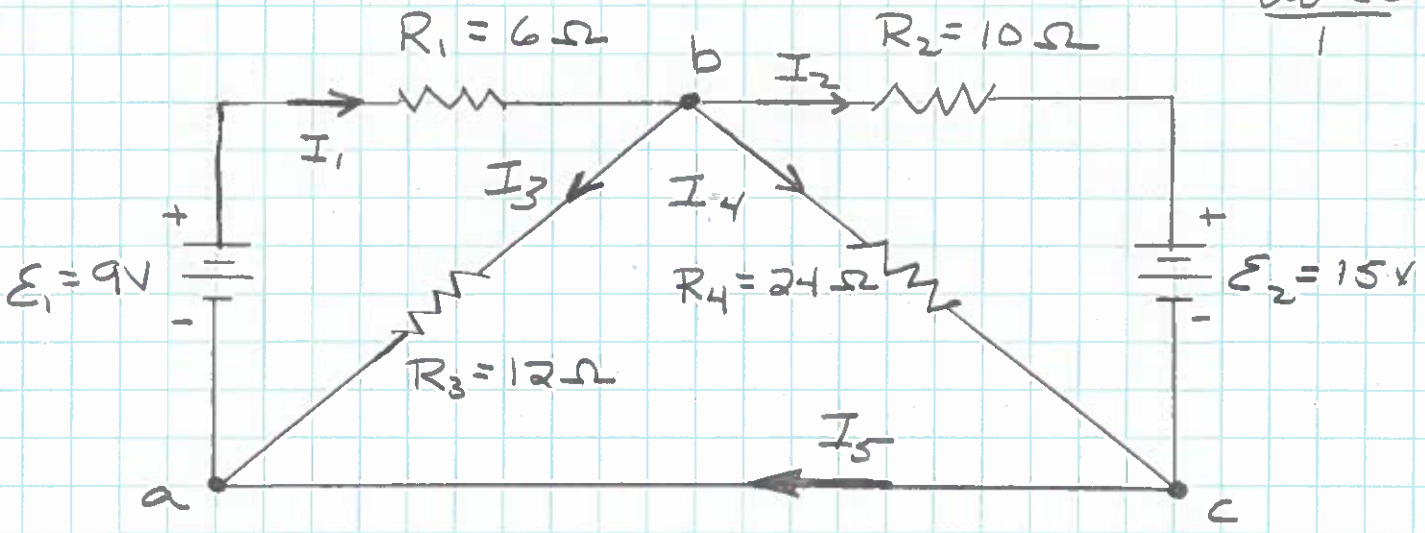

Currents $I_1 \rightarrow I_5$ in assumed direction.

Junction Equations:

Junction b: $I_1 = I_2 + I_3 + I_4$ (1)

Junction a: $I_1 = I_3 + I_5$ (2)

Junction c: $I_5 = I_2 + I_4$ (3)

Note that these are not all independent.

equ'n (1) = equ'n (2) + equ'n (3)

Loop Equations:

left loop @ a: $\mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0$ (4)

middle loop @ b: $-I_4 R_4 + I_3 R_3 = 0$ (5)

right loop @ b: $-I_2 R_2 - \mathcal{E}_2 + I_4 R_4 = 0$ (6)

NOTE: equations (1), (4), (5), & (6) constitute 4 unknown (I_1, I_2, I_3, & I_4) in four independent equations.

SOLVE THE FOUR EQUATIONS:

Subst. eqn'n (1) → eqn'n (4):

$$\mathcal{E}_1 - (I_2 + I_3 + I_4)R_1 - I_3 R_3 = 0 \qquad (7)$$

Now, eqn'n (7), (5), & (6) have 3 equations and 3 unknowns.

EQUATIONS (7), (5), & (6) with numbers:

$$9 - (I_2 + I_3 + I_4)6 - I_3 12 = 0 \qquad (8)$$

$$-I_4 24 + I_3 12 = 0 \qquad (9)$$

$$-I_2 10 - 15 + I_4 24 = 0 \qquad (10)$$

REARRANGE:

$$9 = 6I_2 + 18I_3 + 6I_4 \qquad (11)$$

$$0 = 12I_3 - 24I_4 \qquad (12)$$

$$15 = -10I_2 + 24I_4 \qquad (13)$$

eqn'n (12) $\Rightarrow$ $I_3 = 2I_4$ (14)

eqn'n (13) $\Rightarrow$ $I_2 = \dfrac{24I_4 - 15}{10} = 2.4I_4 - 1.5$ (15)

subst. I_3 & I_2 into eqn'n (11):

$$9 = 6(2.4I_4 - 1.5) + 18(2I_4) + 6I_4$$

Or, $18 = 56.4\, I_4 \Rightarrow \boxed{I_4 = 0.319\,A}$

Subst. I_4 into equtn (14) & (15):

$$I_3 = 2I_4 \Rightarrow \boxed{I_3 = 0.638\ A}$$

$$I_2 = 2.4I_4 - 1.5 \Rightarrow \boxed{I_2 = -0.734\ A}$$

Now, equtn (1):

$$I_1 = I_2 + I_3 + I_3 \Rightarrow \boxed{I_1 = 0.223\ A}$$

And, finally, equtn (3):

$$I_5 = I_2 + I_4 \Rightarrow \boxed{I_5 = -0.415\ A}$$

Negative $\Rightarrow$ I_5 in left $\rightarrow$ right

YOU CAN ALSO SET THIS UP ON THE PHET CIRCUIT SIMULATION: